MATH WHISPERER

Where math makes sense

Research Edition

Number Sense Series

The Number Line and Negative Numbers

Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information, please go to www.mathwhisperer.com

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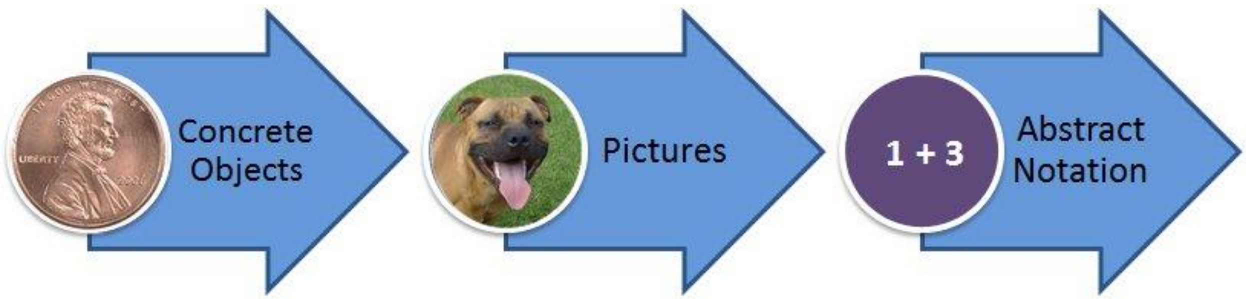
DEDICATION

Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

Contents

1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is supposed to make sense. When you start with hands-on objects, math can make sense.



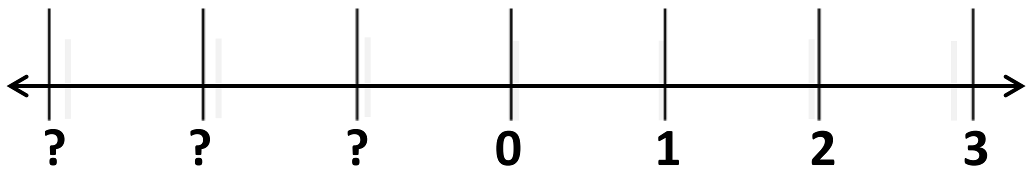
You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three-step progression works for everybody.

It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three-step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. Won't it feel great to never should learn this again? The math will stick with you with the three-step progression.

Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you!

2. The Number Line and Negative Numbers

The number line is a powerful model. It clearly shows that there exist numbers in between the counting numbers: 1, 2, 3 and so on. It makes the existence of negative numbers seem reasonable; on the right side of zero are the (positive) counting numbers and the numbers between them. What happens on the left side of zero? Having another set of numbers there seems reasonable.

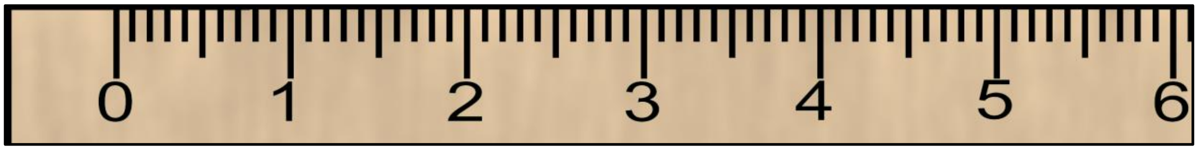


Negative numbers are a critical building block in math for which there is no concrete object. Negative numbers are abstract. But they are highly useful and we need them in math. The best we can do is pictures.

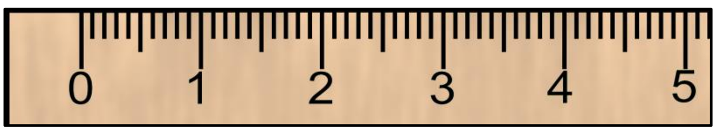
The first step is to understand the concept of a negative number. Then we will compute with negative numbers.

Practice 1: A Ruler Is a Number Line

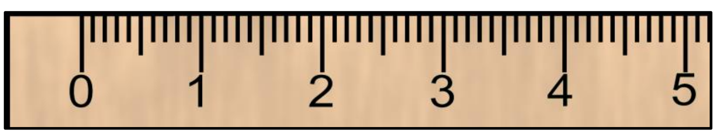
a. What is the purpose of the hatch mark segments on the ruler?



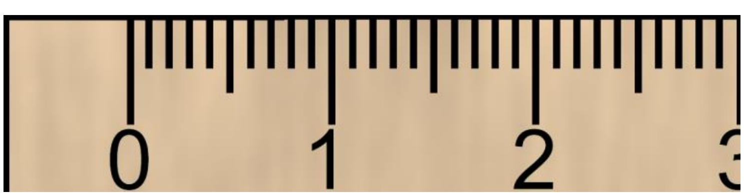
b. Show $\frac{1}{2}$ on the ruler. Explain how you know where it is.



c. Show $2\frac{1}{2}$ on the ruler. Explain how you know where it is.

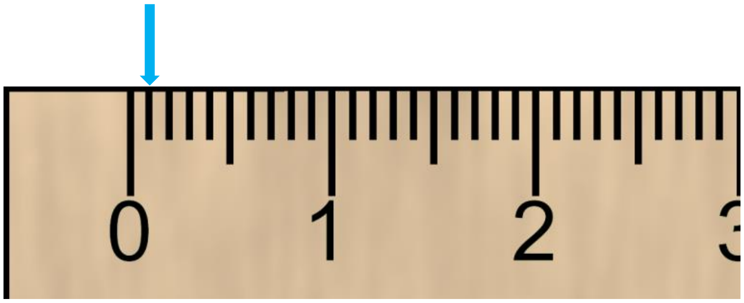


d. Show $\frac{1}{3}$ on the ruler. Explain how you know where it is



Answers page 55

e. What number goes with the first segment on the ruler? Explain how you know.



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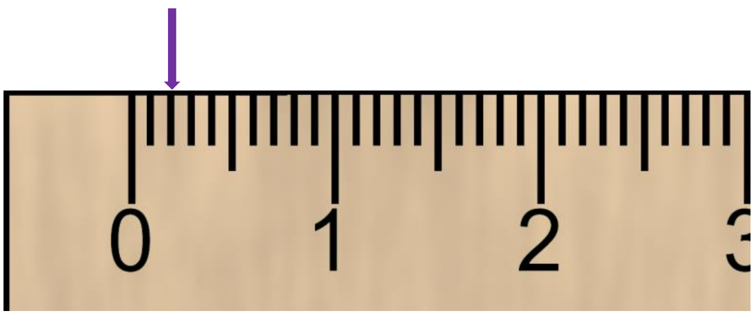
f. What number goes with the second segment on the ruler? Explain how you know.



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Activity 1: Make Your Own Number Line

You will need:

- strips of paper 2 inches or so,
- tape

Now you will make your own number line. Tape your strips of paper together like in the picture.

Then put these numbers on your number line.

0	1	2	3	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$
$\frac{1}{3}$	$1\frac{1}{3}$	$2\frac{1}{3}$	0.75	1.75	2.75	

Hint: Put the whole number 0, 1, 2, 3 on first, and then add the half marks. It will be easier to visualize where to put the fractions on the number line.

Numbers like 1, 2 and 3 are the easiest ones to put on your number line. Do the best you can with fractions and decimals. If you have trouble with either of those, please take the time to do those lessons when you can.

3. Negative Numbers Introduction

If you have experienced cold temperatures, this activity will have meaning for you. If you have only been in warm places, just do your best to imagine the temperatures for cold.

This is a thermometer. Hopefully you have seen one before.

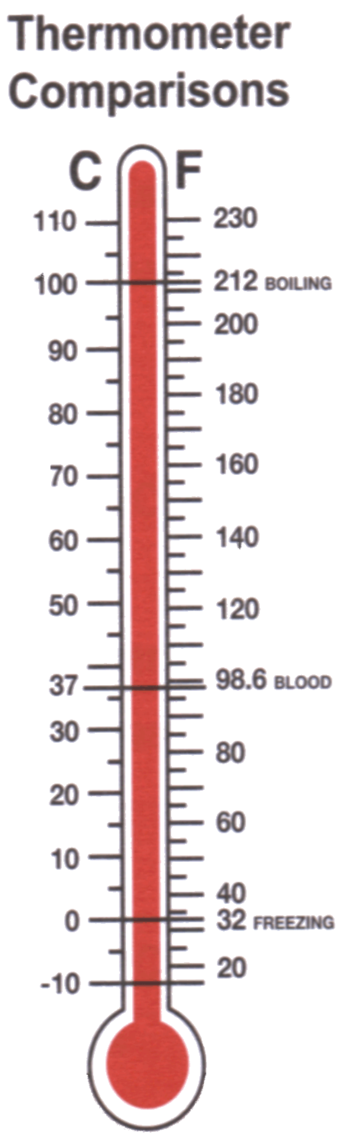


Figure 1

In case you're interested.

In the U.S. most people use the Fahrenheit scale (F), while in Canada and other countries, the Celsius (C) scale is used. For this activity, use the scale you prefer.

For reference, 98.6 ˚F is normal body temperature measured in Fahrenheit. 100˚C is the boiling point of water measured in Celsius. 0˚C is the freezing point of water when measured in Celsius. 32˚F is the freezing point of water when measured in Fahrenheit.

Practice 2: Using Temperatures to Understand Negative Numbers

Beginning Temperature	Ending Temperature	Difference (indicate + or -)	Dropped or Raised?
–3 ˚ F	8 ˚ F	+11	raised
15 ˚ C	3 ˚ C		
21 ˚ F	–2 ˚ F		
–10 ˚ F	–2 ˚ F		
8 ˚ C	–8 ˚ C		
–10 ˚ F	0 ˚ F		
5˚ C	–10˚ C		

Beginning Temperature	Ending Temperature	Difference (indicate + or -)	Dropped or Raised?
-6 ˚ F	8 ˚ F		
1 ˚ C	30 ˚ C		
58 ˚ F	–2 ˚ F		
–10 ˚ C	–2 ˚ C		
40 ˚ C	–10 ˚ C		
–20 ˚ C	0 ˚ C		
43˚ F	–4˚ F		

Which temperature is colder: 3 or –3? How do you know?

____

____

Extra credit: find 2 pairs of temperatures that have a difference of:

a. –6

b. 11

Activity 2: Extend Your Number Line to Include Negative Numbers

You will need:

- Your number line from Activity 1, page 6
- A few more strips of paper, 2 or 3 inches wide, as long as you want
- Tape

Now you will put negative numbers on your number line. These are the numbers to put on your new extended number line

-1 -2 -3 $-\frac{1}{2}$ $-1\frac{1}{2}$ $-2\frac{1}{2}$ $-\frac{1}{3}$

$-1\frac{1}{3}$ $-2\frac{1}{3}$ -0.75 -1.75 -2.75

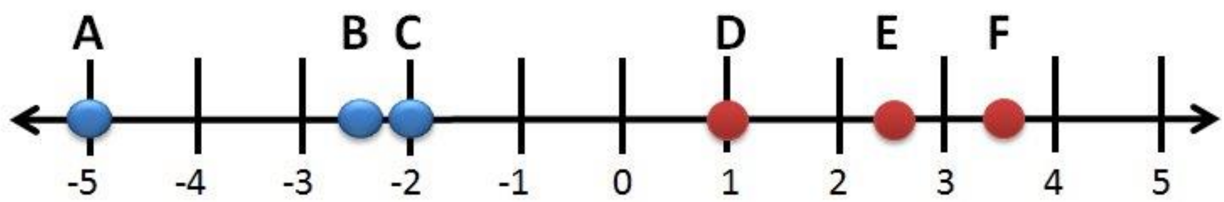
Hint: Since the negative number one includes 0 (zero) as a reference point, remember to align the zero from the positive number line directly on top of the zero on the negative number line.

Hopefully this is at least as easy for you as Activity 1. But if it isn't, stick with it. Negative numbers will make a lot more sense to you after you do this.

Extra credit: Put any other numbers on your number line.

Practice 3: More Practice with the Number Line

What are the numbers that correspond to points A through F?



A ____

B ____

C ____

D ____

E ____

F ____

Which number is greater, 3 or –5? Explain your reasoning.

____

____

Which number is bigger, –3 or –1? Explain your reasoning.

____

____

Answers page 61

Using Money to Understand Negative Numbers

Money is another model for understanding positive and negative numbers.

Example 1: Suppose Ollie has $5. He spends $2. How much money does Ollie have now?

The math expression for this situation is: 5 – 2 = 3.
Ollie has $3.

Now let's get more complicated.

Example 2:

Suppose Ollie has no money.
He wants to go to a movie, and he needs $3.

Before: His mom agrees to lend him the money

After: Ollie owes Mom $3

Ollie has 3 dollars in his hands. But he has to pay this money back. Does he really have $3?

Ollie goes to the movies and spends his 3 dollars.

Now he has no money in his hands.

He stills owes Mom $3.

Here is Ollie's situation in a table:

Ollie has in his hands	He borrows	He earns	Math expression	Ollie's net worth
0	3		0 + (– 3)	–3

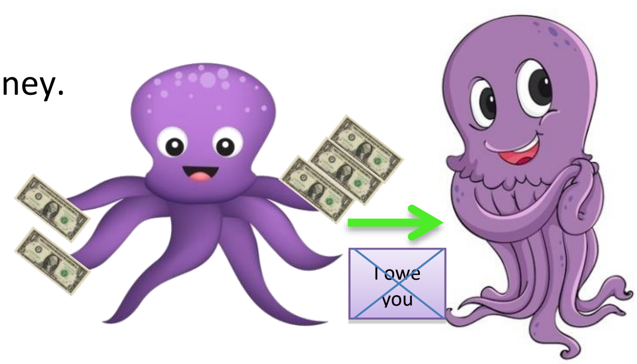
Now suppose Ollie earns $5 babysitting.

Ollie has in his hands	He borrows	He earns	Math expression	Ollie's net worth
0	3	5	5 + (–3)	2

He has to pay his mom back $3 of this money.

How much money does Ollie really have?

His "net worth" is $2.



Practice 4: Situations with Money

Write the correct number in the blank space or spaces on each line.

You have in your hand	You borrow	You earn	Math expression	Your net worth
5	0	4	5 + 4	9
5	4	0		
5	4	3		
0	4	5		
0	4	3		
0	3	4		
3	2	0		
3	2	5		
3	5	2		
8	5	0		
8	0	5		
8	3	5		
8	5	3		

Answers page 62

Practice 5: More Situations with Money

You have	You borrow	You earn	You spend	Math expression	Your net worth
5	6	7	3	5 + (–6) + 7 – 3	3
5	4	0	2		
5	4	3	2		
0	4	5	2		
0	4	3	2		
0	3	4	2		
3	2	0	2		
3	2	5	2		
3	5	2	2		
8	5	0	10		
8	0	5	10		
8	3	5	10		
8	5	3	10		

Practice 6: Ordering Positive and Negative Numbers

If you find these instructions confusing, just do what makes the most sense. It is ok to make mistakes, even many mistakes while learning. Just Try It!!

For each set of numbers, locate each amount on a number line by drawing and labeling a dot where they go. Then write the set of numbers in order from least to greatest.

Numbers: 4, –1, –9, 0	Numbers: 0, 2, –3, –5
⟵⟶ Ordered from least to greatest:	⟵⟶ Ordered from least to greatest:
Numbers: –4, 1, –1.5, –1	Numbers: 0, –2, –1, –2.5
⟵⟶ Ordered from least to greatest:	⟵⟶ Ordered from least to greatest:
Numbers: 3, 0, 1, 2	Numbers: –1, –4, 1.5, 0
⟵⟶ Ordered from least to greatest:	⟵⟶ Ordered from least to greatest:

Answers page 64

Numbers: 6, −3, 1, −4	Numbers: 8, 7, −6, −5
Ordered from least to greatest:	Ordered from least to greatest:
Numbers: 1, −2, 3, −4.5	Numbers: 0, −4, 4, −5, 5
Ordered from least to greatest:	Ordered from least to greatest:
Numbers: 4, 5, 0, 3, 1	Numbers: −4, −5, −1.5, 2.5
Ordered from least to greatest:	Ordered from least to greatest:

Order this list of numbers by writing them from greatest to least:

−5 6 0 −2 1 3

Answers page 65

4. Computation with Positive and Negative Numbers

You will have two ways to understand addition and subtraction of positive and negative numbers– a number line model and a collection model with pennies. You can select the one you prefer, after trying them both. Pennies will be used in future activities.

The next 20 pages will show you how the positive and negative signs work in computation – with addition, subtraction and multiplication. You may have been "taught" before using "rules." Teaching with rules seems easier, and it certainly takes less time up front. But over half of students just don't remember these "rules." And why should they? The rules don't make sense without a deeper way to understand them. That is what the next 20 pages are about.

So, please, do yourself a favor and go through the exercises. They require real thinking. But the benefit to you is that you will actually understand the "rules."

We begin with a number line model for adding whole numbers, like 1, 2, 3. It is called "Walk the Line," and it is a game of chance. So, win or lose, it's a game of luck.

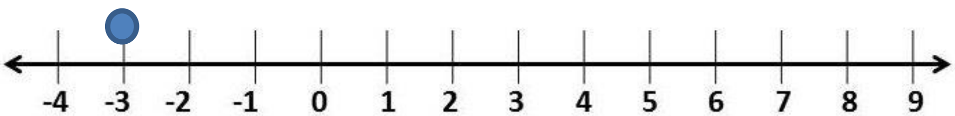
Activity 3: Walk the Line

You will need:

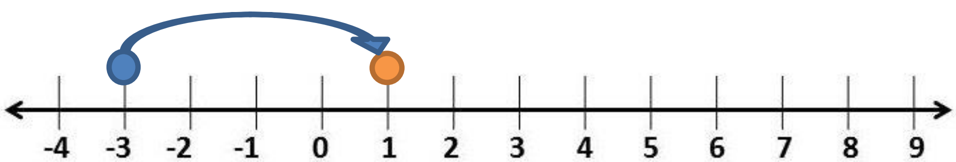
- Walk the Line card decks, on pages 20 and 22.
- A partner
- Number line showing both negative and positive integers (1 per student) page 24.
- Game scoring sheet (Don't use this until the second or third time the game is played) page 22.
- Two different paper clips or other game pieces – different for each

Example: Shuffle the Walk the Line card deck, and place it face down between you and your partner. Each player will have a total of five turns.

Player 1 draws a –3 on his first turn.

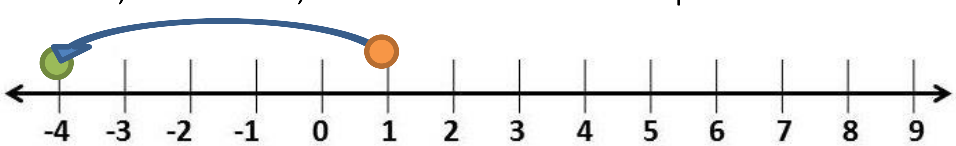


Then on his next turn, turn 2, he draws a 4. He moves 4 places to the right.



He records these first two draws of the cards in the first box.

On Turn 3, his next turn, he draws a –5. He moves 5 places to the left.



Player 1:	Number model	Location
Turn 2	–3 + 4 = 1	1
Turn 3	1 + –5 = –4	–4

On the next turns, each player will take the top card, and then starting from wherever the marker was previously, move his or her marker according to the card just drawn. The winner of the game is the person with the highest score after five rounds.

After playing this game two or three times without using the scoring sheet, now you are ready for it.

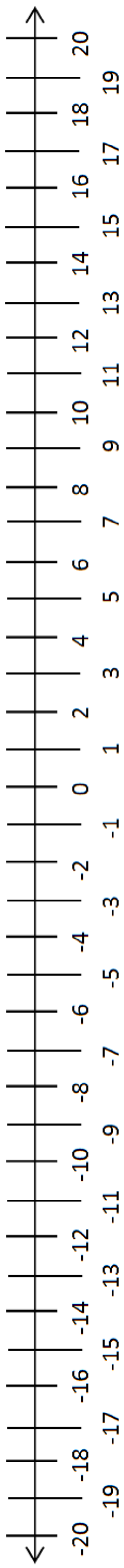
Walk the Line Game Deck

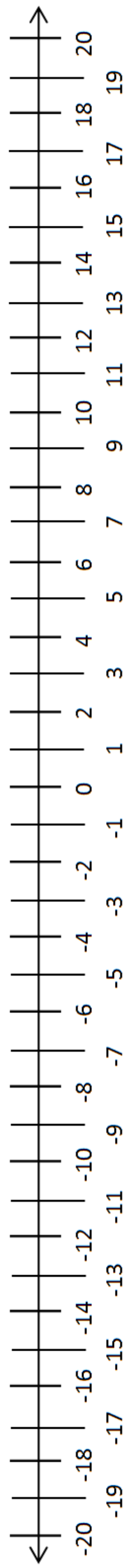
1	2	3
4	5	6
7	8	9
-1	-2	-3
-4	-5	-6

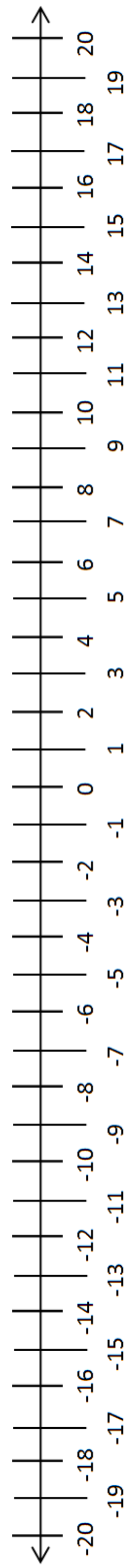
Walk the Line Game Deck, page 2

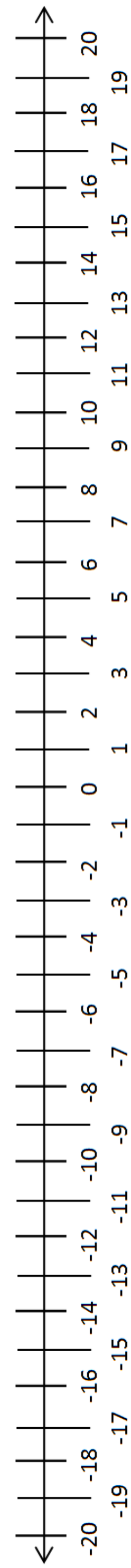
1	2	3
4	5	6
-7	-8	-9
-1	-2	-3
-4	-5	-6

Number Line for Walk the Line

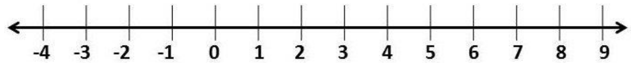


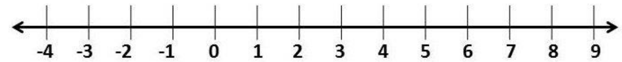






Walk the Line Game Scoring Sheet

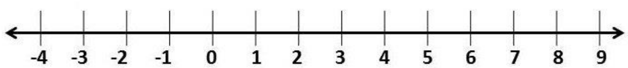


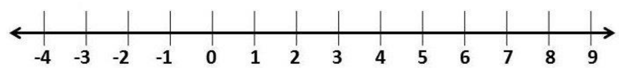


Game 1	**Player 1:**		**Player 2:**	
	Number model	Location	Number model	Location
Turn 1				
Turn 2				
Turn 3				
Turn 4				
Turn 5				

Final Location: ______ Final Location: _____

Who won and by how much? ____

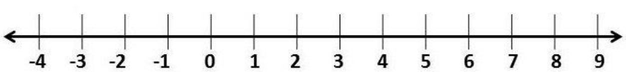


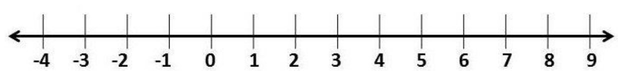


Game 1	**Player 1:**		**Player 2:**	
	Number model	Location	Number model	Location
Turn 1				
Turn 2				
Turn 3				
Turn 4				
Turn 5				

Final Location: ______ Final Location: _____

Who won and by how much? ____





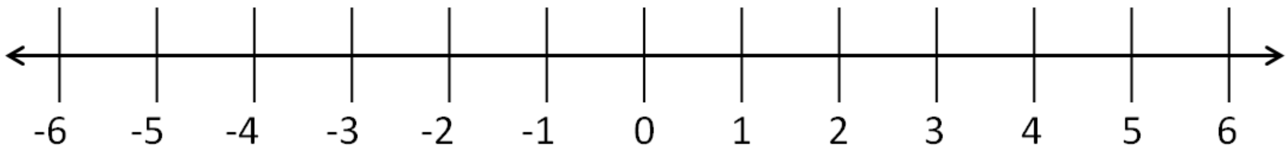
Game 1	**Player 1:**		**Player 2:**	
	Number model	Location	Number model	Location
Turn 1				
Turn 2				
Turn 3				
Turn 4				
Turn 5				

Final Location: ______ Final Location: _____

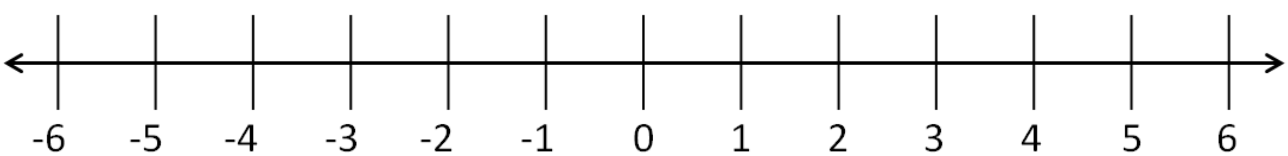
Who won and by how much? ____

Practice 7: Who Won? Who's Right?

1. John and June are playing Walk the Line and they are arguing over who won the game. June's final score is –2. John's final score is –4. June insists that she is the winner, but John thinks he is the winner because 4 is more than 2. Use mathematics to explain who the winner of this round is.



2. John and June are playing another round of Walk the Line. Right now, John's marker is at –4 and June's marker is at 5. What card would John need to draw next in order to move his marker to the same spot where June's is? Explain your reasoning.



Answers page 66

Activity 4: Walk the Line with Subtraction

You will need:

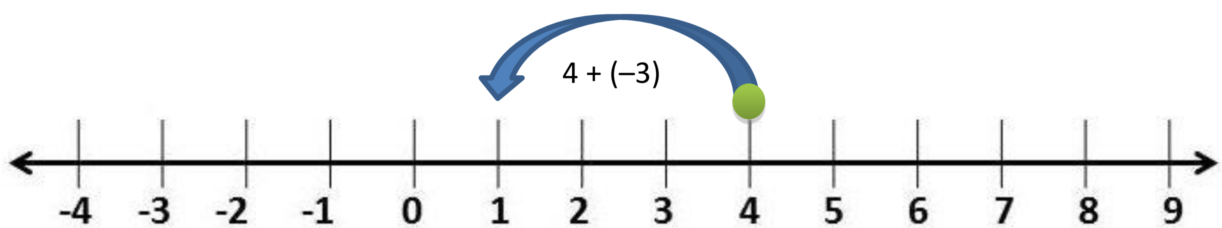
- Same things as for Walk the Line
- A penny
- A teacher to spot check your work

You will play Walk the Line just as in the previous activity, but with a twist. In the last Activity, you always added the numbers. Now you will subtract numbers sometimes. The penny will tell you whether to add (heads) or subtract (tails).

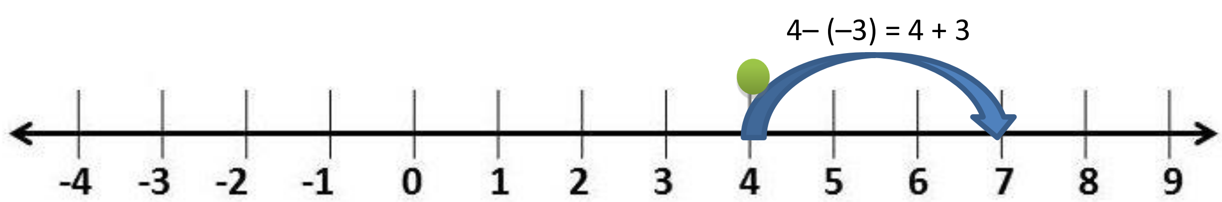
Here is an example. Player 1 draws a 4 on his first turn. Then he draws a –3 on the next card. He flips the penny and it is tails. So, he must subtract –3 from 4. Well, that is different.

–3 means – (–3)

If he was supposed to add –3, that would be easy (hopefully).



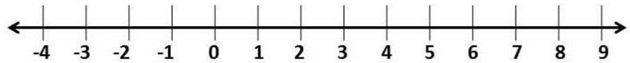
But our player is supposed to subtract –3 from 4. There is only one possibility – move the arrows to the right. So, subtracting –3 must be the same as adding +3. In symbols:

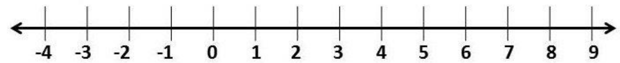


Now it's your turn to play Walk with Line with Subtraction.

Play at least three rounds with your partner.

Walk the Line Game Score Sheet

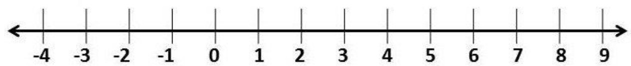


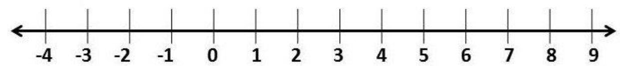


Game 1	**Player 1:**		**Player 2:**	
	Number model	Location	Number model	Location
Turn 1				
Turn 2				
Turn 3				
Turn 4				
Turn 5				

Final Location: ______ Final Location: _____

Who won and by how much? ____

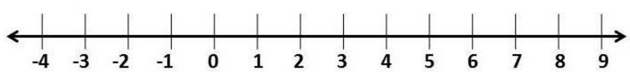


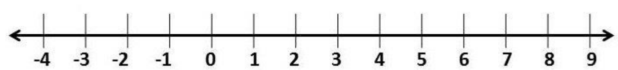


Game 1	**Player 1:**		**Player 2:**	
	Number model	Location	Number model	Location
Turn 1				
Turn 2				
Turn 3				
Turn 4				
Turn 5				

Final Location: ______ Final Location: _____

Who won and by how much? ____





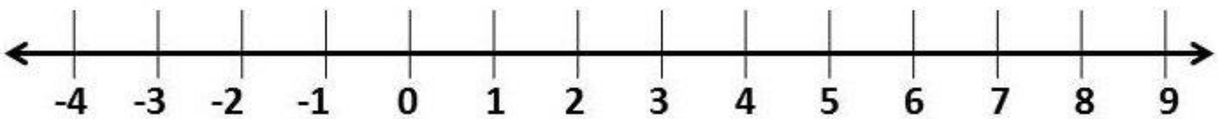
Game 1	**Player 1:**		**Player 2:**	
	Number model	Location	Number model	Location
Turn 1				
Turn 2				
Turn 3				
Turn 4				
Turn 5				

Final Location: ______ Final Location: _____

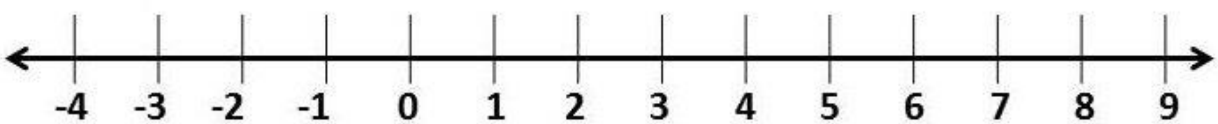
Who won and by how much? ____

Practice 8: Who Won? Who's Right? #2

1. John and June are playing Walk the Line. Right now, both of their markers are on 3. John's next card is 2 and he got "+" when flipped his penny heads. June's next card is –2 and she got "– "tails when she flipped her penny. June says both hers and John's markers will end up at the same place again. Is she right? How do you know?



2. John and June are playing another round of Walk the Line. Right now, John's marker is at –1 and June's marker is at 3. June doesn't have another turn, and this is John's final turn. John's next card is –5. What penny flip would allow John to move his marker past June's and win the game? Explain why this is the way for him to win.



Answers page 67

5. Addition with Negative Numbers

Pennies provide a conceptual model for addition and subtraction of positive and negative numbers, as well as for multiplication. Please take the time to use your pennies so that you can really understand the process.

You may have used pennies before when you studied probability, you may have flipped pennies. That isn't what you are doing here.

In this model, heads equals positive one, and tails equals negative one.

 = +1 and  = –1

What does 1 + (–1) = ? Using the borrowing money model, if I have one penny and owe one penny, my net worth is 0. The penny model shows this:

 ie + 1 + (–1) = 0

For example, the problem – 3 + 4 can be modeled

Each head-tail combination (H T) is equal to zero, the remaining head (H) shows that the answer is +1 or just 1. –3 + 4 = 1

Activity 5: Addition Modeled with Pennies

Numbers	Modeled with pennies	Sum
3 + 4		
–3 + 4		
3 + (–4)		
–3 + (–4)		
6 + 2		
–6 + (–2)		
1 + 5		
–1 + 5		

Numbers	Modeled with pennies	Sum
2 + 3 =		
–2 + 3 =		
2 + (–3) =		
–2 + (–3) =		
3 + (–2) =		
7 + 5 =		
7 + (–5) =		
–7 + 5 =		
–7 + (–5) =		
–5 + 7 =		

Answers page 69

Activity 6: Subtraction Modeled with Pennies

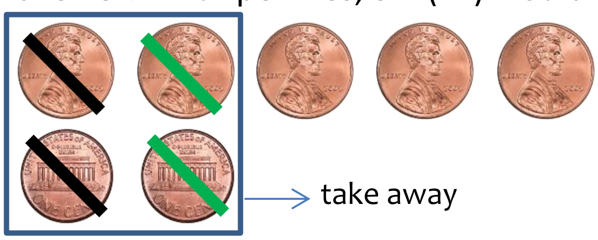
The penny model is a conceptual model for subtracting both positive and negative numbers.

For example, the problem 5 – 2 can be read as "5 take away 2". With pennies, this means

take away

with an answer of 3.

The same problem 5 – 2 can be seen to be the same as 5 + (–2), as they give the same answer. With pennies, 5 + (–2) would be shown as



take away

with the two T canceling the two H for a sum of zero, and the result being +3, shown by the three H.

So, 5 – 2 gives the same answer as 5 + (–2).

Here is a question: Does a – b always equal a + (–b)?

In this example, a = 5 and b = 2.

You may be thinking: Who cares?! And right now, probably you don't care. That is ok. But I guarantee you that your life in math will be easier when you know the answer to this question, and can move back and forth with the form of the math expression that works best for you. So please stick it out here!

Practice 9: Does a – b = a + (-b)?

Use pennies to model each of these problems. Then draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
6 – 4 = 2 H H H H H H	6 + (–4) = 2 H H H H H H T T T T	6 – 4 = 6 + (–4) = 2
7 – 1 =		
	6 + (–5) =	
8 – 6 =		
	3 + (–5) =	
	4 + (–3) =	
14 – 5 =		
5 – 2 =		
	7 + (–8) =	

You can use pennies to model each of these problems if you want. Draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
12 – 4 =		
5 – 4 =		
	4 + (–5) =	
2 – 5 =		
7 - 5 =		
	3 + (–2) =	
12 – 3 =		
	11 + (–4)	
	8 + (–5) =	

Practice 10: Does 5 – 3 = 3 – 5 ?

Here are two problems: 5 – 3 = ? and 3 – 5 = ?

Do they have the same answer?

With the pennies, we can figure it out easily!

5 – 3 is "5 take away 3."

Here is 5:

Taking away 3 is

So, 5 – 3 = 2.

3 – 5 is more difficult. This is where a – b = a + (–b) really helps.

3 – 5 = 3 + (–5) with pennies:

Every H T combination equals 0. Woo hoo. All that is left is T T, which is –2.

So, 3 – 5 = –2.

So, 5 – 3 = 2 and 3 – 5 = –2. 5 – 3 does not equal 3 – 5.

Now you practice!

Practice 11: a – b Compared to b - a

a. Does 6 – 4 = 4 – 6?

Explain how you know.

b. What can x be to satisfy: $x - 2 = 2 - x$?

c. What can z be to satisfy: 5 – z = z – 5?

Answers page 72

Practice 12: Addition with Negative Numbers

a. –3 + 5 =

b. –4 + 2 =

c. –4 + 5 =

d. –2 + 1 =

e. –2 + 3 =

f. 4 + (–3) =

g. 5 + (–2) =

h. 7 + (–3) =

i. 8 + (–2) =

j. 5 + (–4) =

k. –2 + (–5) =

l. –4 + (–2) =

m. –1 + 5 =

n. –12 + 5 =

o. –18 + 3 =

p. 6 + (–3) =

q. 5 + (–1) =

r. 7 + (–7) =

Answers page 73

s. 8 + (–7) =

t. –5 + (–4) =

u. 8 – 2 =

v. 15 – 3 =

w. 8 – 5 =

x. 5 – 1 =

y. 4 – 5 =

z. 3 – 19 =

aa. 5 – 7 =

bb. 9 – 11 =

cc. 6 – 4 =

dd. Which is greater –1 or –3 ? Explain how you know.

6. Multiplication with Negative Numbers

For the rest of your life in math classes, you will be multiplying with positive and negative numbers. There are four possibilities (positive times positive, positive times negative, negative times positive, negative times negative). Eventually, you will want to memorize how the signs of the answers work. But the signs make sense, and have to be the way they are. That is what these next four pages are about.

There are four combinations of multiplication with positive and negative numbers, with 3 ∗ 2 as an example:

1. (3) ∗ (2)

2. (3) ∗ (–2)

3. (–3) ∗ (2)

4. (–3) ∗ (–2)

1. (3) ∗ (2) means three groups of 2

So, the answer is (3) ∗ (2) = 6.

2. (3) ∗ (–2) means three groups of –2

So, the answer is (3) ∗ (–2) = –6.

3. There are two ways to show (–3) ∗ (2).

The first way is to use one of the fundamental rules of our number system, that

(a) ∗ (b) = (b) ∗ (a)

(b) Using this (–3) ∗ (2) = (2) ∗ (–3) which means 2 groups of –3

So (–3) ∗ (2) = (2) ∗ (–3) = –6.

The second way is a lot of trouble, and you wouldn't want to use it very often, but it will help you with the next problem, with two negative factors.

In this more complicated way, (–3) ∗ (2) can be read "take away 3 groups of 2." To do this, we start with a very complicated form of zero:

0 is the same as

Now we can "take away 3 groups of 2"

What remains is –6

So (–3) ∗ (2) = –6.

4. Now we are ready for (–3) ∗ (–2).

(–3) ∗ (–2) can be read as "take away 3 groups of –2." To do this, we start with zero.

Once again,

0 is the same as

Now we can "take away 3 groups of –2"

What remains is 6.

So (–3) ∗ (–2) = 6.

Activity 7: Multiplication Modeled with Pennies

Materials:

- 30 pennies 

Lay out the pennies and then draw.

#1

3 ∗ 5 means: put on 3 groups of 5

3 ∗ (–5) means: put on 3 groups of –5

–3 ∗ 5 means: Do this two ways.

Way 1:
–3 ∗ 5 = 5∗ (–3), so put on 5 groups of –3

Way 2:
–3 ∗ 5 means "take away 3 groups of 5." To do this, you must use a complicated form of zero, namely 3 groups of 5 and 3 groups of –5. Now you can "take away" 3 groups of 5.

(–3) ∗ (–5) = This problem corresponds to the second way to solve –3 ∗ 5, using a complicated form of zero, , namely 3 groups of 5 and 3 groups of –5. Now you can "take away" 3 groups of –5.

---

2 ∗ 4 means: put on 2 groups of 4

2 ∗ (–4) means: put on 2 groups of –4

Answers page 76

$-2 * 4$ can be done two different ways.

Way 1:

$-2 * 4 = 4 * (-2)$, so put on 4 groups of -2

Way 2:

$-2 * 4$ means "take away 2 groups of 4." To do this, you must use a complicated form of zero, namely 2 groups of 4 and 2 groups of -4. Then they can "take away" 2 groups of 4.

$(-2) * (-4) =$ means "take away 2 groups of -4. To do this, you must use a complicated form of zero, , namely 2 groups of 4 and 2 groups of -4.
Now you can "take away" 2 groups of -4.

Answers page 77

1 ∗ 5 means: put on 1 group of 5

1 ∗ (–5) means: put on 1 group of –5

–1 ∗ 5 means: Do this two ways.

Way 1:
–1 ∗ 5 = 5 ∗ (–1), so put on 5 groups of –1

Way 2:
$-1 * 5$ means “take away 1 groups of 5 To do this, you must use a complicated form of zero, namely 1 group of 5 and 1 group of –5. Then they can “take away” 1 group of 5.

$(-1) * (-5) =$ This problem corresponds to the second way to solve $-1 * 5$, using a complicated form of zero, , namely 1 group of 5 and 1 group of –5. Now you can “take away” 1 group of –5.

Practice 13: Multiplication with Negative Numbers

Symbols	Words	Picture	Result
2 ∗ 4	put on 2 groups of 4		8
2 ∗ 5			
–2 ∗ 5			
2 ∗ (–4)			
1 ∗ (–7)			
–2 ∗ 6			
–2 ∗ (–5)			
–4 ∗ (–3)			

I hope the signs, i.e. positive or negative, make sense to you now. Many students start with memorizing these signs in a regular math class. Now you have seen that signs make sense – they must be the way they are.

Answers page 81

Symbols	Words	Picture	Result
7 * 4			
0 * (–5)			
–3 * 2			
5 * (–4)			
–1 * (–3)			
–8 * 2			
–2 * (–3)			
–4 * (–2)			

Answers page 82

7. Generalizing

Now we will do something mathematicians do – generalize. We will go from the specific problem to all numbers.

(a) ∗ (b) = + i.e. positive

(a) ∗ (–b) = – i.e. negative

(–a) ∗ (b) = – i.e. negative

(–a) ∗ (–b) = + i.e. positive

This is the same thing as "memorizing the rules". By now, you have seen why and that it is important. These aren't really "rules" they just make sense. If you ever forget, and most of us do, you can reinvent them.

I.E. stands for the Latin words id est. This translates to English as "that is, or that is to say." For example, you might use the phrase "VR ie virtual reality."

Optional Practice for a-b = a+(-b)

You can use pennies to model each of these problems if you want. Draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
6 – 14 =	6 + (–14) = –8 H H H H H H T T T T T T T T T T T T T T	6 – 14 = 6 + (–14) = –8
7 – 11 =		
6 – 9 =		
8 – 15 =		
	3 + (–12) =	
	4 + (–13) =	
4 – 5 =		
5 – 12 =		

Optional Practice for a-b = a+(-b)

You can use pennies to model each of these problems if you want. Draw the H's and T's. Fill in the other blanks.

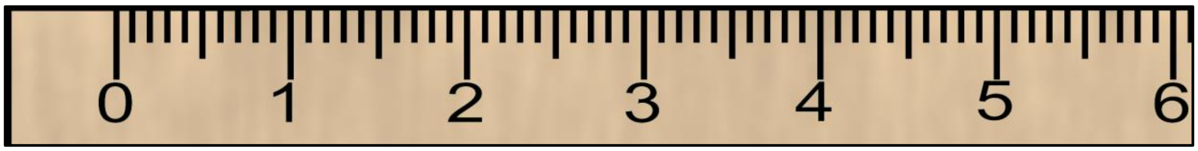
a – b =	a + (–b) =	a – b = a + (–b) =
12 – 14 =		
7 – 18 =		
	6 + (–15) =	
1 – 6 =		
2 – 5 =		
	5 + (–11) =	
12 – 14 =		
	11 + (–14)	

Answers page 85

Solutions / Answers

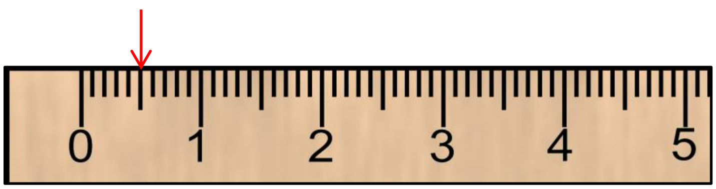
Practice 1: A Ruler Is a Number Line

g. What is the purpose of the hatch marks on the ruler?



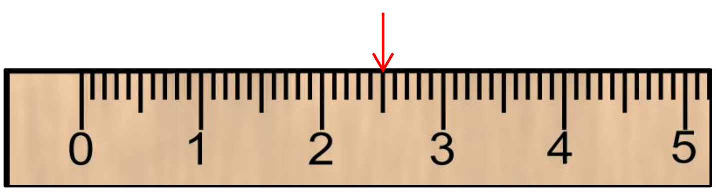
To be able to measure numbers in between the whole numbers

h. Show $\frac{1}{2}$ on the ruler. Explain how you know where it is.



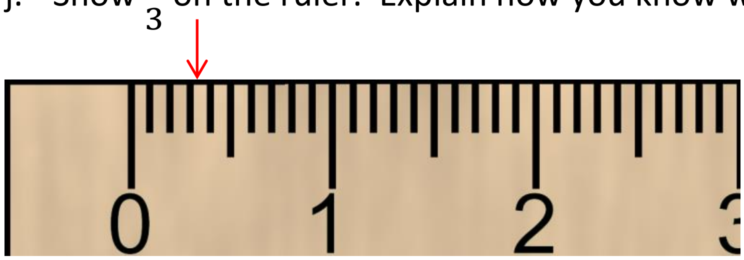
It is half way between 0 and 1. It is the medium sized hatch mark between 0 and 1.

i. Show $2\frac{1}{2}$ on the ruler. Explain how you know where it is.



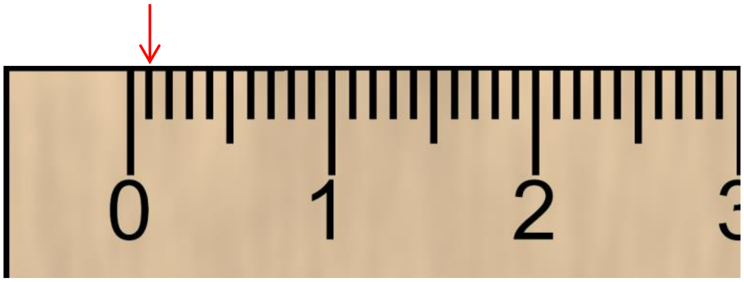
The 2 is marked on the ruler. The $\frac{1}{2}$ is in the middle between the 2 and the 3.

j. Show $\frac{1}{3}$ on the ruler. Explain how you know where it is



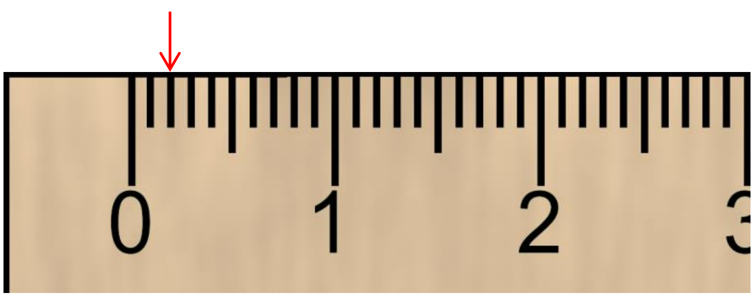
It is $\frac{1}{3}$ of the distance between the 0 and the 1. There are 10 hatch marks, so it is just past the third hatch mark.

k. What number goes with the first segment on the ruler? Explain how you know.



$\frac{1}{10}$. There are ten divisions between each whole number, so each hatch mark (division) is $\frac{1}{10}$.

l. What number goes with the second segment on the ruler? Explain how you know.



There are 10 hatch marks between the whole numbers, so each hatch mark represents $\frac{1}{10}$. Two hatch marks are $\frac{2}{10}$ or $\frac{1}{5}$.

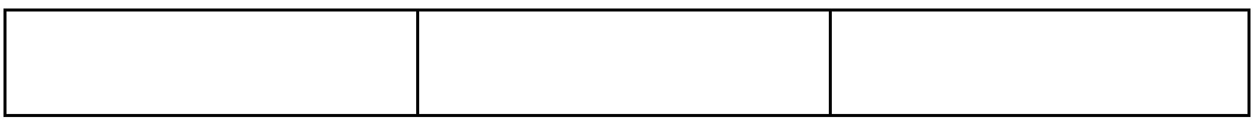
Did you follow the hint? Was that easier?

Activity 1: Make Your Own Number Line

You will need:

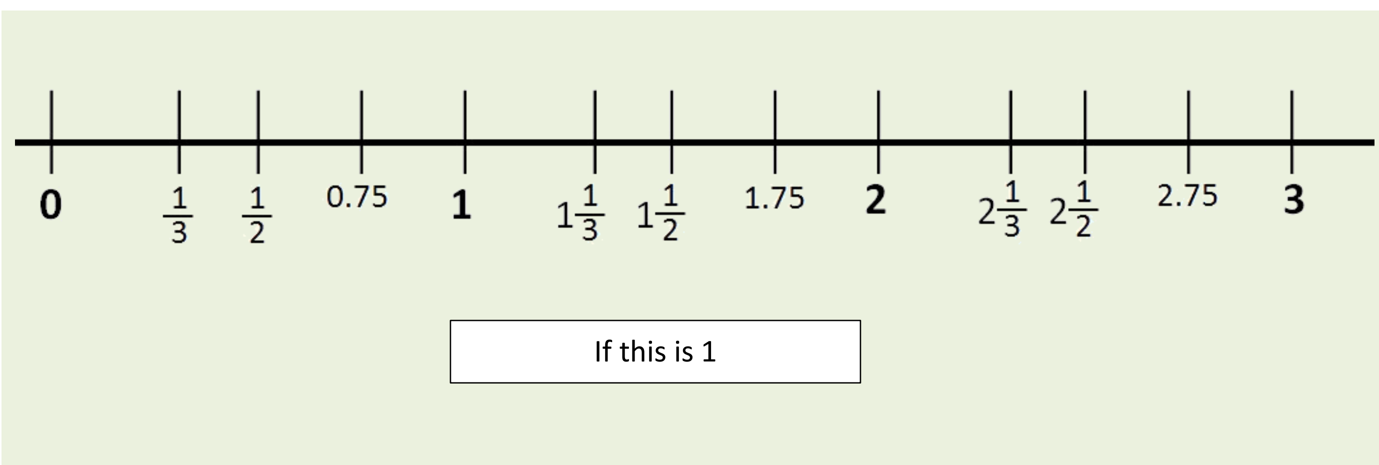
- strips of paper 2 inches or so,
- tape

Now you will make your own number line. Tape your strips of paper together like in the picture.



Then put these numbers on your number line.

0	1	2	3	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$
$\frac{1}{3}$	$1\frac{1}{3}$	$2\frac{1}{3}$	0.75	1.75	2.75	



Practice 2: Using Temperatures to Understand Negative Numbers

Beginning Temperature	Ending Temperature	Difference (indicate + or -)	Dropped or Raised?
–3 ˚ F	8 ˚ F	+11	raised
15 ˚ C	3 ˚ C	–12	dropped
21 ˚ F	–2 ˚ F	–23	dropped
–10 ˚ F	–2 ˚ F	+8	raised
8 ˚ C	–8 ˚ C	–16	dropped
–10 ˚ F	0 ˚ F	+10	raised
5˚ C	–10˚ C	–15	dropped

Beginning Temperature	Ending Temperature	Difference (indicate + or -)	Dropped or Raised?
–6 ˚ F	8 ˚ F	+14	raised
1 ˚ C	30 ˚ C	+29	raised
58 ˚ F	–2 ˚ F	–60	dropped
–10 ˚ C	–2 ˚ C	+8	raised
40 ˚ C	–10 ˚ C	–50	dropped
–20 ˚ C	0 ˚ C	+20	raised
43˚ F	–4˚ F	–47	dropped

Which temperature is colder: 3 or –3? How do you know?

–3 is colder. –3 + 6 = 3 So 3 is 6 degrees warmer than –3

Extra credit: find 2 pairs of temperatures that all have a difference of:

a. –6 b. 11

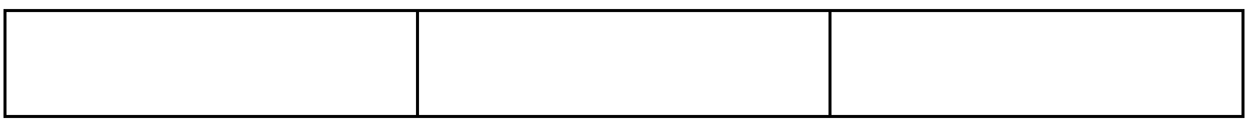
There is an infinity of answers – Here are some

a. 0 and –6 20 and 14 12 and 6 19 and 13
b. 1 and 12 20 and 31 21 and 32 0 and 11

Activity 2: Extend Your Number Line to Include Negative Numbers

You will need:

- Your number line from Activity 1, page 6
- A few more strips of paper, 2 or 3 inches wide, as long as you want
- Tape

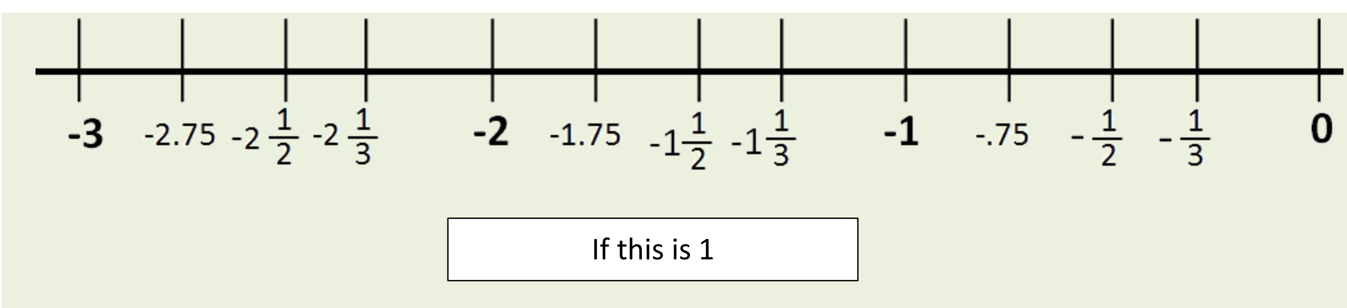


Now you will put negative numbers on your number line.   These are the numbers to put on your new extended number line

-1 -2 -3 $-\frac{1}{2}$ $-1\frac{1}{2}$ $-2\frac{1}{2}$ $-\frac{1}{3}$

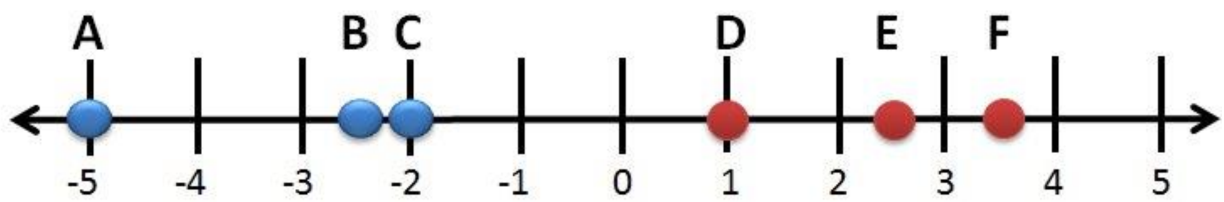
$-1\frac{1}{3}$ $-2\frac{1}{3}$ -0.75 -1.75 -2.75

Hint: Sine the negative number one includes 0 (zero) as a reference point, remember to align the zero from the positive number line directly on top of the zero on the negative number line.



Practice 3: More Practice with the Number Line

What are the numbers that correspond to points A through F?



A	–5
B	–2.5 or $-2\frac{1}{2}$
C	–2
D	1
E	2. 5 or $2\frac{1}{2}$
F	3. 5 or $3\frac{1}{2}$

Which number is greater, 3 or –5? Explain your reasoning.

3 is greater. There are several possible explanations why. Here are two: 5 is greater than 3 – it is to the right of 3 on the number line. Since 3 is to the right of –5, it must be greater also. Second reasoning: If I have –5 dollars, I owe $5. It takes $8 to get to $3. So, I have more money if I have $3 compared to –$5.

Which number is bigger, –3 or –1? Explain your reasoning.

–1 is bigger. To go from –3 to –1, you have to add 2. –3 + 2 = –1. So –1 is bigger.

Note: This is not a reason: Because –1 is closer to 0. That doesn't explain anything. Comparing 1 and 3, 1 is closer to 0, but 3 is larger.

Practice 4: Situations with Money

You have in your hand	You borrow	You earn	Math expression	Your net worth
5	0	4	5 + 4	9
5	4	0	5 + (–4)	1
5	4	3	5 + (–4) + 3	4
0	4	5	0 + (–4) + 5	1
0	4	3	0 + (–4) + 3	–1
0	3	4	0 + (–3) + 4	1
3	2	0	3 + (–2)	1
3	2	5	3 + (–2) + 5	6
3	5	2	3 + (–5) + 2	0
8	5	0	8 + (–5)	3
8	0	5	8 + 5	13
8	3	5	8 + (–3) + 5	10
8	5	3	8 +(–5) + 3	6

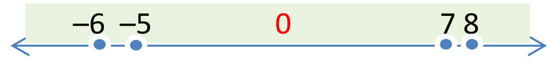
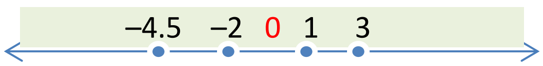
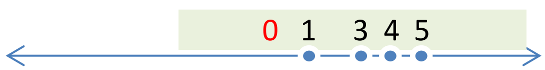
Practice 5: More Situations with Money

You have	You borrow	You earn	You spend	Math expression	Your net worth
5	6	7	3	5 + (–6) + 7 – 3	3
5	4	0	2	5 + (–4) –2	–1
5	4	3	2	5 + (–4) + 3 –2	2
0	4	5	2	0 + (–4) + 5 – 2	–1
0	4	3	2	0 + (–4) + 3 – 2	–3
0	3	4	2	0 + (–3) + 4 – 2	–1
3	2	0	2	3 + (–2) –2	–1
3	2	5	2	3 + (–2) + 5 – 2	4
3	5	2	2	3 + (–5) + 2 – 2	–2
8	5	0	10	8 + (–5) – 10	–7
8	0	5	10	8 + 5 – 10	3
8	3	5	10	8 + (–3) +5 – 10	0
8	5	3	10	8 + (–5) + 3 – 10	–4

Practice 6: Ordering Positive and Negative Numbers

For each set of numbers, locate each amount on a number line by drawing and labeling a dot where they go. Then write the set of numbers in order from least to greatest.

Numbers: 4, –1, –9, 0	Numbers: 0, 2, –3, –5
–9 –1 0 4	–5 –3 0 2
Ordered from least to greatest:	Ordered from least to greatest:
–9, –1, 0, 4	–5, –3, 0, 2
Numbers: –4, 1, –1.5, –1	Numbers: 0, –2, –1, –2.5
–4 –1.5 –1 0 1	–2.5 –2 –1 0
Ordered from least to greatest:	Ordered from least to greatest:
–4, –1.5, –1, 1	–2.5, –2, –1, 0
Numbers: 3, 0, 1, 2	Numbers: –1, –4, 1.5, 0
0 1 2 3	–4 –1 0 1.5
Ordered from least to greatest:	Ordered from least to greatest:
0, 1, 2, 3	–4, –1, 0, 1.5

Numbers: 6, –3, 1, –4	Numbers: 8, 7, –6, –5
	
Ordered from least to greatest:	Ordered from least to greatest:
–4, –3, 1, 6	–6, –5, 7, 8
Numbers: 1, –2, 3, –4.5	Numbers: 0, –4, 4, –5, 5
	
Ordered from least to greatest:	Ordered from least to greatest:
–4.5, –2, 1, 3	–5, –4, 4, 5
Numbers: 4, 5, 0, 3, 1	Numbers: –4, –5, –1.5, 2.5
	
Ordered from least to greatest:	Ordered from least to greatest:
0, 1, 3, 4, 5	–5, –4, –1.5, 2.5

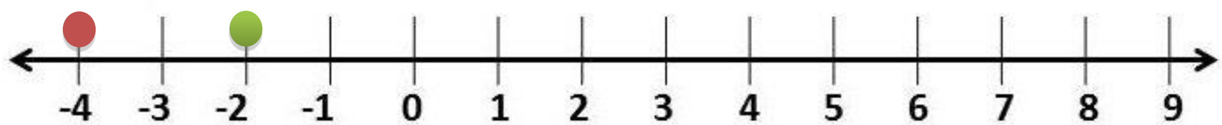
Order this list of numbers by writing them from greatest to least:

–5 6 0 –2 1 3

6 3 1 0 -2 -5

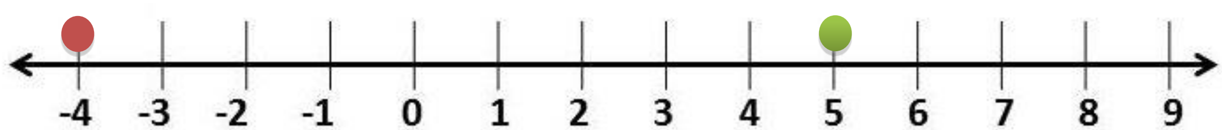
Practice 7: Who Won? Who's Right?

3. John and June are playing Walk the Line and they are arguing over who won the game. June's final score is –2. John's final score is –4. June insists that she is the winner, but John thinks he is the winner because 4 is more than 2. Use mathematics to explain who the winner of this round is.



June is the winner. –2 is greater than –4, so June has the higher score. John is right that 4 is more than 2, but that is not relevant.

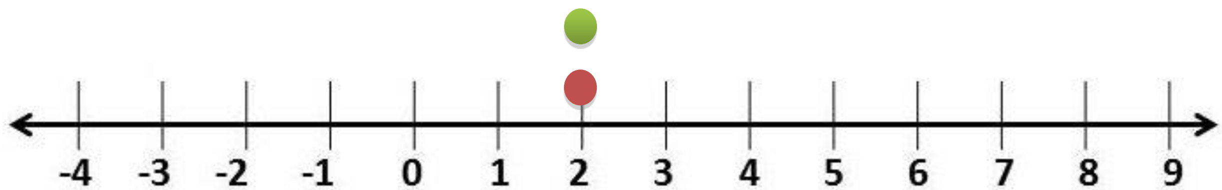
4. John and June are playing another round of Walk the Line. Right now, John's marker is at –4 and June's marker is at 5. What card would John need to draw next in order to move his marker to the same spot where June's is? Explain your reasoning.



There are nine spaces between –4 and 5, so John would need to draw a 9. –4 + 9 = 5.

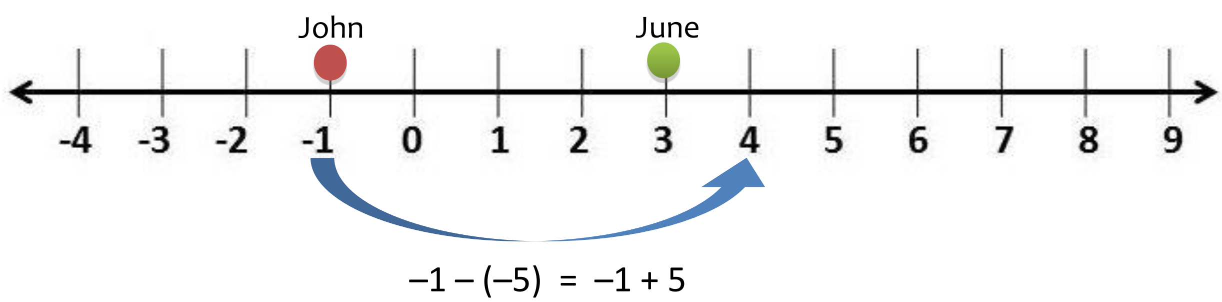
Practice 8: Who Won? Who's Right? #2

1.   John and June are playing Walk the Line.   Right now, both of their markers are on 3. John's next card is 2 and he got "+" when he flipped the penny. June's next card is -2 and she got "– " when she flipped her penny. June says both hers and John's markers will end up at the same place again. Is she right? How do you know?



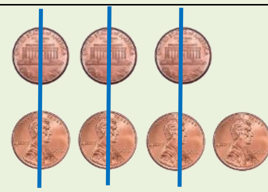
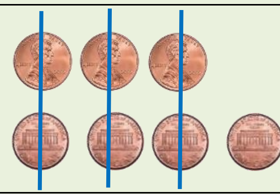
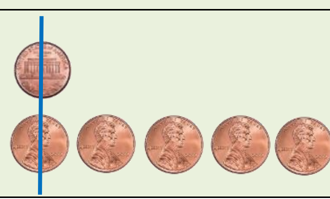
John will move to 5, because he was on 3, and will add 2 more. June will also move to 5, because 3 – (–2) = 5. So, June is right.

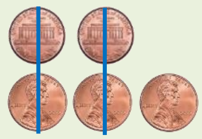
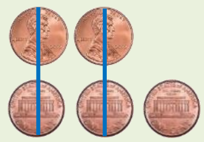
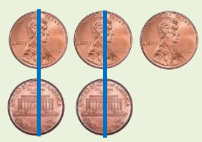
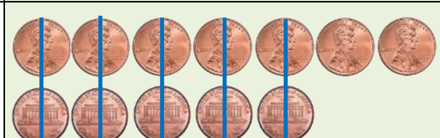
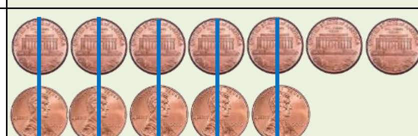
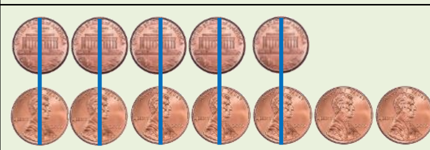
2. John and June are playing another round of Walk the Line. Right now, John's marker is at –1 and June's marker is at 3. June doesn't have another turn, and this is John's final turn. John's next card is –5. How can John move his marker past June's and win the game?



John is at –1. If he can add 5 to –1, he will end up on 4, because –1 + 5 = 4. Since he drew a –5, he needs to get tails, so that he can subtract –5. –1 – (–5) = –1 + 5 = 4. John will win with a 4, as June's marker is on 3.

Activity 5: Addition Modeled with Pennies

Numbers	Modeled with pennies	Sum
3 + 4		7
–3 + 4		1
3 + (–4)		-1
–3 + (–4)		-7
6 + 2		8
–6 + (–2)		-8
1 + 5		6
–1 + 5		4

Numbers	Modeled with pennies	Sum
2 + 3 =		5
–2 + 3 =		1
2 + (–3) =		–1
–2 + (–3) =		–5
3 + (–2) =		1
7 + 5 =		12
7 + (–5) =		2
–7 + 5 =		–2
–7 + (–5) =		–12
–5 + 7 =		2

Practice 9: Does a – b = a + (-b)?

Use pennies to model each of these problems. Then draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
6 – 4 = 2 H H H H H H	6 + (–4) = 2 H H H H H H T T T T	6 – 4 = 6 + (–4) = 2
7 – 1 = H H H H H H H	7 + (-1) = H H H H H H H T	7 – 1 = 7 + (–1) = 6
6 – 5 = H H H H H H	6 + (–5) = H H H H H H T T T T T	6 – 5 = 6 + (–5) = 1
8 – 6 = H H H H H H H H	8 + (–6) = H H H H H H H H T T T T T T	8 – 6 = 8 + (–6) = 2
3 – 5 = H H H H H T T	3 + (–5) = H H H T T T T T	3 – 5 = 3 + (–5) = –2
4 – 3 = H H H H	4 + (–3) = H H H H T T T	4 – 3 + 4 + (–3) = 1
14 – 5 = H H H H H H H H H H H H H H H	14 + (–5) = H H H H H H H H H H H H H H T T T T T	14 – 5 = 14 + (–5) = 9
5 – 2 = H H H H H	5 + (–2) = H H H H H T T	5 – 2 = 5 + (–2) = 3
7 – 8 = H H H H H H H H T	7 + (–8) = H H H H H H H T T T T T T T T	7 – 8 = 7 + (–8) = –1

You can use pennies to model each of these problems if you want. Draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
12 – 4 = ~~H H H H~~ H H H H H H H H	12 + (–4) = H H H H H H H H H H H H T T T T	12 – 4 = 12 + (–4) = 8
5 – 4 = ~~H H H H~~ H	5 + (–4) = H H H H H T T T T	5 – 4 = 5 + (–4) = 1
4 – 5 = H H H H H H T	4 + (–5) = H H H H T T T T T	4 – 5 = 4 + (–5) = –1
2 – 5 = ~~H H H H H H~~ T T T	2 + (–5) = H H T T T T T	2 – 5 = 2 + (–5) = –3
7 – 5 = ~~H H H H H~~ H H	7 + (-5) = H H H H H H H T T T T T	7 – 5 = 7 + (–5) = 2
3 – 2 = ~~H H~~ H	3 + (–2) = H H H T T	3 – 2 = 3 + (–2) = 1
12 – 3 = ~~H H H~~ H H H H H H H H H	12 + (–3) = H H H H H H H H H H H H T T T	12 – 3 = 12 + (–3) = 9
11 – 4 = ~~H H H H~~ H H H H H H H	11 + (–4) = H H H H H H H H H H H T T T T	11 – 4 = 11 + (–4) = 7
8 – 5 = ~~H H H H H~~ H H H	8 + (–5) = H H H H H H H H T T T T T	8 – 5 = 8 + (–5) = 3

Practice 11: a – b Compared to b - a

a. Does 6 – 4 = 4 – 6?

Explain how you know.

No. 6 – 4 means “6 take away 4” and the answer is 2.

“4 – 6” means “4 take away 6” and the answer is –2

b. What can x be to satisfy: $x - 2 = 2 - x$? The only value x can be is 2.

In that case, $x - 2 = 2 - 2 = 0$. And $2 - x = 2 - 2 = 0$.

c. What can z be to satisfy: 5 – z = z – 5?

z can only equal 5, because for any other value of z there would be positive number on one side and a negative number on the other side.

5 – z = 5 – 5

2 – 5 = 5 – 5

Both = 0

Practice 12: Addition with Negative Numbers

a. $-3 + 5 =$ 2

b. $-4 + 2 =$ -2

c. $-4 + 5 =$ 1

d. $-2 + 1 =$ -1

e. $-2 + 3 =$ 1

f. $4 + (-3) =$ 1

g. $5 + (-2) =$ 3

h. $7 + (-3) =$ 4

i. $8 + (-2) =$ 6

j. $5 + (-4) =$ 1

k. $-2 + (-5) =$ -7

l. $-4 + (-2) =$ -6

m. $-1 + 5 =$ 4

n. $-12 + 5 =$ -7

o. $-18 + 3 =$ -15

p. $6 + (-3) =$ 3

q. $5 + (-1) =$ 4

r. $7 + (-7) = 0$

s. $8 + (-7) = 1$

t. $-5 + (-4) = -9$

u. $8 - 2 = 6$

v. $15 - 3 = 12$

w. $8 - 5 = 3$

x. $5 - 1 = 4$

y. $4 - 5 = -1$

z. $3 - 19 = -16$

aa. $5 - 7 = -2$

bb. $9 - 11 = -2$

cc. $6 - 4 = 2$

dd. Which is greater -1 or -3 ? Explain how you know.

-1 is greater. I know because $-3 + 2 = -1$, i.e. I have to add 2 positive numbers to -3 to get to -1. This is just like 5 is greater than 3 because I have to add 2 positive numbers to 3 to get to 5.
(Note: -1 is closer to 0 is not a reason. 3 is closer to 0 than 5, and yet 5 is greater than 3).

Activity 7: Multiplication Modeled with Pennies

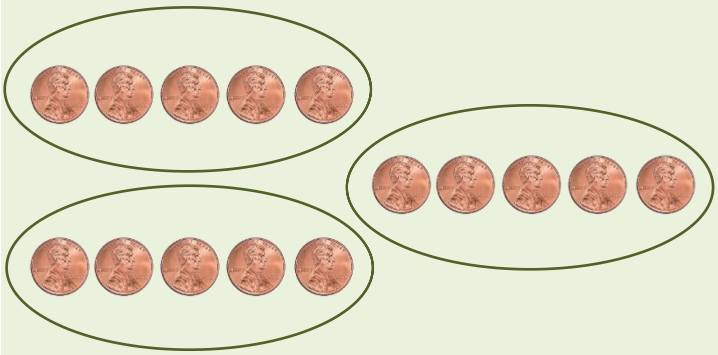
Materials:

- 30 pennies 

Lay out the pennies and then draw.

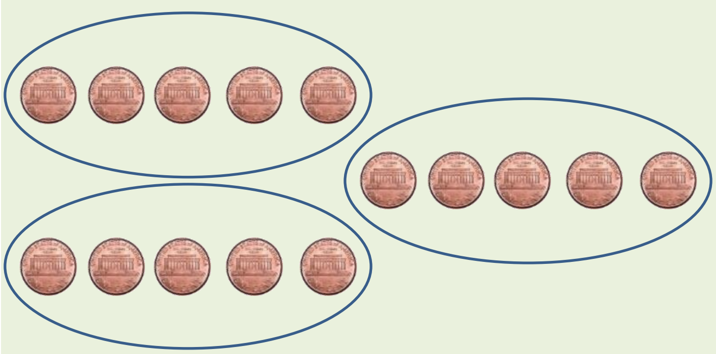
#1

3 ∗ 5 means: put on 3 groups of 5



Result 15

3 ∗ (–5) means: put on 3 groups of –5

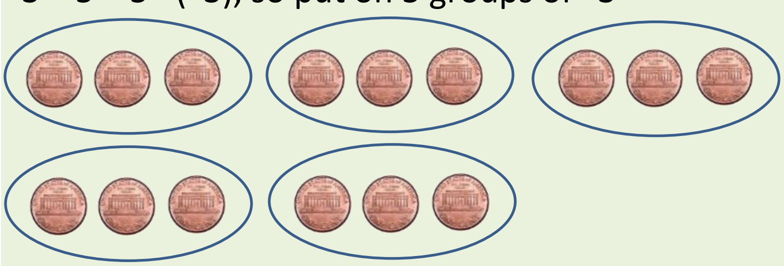


Result - 15

–3 ∗ 5 means: Do this two ways.

Way 1:

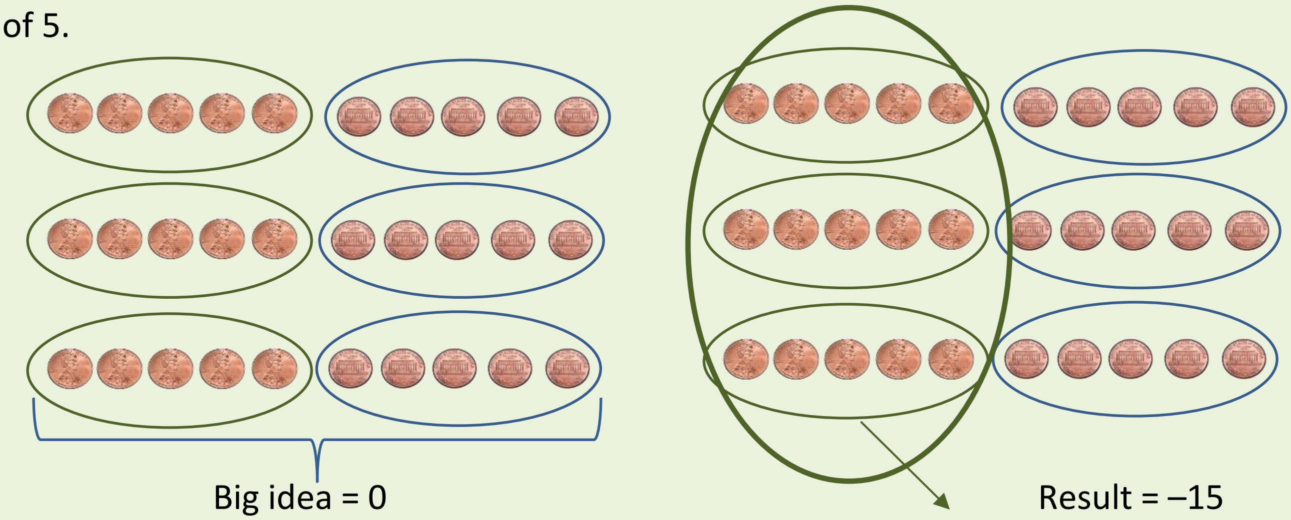
–3 ∗ 5 = 5∗ (–3), so put on 5 groups of –3



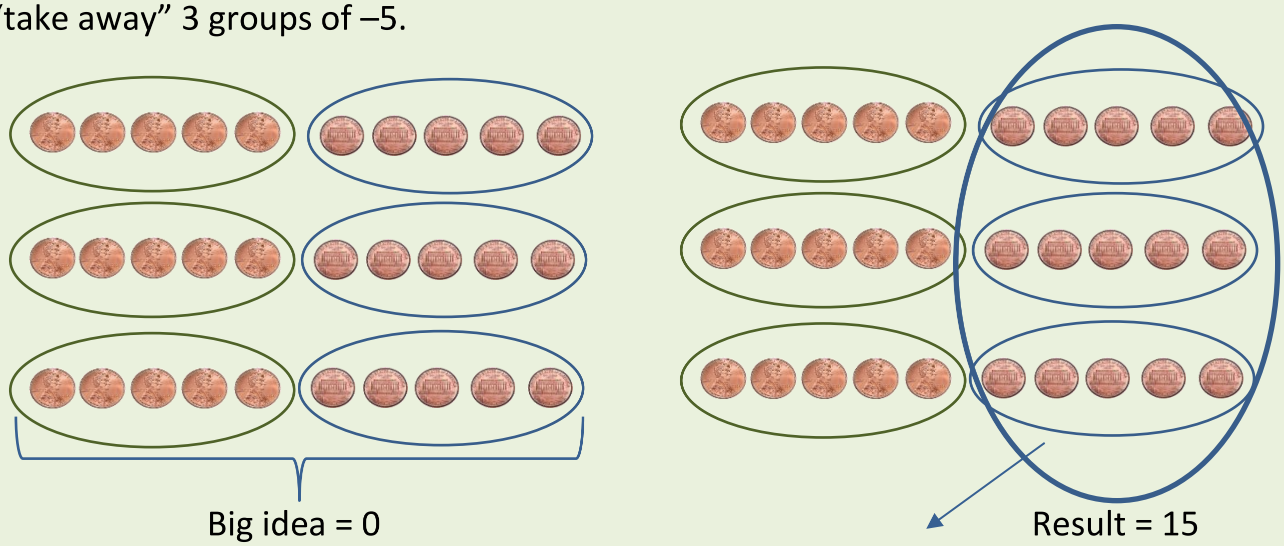
Result -15

Way 2:

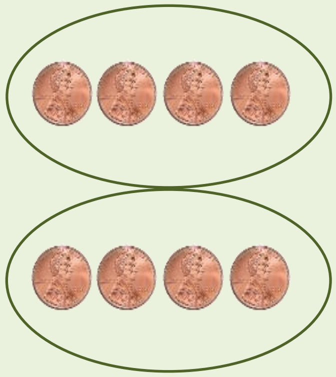
–3 ∗ 5 means "take away 3 groups of 5." To do this, you must use a complicated form of zero, namely 3 groups of 5 and 3 groups of –5. Now you can "take away" 3 groups of 5.



–2(–3) ∗ (–5) = This problem corresponds to the second way to solve –3 ∗ 5, using a complicated form of zero, , namely 3 groups of 5 and 3 groups of –5. Now you can "take away" 3 groups of –5.

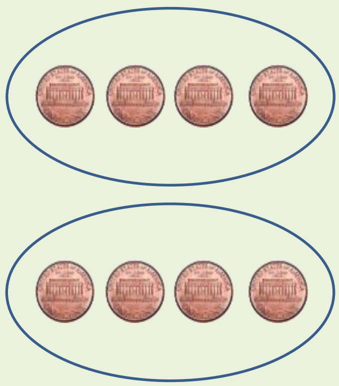


2 ∗ 4 means: put on 2 groups of 4



Result 8

2 ∗ (–4) means: put on 2 groups of –4

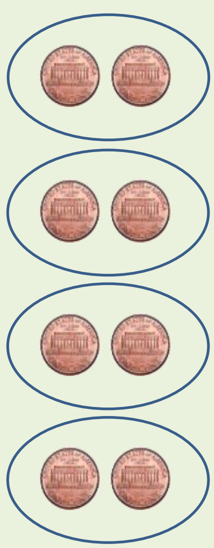


Result –8

–2 ∗ 4 can be done two different ways.

Way 1:

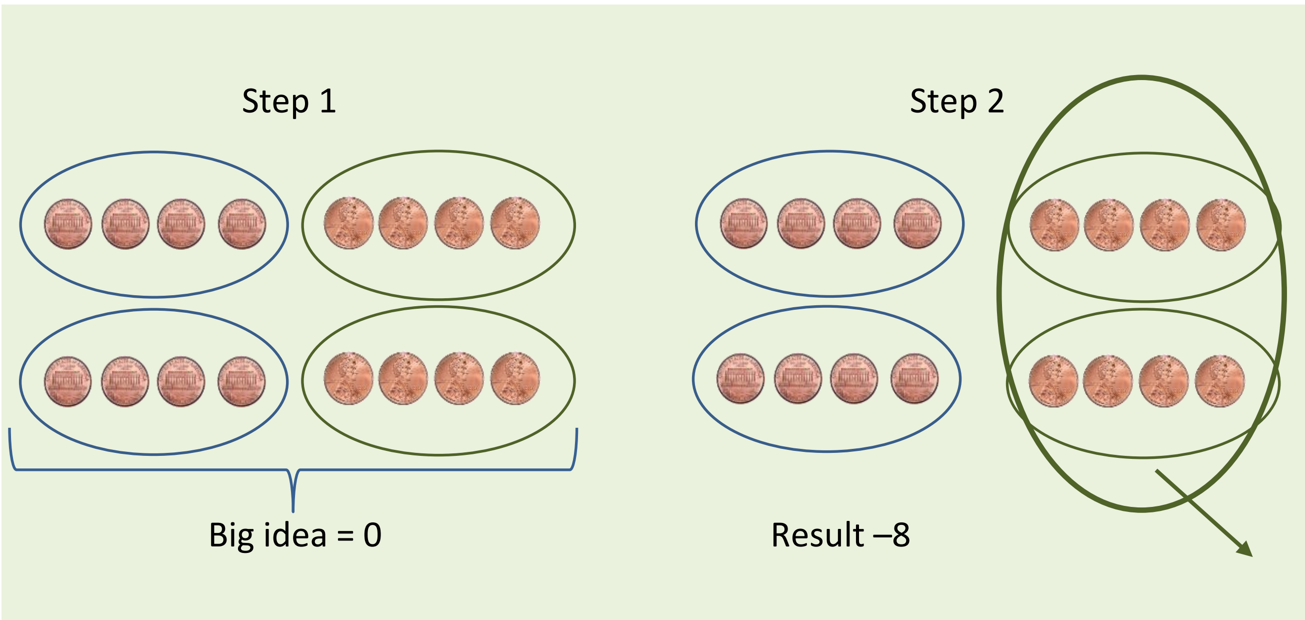
–2 ∗ 4 = 4∗ (–2), so put on 4 groups of –2



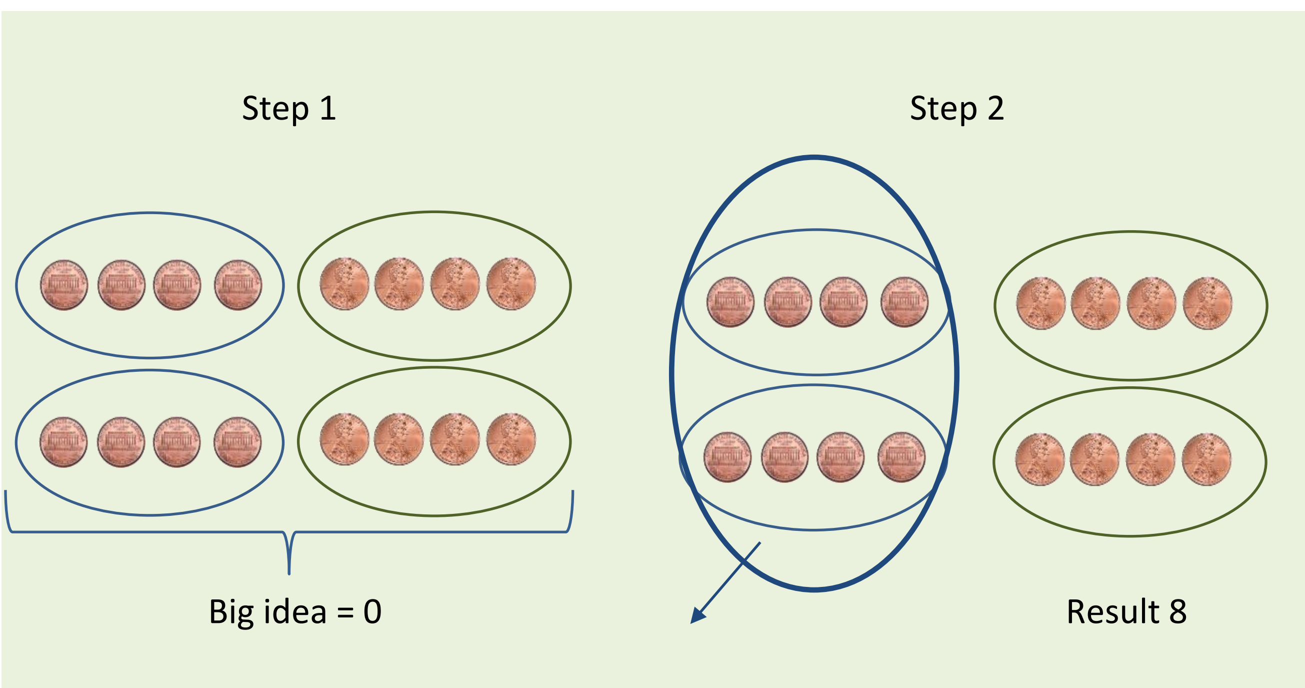
Result –8

Way 2:

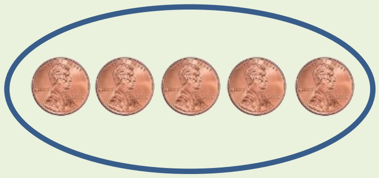
−2 ∗ 4 means “take away 2 groups of 4.” To do this, you must use a complicated form of zero, namely 2 groups of 4 and 2 groups of –4. Then they can “take away” 2 groups of 4.



(–2) ∗ (–4) = means “take away 2 groups of –4. To do this, you must use a complicated form of zero, , namely 2 groups of 4 and 2 groups of –4.
Now you can “take away” 2 groups of –4.

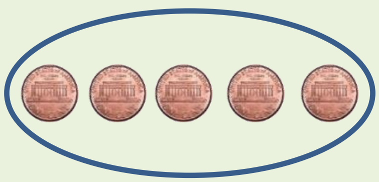


$1 * 5$ means: put on 1 group of 5



Result 5

$1 * (-5)$ means: put on 1 group of -5



Result -5

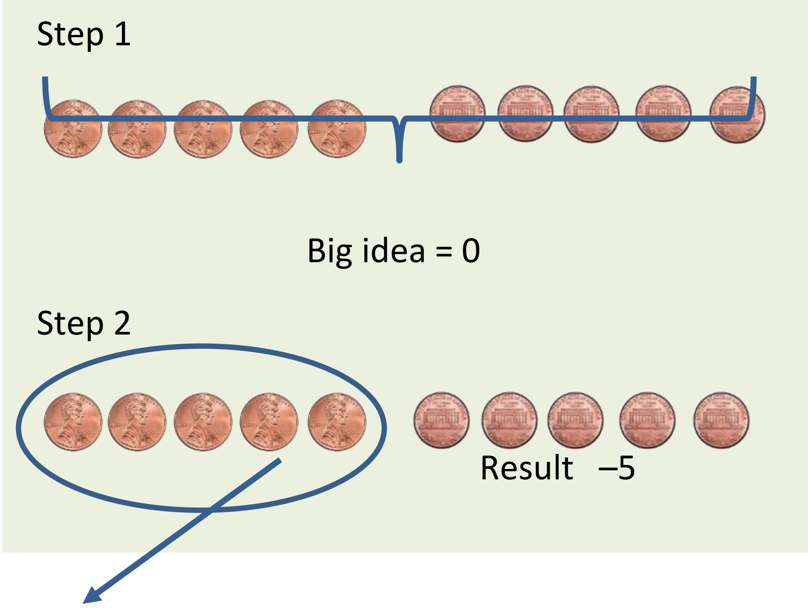
$-1 * 5$ means: Do this two ways.

Way 1:
$-1 * 5 = 5 * (-1)$, so put on 5 groups of -1

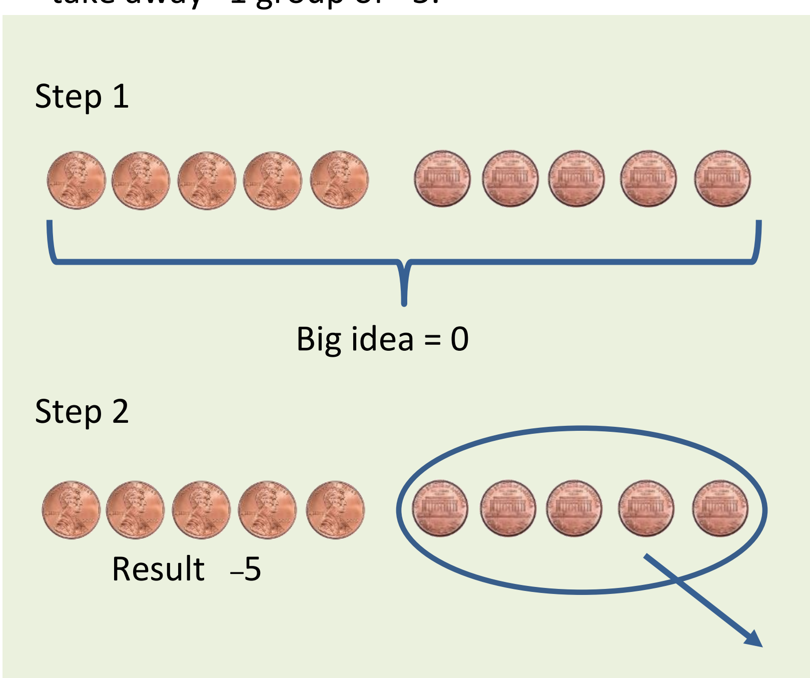
Result -5

Way 2:

$-1 * 5$ means "take away 1 groups of 5." To do this, you must use a complicated form of zero, namely 1 group of 5 and 1 group of –5. Then they can "take away" 1 group of 5.



$(-1) * (-5) =$ This problem corresponds to the second way to solve $-1 * 5$, using a complicated form of zero, , namely 1 group of 5 and 1 group of –5. Now you can "take away" 1 group of –5.



Practice 13: Multiplication Modeled with Pennies

Symbols	Words	Picture	Result
2 ∗ 4	put on 2 groups of 4		8
2 ∗ 5	Put on 2 groups of 5		10
2 ∗ (–4)	Put on 2 groups of –4		–8
1 ∗ (–7)	Put on one group of –7		–7
–2 ∗ 5	Take off 2 groups of 5	Start with 0 This is also 0 H H H H H T T T T T H H H H H T T T T T Now take away 2 groups of 5 H H H H H T T T T T H H H H H T T T T T Result is 2 groups of –5, or –10 –2 ∗ 5 = –10 Or –2 ∗ 5 = 5 ∗ –2 Put on 5 groups of -2 T T T T T T T T T T –2 ∗ 5 = 5 ∗ –2 = 10	–10

–2 ∗ 6	Take off 2 groups of 6	Start with 0 This is also 0 H H H H H H T T T T T T H H H H H H T T T T T T Now take away 2 groups of 6 H H H H H H T T T T T T H H H H H H T T T T T T Result is 2 groups of –6, or –12 –2 ∗ 6 = –12 Or –2 ∗ 6 = 6 ∗ –2 Put on 6 groups of -2 T T T T T T T T T T T T –2 ∗ 6 = 6 ∗ –2 = 12	–12
–2 ∗ (–5)	Take off 2 groups of –5	Start with 0 This is also 0 H H H H H T T T T T H H H H H T T T T T Now take away 2 groups of –5 H H H H H T T T T T H H H H H T T T T T Result is 2 groups of 5, or 10 –2 ∗ (–5) = 10	10
–4 ∗ (–3)	Take off 4 groups of –3	Start with 0 This is also 0 H H H T T T H H H T T T H H H T T T H H H T T T Now take away 4 groups of –3 H H H T T T H H H T T T H H H T T T H H H T T T Result is 4 groups of 3, or 12 –4 ∗ (–3)	12

Symbols	Words	Picture	Result
7 ∗ 4	Put on 7 groups of 4	H H H H H H H H H H H H H H H H H H H H H H H H H H H H	28
0 ∗ (–5)	Put on 0 (zero, none) groups of –5		0
–3 ∗ 2	Take off 3 groups of 2	HH TT HH TT HH TT	–6
5 ∗ (–4)	Put on 5 groups of –4	T T T T T T T T T T T T T T T T T T T T	–20
–1 ∗ (–3)	Take off 1 group of –3	H H H T T T	3
–8 ∗ 2	Take off 8 groups of 2	H H H H T T T T H H H H T T T T H H H H T T T T H H H H T T T T	–16
–2 ∗ (–3)	Take off 2 groups of –3	H H H T T T H H H T T T	6
–4 ∗ (–2)	Take off 4 groups of –2	H H T T H H T T H H T T H H T T	8

Optional Practice for a-b = a+(-b)

You can use pennies to model each of these problems if you want. Draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
6 – 14 = H H H H H H H H H H H H H H H H H H H H T T T T T T T T	6 + (–14) = –8 H H H H H H T T T T T T T T T T T T T T	6 – 14 = 6 + (–14) = –8
7 – 11 = H H H H H H H H H H H T T T T	7 + (–11) H H H H H H H T T T T T T T T T T T	7 – 11 = 7 + (–11) + –4
6 – 9 = H H H H H H H H H T T T	6 + (–9) H H H H H H T T T T T T T T T	6 – 9 = 6 + (–9) = –3
8 – 15 = H H H H H H H H H H H H H H H H H T T T T T T T	8 + (–15) H H H H H H H H T T T T T T T T T T T T T T T	8 – 15 = 8 + (–15) = –7
3 – 12 H H H H H H H H H H H H H H T T T T T T T T T	3 + (–12) = H H H T T T T T T T T T T T T	3 – 12 = 3 + (–12) = –9
4 – 13 H H H H H H H H H H H H H H T T T T T T T T T	4 + (–13) = H H H H T T T T T T T T T T T T T	4 – 13 = 4 + (–13) = –9
4 –5 = H H H H H T	4 + (–5) H H H H T T T T T	4 – 5 = 4 + (–5) = –1
5 – 12 = H H H H H H H H H H H H H H H T T T T T T T	5 + (–12) H H H H H T T T T T T T T T T T T	5 – 12 = 5 + (–12) = –7

Optional Practice for a-b = a+(-b)

You can use pennies to model each of these problems if you want. Draw the H's and T's. Fill in the other blanks.

a – b =	a + (–b) =	a – b = a + (–b) =
12 – 14 = –2	12 + (–14) = –2	12 – 14 = 12 + (–14)
7 – 18 = –11	7 + (–18) = –11	7 – 18 = 7 + (–18)
6 – 15 = –9	6 + (–15) = -9	6 – 15 = 6 + (–15)
1 – 6 = –5	1 + (-6) = –5	1 – 6 = 1 + (–6)
2 – 5 = –3	2 + (–5) = –3	2 – 5 = 2 + (–5)
5 – 11 = –6	5 + (–11) = –6	5 – 11 = 5 + (–11)
12 – 14 = –2	12 + (–14) = –2	12 – 14 = 12 + (–14)
11 – 14 = –3	11 + (–14) = –3	11 – 14 = 11 + (–14)

7. Help for Helpers

I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as "I know you can do this. I believe in you." And then leave the student alone to do the work.

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn't going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don't generally want to ask our boss or friend: "Am I right? Am I right?" As an adult, we have to know the answer is right ourselves. The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student's life. You will gain confidence in your students as you watch them be successful on their own.

The overall goal of this lesson collection is for the student to understand the abstract concept of negative numbers.

In real life, negative numbers can be useful as a way to look at owing money.

But the real reason to study negative numbers is that they are a critical part of algebra.