



**MATH
WHISPERER**
Where math makes sense

Research Edition

What is x ?

Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information please go to www.mathwhisperer.com

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DEDICATION

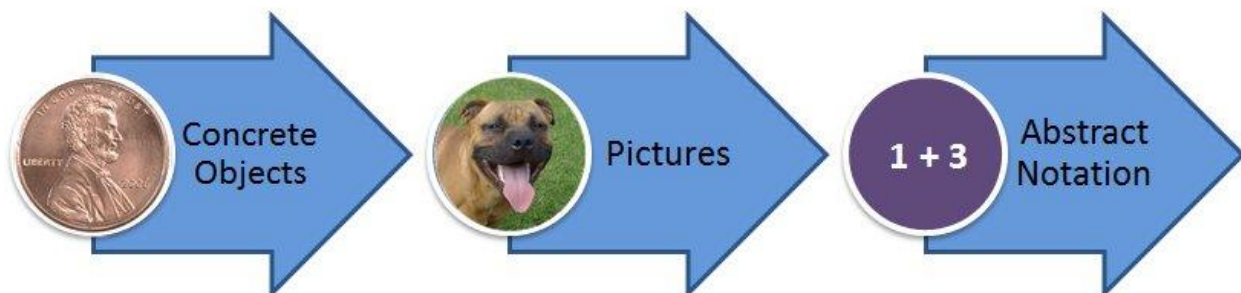
Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

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1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is actually supposed to make sense. When you start with hands-on objects, math can make sense.



You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three step progression works for everybody.

It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. Won't it feel great to never have to learn this again? The math will stick with you with the three step progression.






Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you.

Supplies: For penny holders, I prefer Wilton mini-baking cups. For pennies, please use shiny pennies, if at all possible. They are so much nicer to work with.

2. Welcome to What is x ?

This set of exercises will make sure you can reliably solve algebraic equations such as $3x - 1 = 8$ on your own. This will put you well on the road to success in algebra.

You will need:

- 15 pennies 
- 10 penny holders of one color 
- 10 penny holders of another color 



The reason you need these items is that you will learn far better by beginning with the hands-on objects. Please trust me on this. (After all, I am the Math Whisperer!) Educational research for over 100 years has shown that people learn best by beginning with hands-on objects, then drawing and finally in step three, using the abstract math symbols. As a typical student, you may have only used abstract math symbols. You will be so happy to see how much easier it is to learn math when you start with the hands-on objects.

Activity 1: What is x ?

You will need:

- 5 or more pennies
- 5 or more penny holders of a single color

Take out one of your pennies.
This is “one”.



Now take out one of your “penny holders”.
This is used to represent an unknown quantity.



We can give it any name we want. However, we are going to call it “ x .”

- “ x ” is a popular name for an unknown quantity.
- The name doesn’t matter, as long as we all know what we are talking about.
- We just need to all agree to the same name. In this case, it will be “ x .”

Now lay out two pennies and a penny holder like this:



One name of this collection is “one and one and x ” which can be written $1 + 1 + x$.

What else can it be called?



Answers: $2 + x$ $x + 2$ $1 + x + 1$ and more.

Now you try some. You can work on Practice 1 by yourself or with a partner.

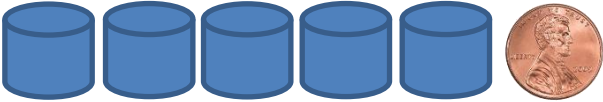


Practice 1: Naming Collections Round 1






Here's a hint: You may not be sure what to do on Practice 1. Just try your best. What's the worst thing that could happen to you? I think you will do just fine.

Algebraic Expression	Pictures of Pennies and Penny Holders
$1 + x + 1 + x$	
	
	
$x + x + x + x$	
$x + 2 + x + x$	

Practice 2: Naming Collections Round 2

Algebraic Expression	Pictures of Pennies and Penny Holders
$x + x + x + 2$	
	
	
$x + 3 + x + 1$	
$5 + x + x + x$	
$x + 2 + x + x + 1$	
	
$2 \text{ } x\text{'s} + 3$	

3. Activity 2: Substitution

- You will need: 5 or more pennies 
- 5 or more penny holders 
- 2 or more different color penny holders 

Let's review. This is "**one**."  And this is "**x**". 

Now you lay out one penny and penny holder like this.

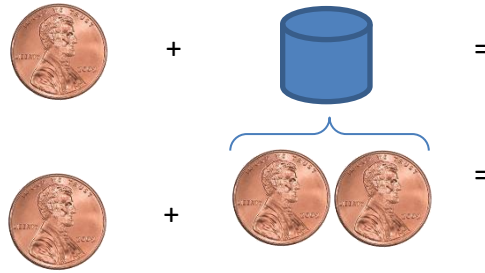


What are some math expressions that describe this situation?

Now suppose that suddenly we are given a value, or number, for x .

Suppose we are told that $x = 2$.

We can substitute 2 pennies every time we see x .



Equals 3

The big idea is that we **substituted** two “ones” for the x .

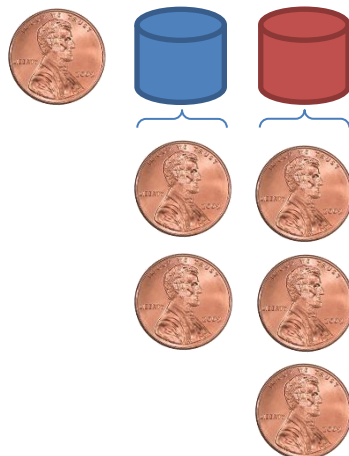
We also want to introduce another unknown, “ y .”



Let’s do one more example together. Lay out your holders and pennies like this:



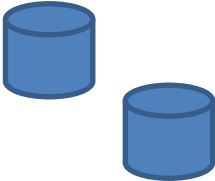
Now say we are told that $x = 2$ and $y = 3$. What is the value of this expression?



Now we have a total of 6. So: $1 + x + y = 1 + 2 + 3 = 6$

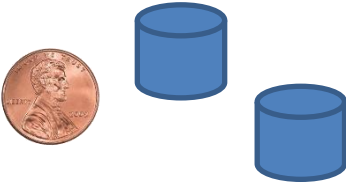
Practice 3: Substitution

1.



= ? when $x = 3$ _____
 = ? when $x = 0$ _____
 = ? when $x = 4$ _____

2.



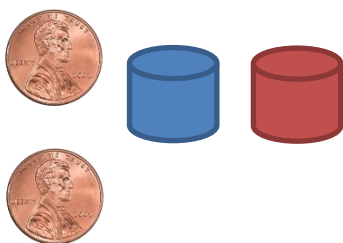
= ? when $x = 3$ _____
 = ? when $x = 0$ _____
 = ? when $x = 4$ _____

3.



= ? when $x = 3$ and $y = 2$ _____
 = ? when $x = 0$ and $x = 1$ _____
 = ? when $x = 4$ and $x = 1$ _____

4.

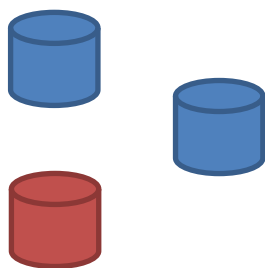


= ? when $x = 3$ and $y = 2$

= ? when $x = 0$ and $y = 1$

= ? when $x = 4$ and $y = 1$

5.



= ? when $x = 3$ and $y = 2$

= ? when $x = 0$ and $y = 1$



= ? when $x = 4$ and $y = 1$

Now lay out the penny holders and pennies and sketch them to show:

6.	$x + 1$	
7.	$x + y + 4$	
8.	$2x + 1$	
9.	$x + y + x + 3$	
10.	$3 + x$	





Activity 3: Showing that $x + x = 2x$

You will need:

- 5 or more pennies 
- 5 or more penny holders 



Let's review. This is "one."  And this is " x ". 

Suppose we find out that $x = 3$. Then every time we see x , we can **substitute** 3 pennies, or 3 ones.

So:  is the same as   

Our first example is $2x$, and our second will be $5x$.

Step #1: Show that $x + x = 2x$

Let's look at 2 penny holders:  

- An earlier lesson showed that we could name this $x + x$ or x and x .
- There is another way to name these two penny holders, and that is **$2x$** , or **two times x** , read as **two x** .
- In the expression **$2x$** , the tradition is to leave out the times sign.

Example #1:

Let's prove that by filling out the following table that has been started for you.

Note: You will need to use * for the multiplication sign so as not to confuse it with x , the variable.

x	$x + x$	$2 * x = 2x$
1	$1 + 1 = 2$	$2 * 1 = 2$
2	$2 + 2 = 4$	$2 * 2 = 4$
3		
4		
5		
6		

Example #2:

Show that $x + x + x + x + x = 5x$



We know that this can be called $x + x + x + x + x$

After the example in Step 1, where you saw that $x + x = 2x$, would you be surprised to see that $x + x + x + x + x = 5x$?

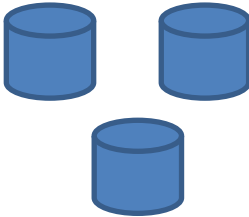
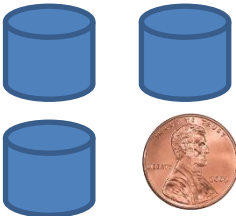

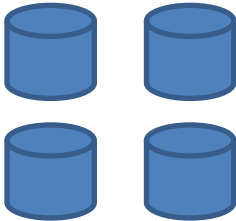

Let's show that this is true with a table again.

x	$x + x + x + x + x$	$5 * x = 5x$
1		
2		
3		
4		
5		
6		

Practice 4: Advanced Naming Collections



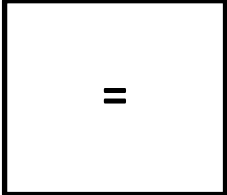
Use these pictures to lay out the pennies and penny holders. Find two expressions for each collection and evaluate with the listed values of x .

Note: If you are confused about how to proceed, just figure out what makes the most sense to you. What's the worst that could happen?

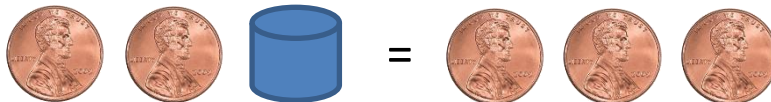
Picture	Expression 1	Expression 2	Evaluate for $x = ?$
			$x = 1$
			$x = 0$
			$x = 1$
			$x = 2$
			$x = 3$
			$x = 5$
			$x = 5$
			$x = 1$
			$x = 1$
			$x = 7$

4. Solving for x - Activity 4

You will need:

- 10 or more pennies 
- 5 or more penny holders 
- A sheet of paper with “ = ” drawn in the middle 

The penny holders and pennies can be used to solve equations such as:



Can you figure out that x must = 1?

Here are two paths of reasoning to show this:

Reasoning 1:

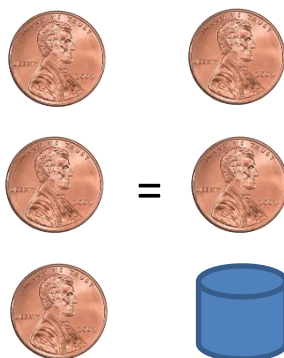
I'll cross out what is the same on both sides



From this, I can see that $x = 1$

Reasoning 2:

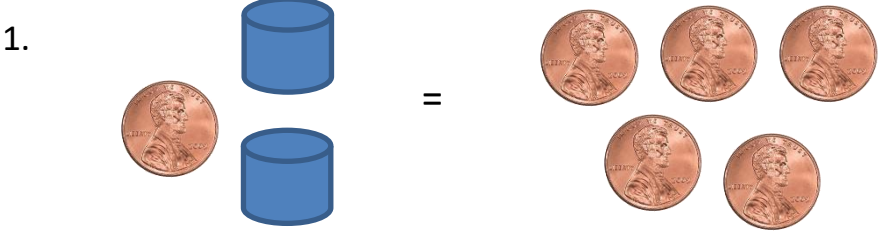
I'll line up both sides



From this, I can see that $x = 1$

Practice 5: Solving Equations

Solve for x :



$x =$ _____ Explain how you know:



$x =$ _____ Explain how you know:

Use the penny holders and pennies to solve these equations for x

Sketch each equation.

3. $2x + 1 = 5$

4. $x + 5 = 6$

5. $3x + 2 = 8$

6. $x + 4 = 2x$

7. $2x + 1 = x + 5$




Answers page 42

5. Activity 5: Functions

What you need:

- 15 pennies
- 10 penny holders (x 's)
- 10 penny holders of a different color (y 's)

Let's review the objects we are using.

This is **1**,  and this is **x** ,  and this is **y** 

Now we are off to the concept of a function.

Lay out: $2x + 1 = y$

Are you able to solve for y ? Why or why not?

Try this: Solve for y if $x = 1$ $y =$ _____

Next try this: Solve for y if $x = 3$ $y =$ _____

Let's look at $2x + 1 = y$ again

Is there only one number that y can be? Why or why not?

A **function** is a relationship between two variables like x and y . Once we know what number x is, we know what number y is.

Practice 6: Functions

Complete the tables.

1.  = 

$$2x + 1 = y$$

x	$2x + 1$	y
1	$2 * 1 + 1 = 3$	3
2	$2 * 2 + 1 = 5$	5
3		
4		
5		
6		

2.



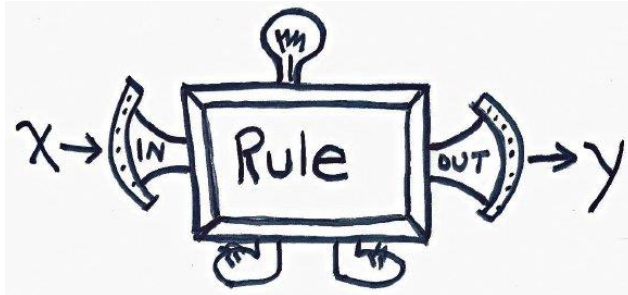
$$x + 2 = y$$

x	$x + 2$	y
1	$1 + 2 = 3$	3
2	$2 + 2 = 4$	4
3		
4		
5		
6		

Activity 6: Function Machines



The following pages are one way to show functions using the popular function machines. You have probably seen them in textbooks and school. They are a lot of fun for many students. If they don't seem like fun to you, that is okay for now. After you are successful with them I predict that you will enjoy them. Do the best you can.



For this function machine 1 is added to x . In other words, $y = x + 1$.
Fill in the blanks.

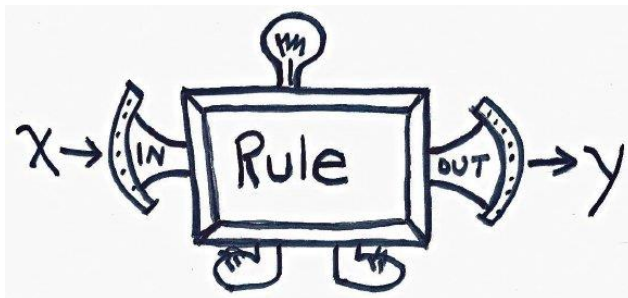
x	y
1	2
2	3
3	4
4	
5	
10	
20	

What does the machine do to the number?

Function Machine 2

For this function machine x is multiplied by 2 and 1 is added. In other words, $y = 2x + 1$.

Fill in the blanks.

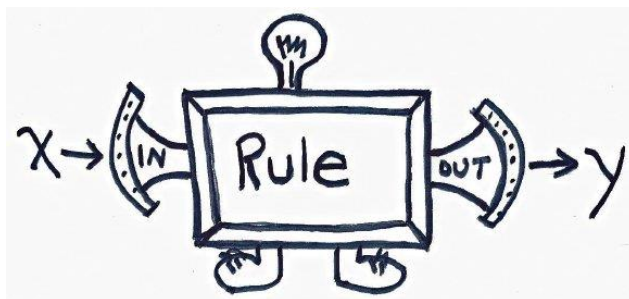


$$2x + 1$$

x	y
1	1
2	3
3	3
4	7
5	
10	
20	

What does the machine do to the number?

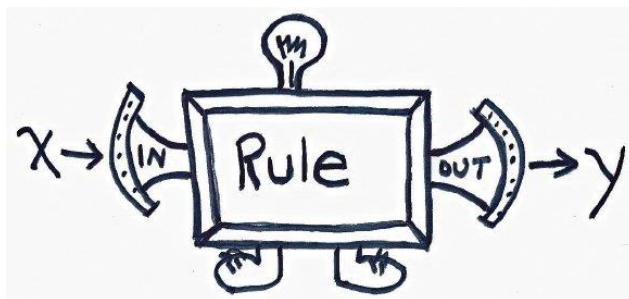
Function Machine 3



x	y
1	5
2	10
3	15
4	
5	
10	
20	

What does the machine do to the number?

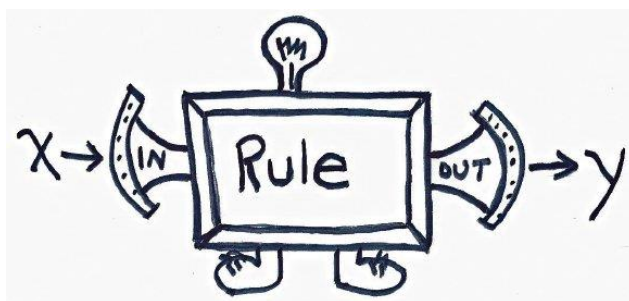
Function Machine 4



x	y
0	2
1	5
2	8
3	11
4	
5	
10	
20	

What does the machine do to the number?

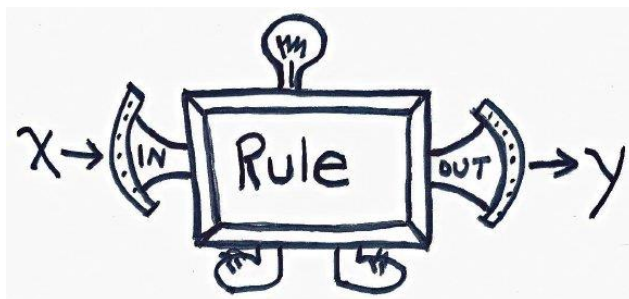
Function Machine 5



x	y
0	-3
1	2
2	7
3	12
4	
5	
10	
20	

What does the machine do to the number?

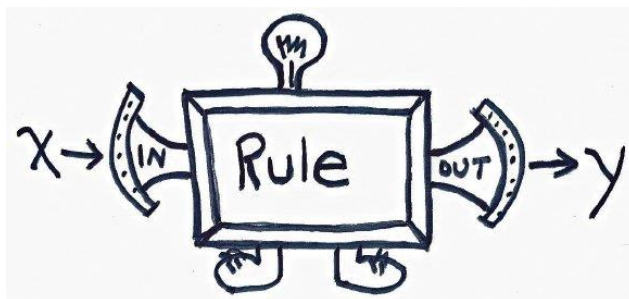
Function Machine 6



x	y
0	6
1	13
2	20
3	27
4	
5	
10	
20	

What does the machine do to the number?

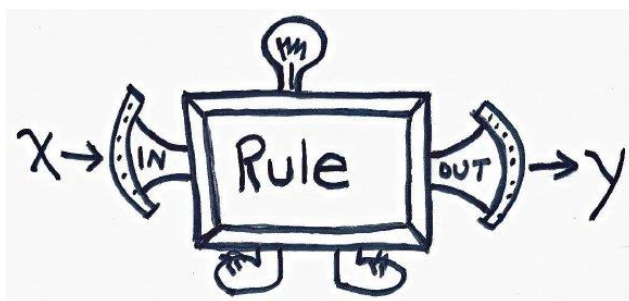
Function Machine 7



x	y
0	1
1	5
2	9
3	13
4	
5	
10	
20	

What does the machine do to the number?

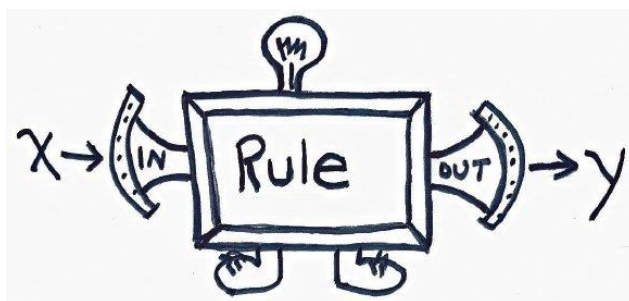
Function Machine 8



x	y
0	0
1	.5
2	1
3	1.5
4	
5	
10	
20	

What does the machine do to the number?

Function Machine 9



x	y
0	1
1	1.5
2	2
3	2.5
4	
5	
10	
20	

What does the machine do to the number?






Solutions / Answers




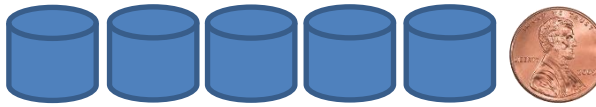



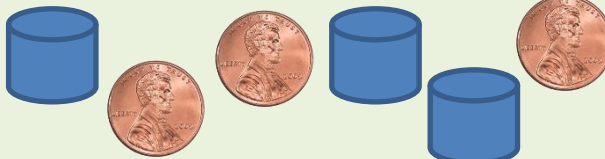


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Practice 1: Naming Collections Round 1

Please note: order is not important. Both $a + b$ or $b + a$ are the same.

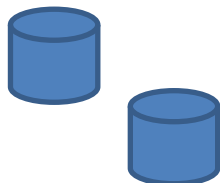
Algebraic Expression	Pictures of Pennies and Penny Holders
$1 + x + 1 + x$	
$x + x + 2$ or $2x + 1 + 1$ or $x + x + 1 + 1$	
$x + 1 + 1 + 1 + 1$ or $x + 4$	
$x + x + x + x$	
$x + 2 + x + x$	

Practice 2: Naming Collections Round 2

Algebraic Expression	Pictures of Pennies and Penny Holders
$x + x + x + 2$	
$x + x + x + x + x + 1$ or $5x + 1$ or $1 + 5x$	
$x + 3$ or $x + 1 + 1 + 1$ or $x + 2 + 1$	
$x + 3 + x + 1$	
$5 + x + x + x$	
$x + 2 + x + x + 1$	
$x + 1 + x + x$ or $3x + 1$ or $x + x + x + 1$	
$2x's + 3$	<p data-bbox="820 1533 1404 1648">You may not be able to do this one, as $2x$ may not be accessible yet. If you can't figure it out, that's ok – you will be able to soon.</p>  <p data-bbox="820 1764 1404 1869">It's not as important that you get this correct right now as it is important that you see the issue, namely, what is $2x$?</p>

Practice 3: Substitution

1.



= ? when $x = 3$

6

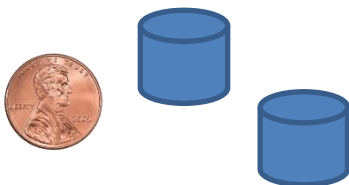
= ? when $x = 0$

0

= ? when $x = 4$

8

2.



= ? when $x = 3$

7

= ? when $x = 0$

1

= ? when $x = 4$

9

3.



= ? when $x = 3$ and $y = 2$

12

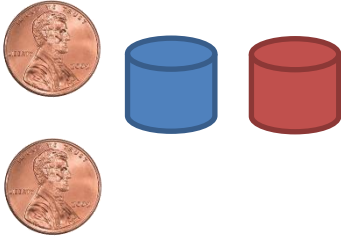
= ? when $x = 0$ and $y = 1$

5

= ? when $x = 4$ and $y = 1$

13

4.



= ? when $x = 3$ and $y = 2$

7

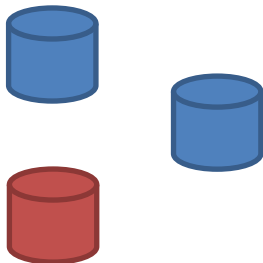
= ? when $x = 0$ and $y = 1$

3

= ? when $x = 4$ and $y = 1$

7

5.



= ? when $x = 3$ and $y = 2$

8






= ? when $x = 0$ and $y = 1$

1

= ? when $x = 4$ and $y = 1$

9

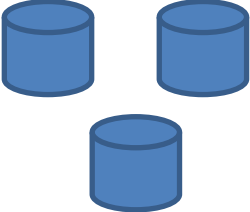


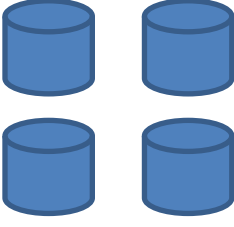

Now lay out the penny holders and pennies and sketch them to show:

6.	$x + 1$	
7.	$x + y + 4$	
8.	$2x + 1$	
9.	$x + y + x + 3$	
10.	$3 + x$	

Practice 4: Advanced Naming Collections

Use these pictures to lay out the blocks. Find two expressions for each collection and evaluate with the listed values of x .

Note: If you are confused about how to proceed, just figure out what makes the most sense to you. What's the worst that could happen?

Picture	Expression 1	Expression 2	Evaluate for $x = ?$
	$x + x + x$ or $2x + x$ or $x + 2x$ or $3x$	Same, but must be different than Expression 1	$x = 1$ 3
	$x + x + x + 1$ or $2x + x + 1$ or $1 + x + x + x$ or $1 + 2x + 1$ or $2x + 1 + x$	Same, but must be different than Expression 1	$x = 1$ 4
	$x + 2$ or $1 + x + 1$ or $2 + x$ or $1 + 1 + x$	Same, but must be different than Expression 1	$x = 3$ 5
	$x + x + x + x$ or $4x$ or $3x + x$ or $2x + 2x$ or $x + 3x$ or $x + 2x + x$ or $x + x + 2x$	Same, but must be different than Expression 1	$x = 5$ 20
	$x + 3$ or $3 + x$ or $2 + x + 1$ or $1 + 1 + 1 + x$	Same, but must be different than Expression 1	$x = 1$ 4
			$x = 7$ 10

Practice 5: Solving Equations

Solve for x :

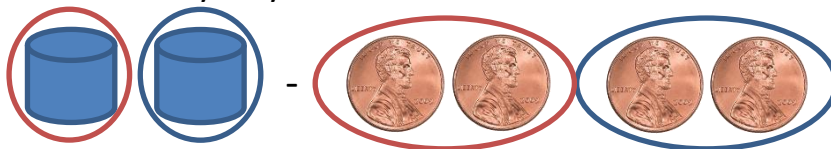
1. $x = 2$

Explain how you know.

Possible response: I took away a penny from each side. This left me with



I could see the only way to do this was:



So each x has to equal 2.

2. $x = 5$

Explain how you know.

Possible response: I took away a penny from each side. This left me with



So each x has to equal 5.

The following items asked you to use penny holders and pennies to solve the equations for x .

Sketch each equation.

3. Solve $2x + 1 = 5$

Solution:



One way to solve

Take "one" away from each side



The only way for each x to equal the same this is for each x to equal 2



4. Solve $x + 5 = 6$

Solution:



One way to solve

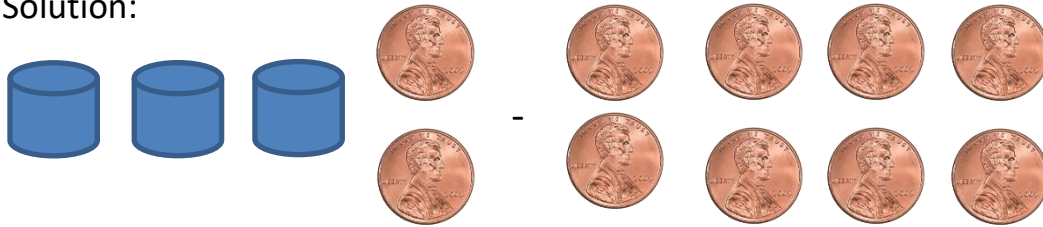
Take "five" away from each side



So x must equal 1

5. Solve $3X + 2 = 8$

Solution:

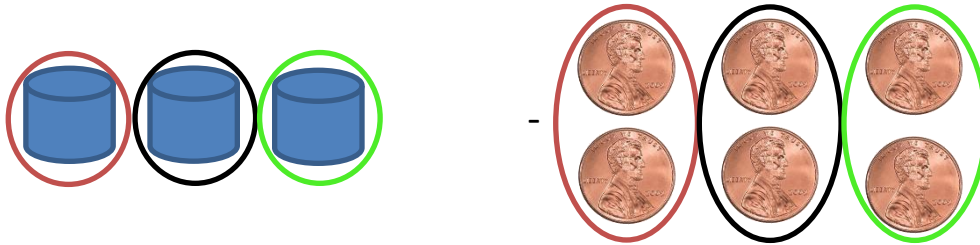


One way to solve

Take "two" away from each side.

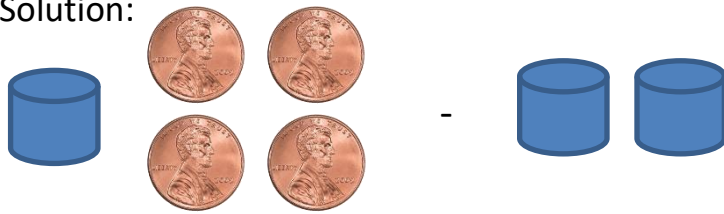


The only way for each X to equal the same thing is for each X to equal 2.



6. Solve $x + 4 = 2x$

Solution:



One way to solve

Take “ x ” away from each side.



So x must equal 4.

7. Solve $2x + 1 = x + 5$



One way to solve:

Take an x and a 1 away from each side:



So x must equal 4.

Practice 6: Functions

Complete the tables.

1.  = 

$$2x + 1 = y$$

x	$2x + 1$	y
1	$2 * 1 + 1 = 3$	3
2	$2 * 2 + 1 = 5$	5
3	$2 * 3 + 1 = 7$	7
4	$2 * 4 + 1 = 9$	9
5	$2 * 5 + 1 = 11$	11
6	$2 * 6 + 1 = 13$	13

2.  = 

$$x + 2 = y$$

x	$x + 2$	y
1	$1 + 2 = 3$	3
2	$2 + 2 = 4$	4
3	$3 + 2 = 5$	5
4	$4 + 2 = 6$	6
5	$5 + 2 = 7$	7
6	$6 + 2 = 8$	8

Function Machines

Function Machine 1: $y = x + 1$ (this was provided as an example)

Function Machine 2: $y = 2x + 1$ (this was provided as an example)

Function Machine 3: $y = 5x$

Function Machine 4: $y = 3x + 2$

Function Machine 5: $y = 5x - 3$

Function Machine 6: $y = 7x + 6$

Function Machine 7: $y = 4x + 1$

Function Machine 8: $y = 0.5x$ or $y = .5x$

Function Machine 9: $y = (0.5)x + 1$ or $y = .5x + 1$

7. Help for Helpers



I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as “I know you can do this. I believe in you.” And then leave the student alone to do the work.

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn’t going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don’t generally want to ask our boss or friend: “Am I right? Am I right?” As an adult, we have to know the answer is right ourselves.

The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student’s life. You will gain confidence in your students as you watch them be successful on their own.

The overall goal for this lesson set is for the student to understand the abstract concept of “variable.” Our students are exposed to this concept of variable at a fairly early grade and yet many do not yet actually understand what a variable, such as “ x ” or “ y ” actually means. Even so, they may be successful at getting correct answers using variables. Or they may stop in their mental tracks when they see a variable. After this set of lessons, students will understand what a variable means on a deep level. And that means they will be prepared to move ahead in algebra.

The other critical goal for the lesson set is that the students become familiar with the process of

Concrete objects → Pictures → Abstract notation. We take the students into balancing equations very quickly and students generally are very proud of themselves once they have done so successfully. It is a big thing for students to solve problems they may not have been able to solve before. It changes their relationship to math, and math then starts to make sense.

The learning objectives in each of the lessons are these:

- What is x ?: The abstract concept of variable is introduced.
- Substitution: The idea that a number can be exchanged for a variable is introduced.
- Showing that $x + x = 2x$: Algebraic expressions like “ $2x$ ” make complete sense to those of us who have done algebra for years. But understanding that an expression like this means “2 multiplied by x ” is not always obvious.
- Solving for x : This is where students solve the first set of actual problems. Actually solving reasonably complicated equations with reliability and confidence feels good!
- Functions: These exercises/problems build confidence in the principles in the prior lessons. And problems like these appear on all sorts of state and other tests—now your student has a chance to solve them, when he or she may not have been able to do so before. But that is not the goal of this lesson, just a positive consequence.