MATH WHISPERER

Where math makes sense

Research Edition

Number Sense Series

Fractions 2

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Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information, please go to www.mathwhisperer.com

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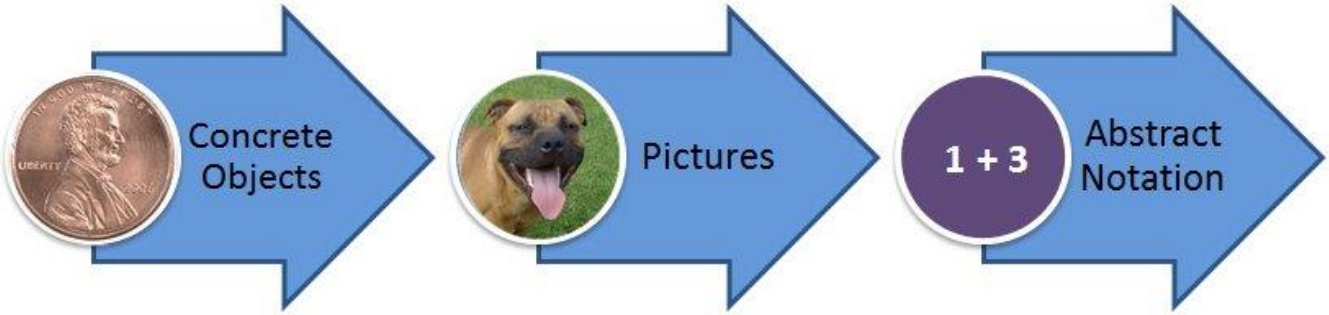
DEDICATION

Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

Contents

1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is actually supposed to make sense. When you start with hands-on objects, math can make sense.



You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three step progression works for everybody.

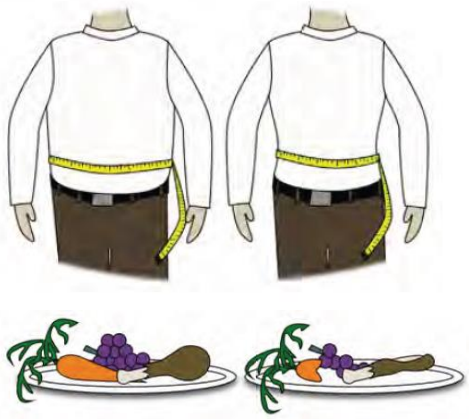
It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. Won't it feel great to never have to learn this again? The math will stick with you with the three step progression.

Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you.

2. Equivalent Fractions, Part Two

In Part 1 you learned a lot about equivalent fractions — what they mean and two ways to find them. It turns out there is a third way to find equivalent fractions, and that is the way that Einstein did. It's the way all the world's best mathematicians and scientists find equivalent fractions. It uses multiplication of fractions, and you are ready for it!

We'll start with the new word phrases first. (The words are my least favorite part. I'd rather be doing it, compared to talking about. But we do need the words so that we are talking about the same thing.)



Reducing a fraction means to find an equivalent fraction that has smaller numbers. It is like a person who loses weight and **reduces** his or her size. It's the same person, just smaller.

Another example of the word **reduced** is in the phrase "reduced portions." This means the same food, but smaller amounts.

In the reduced weight, it's the same person. In the reduced portions, it's the same food.

$\frac{1}{2}$ is a reduced version of $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, and so many more.

$\frac{1}{3}$ is a reduced version of $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, and so many more.

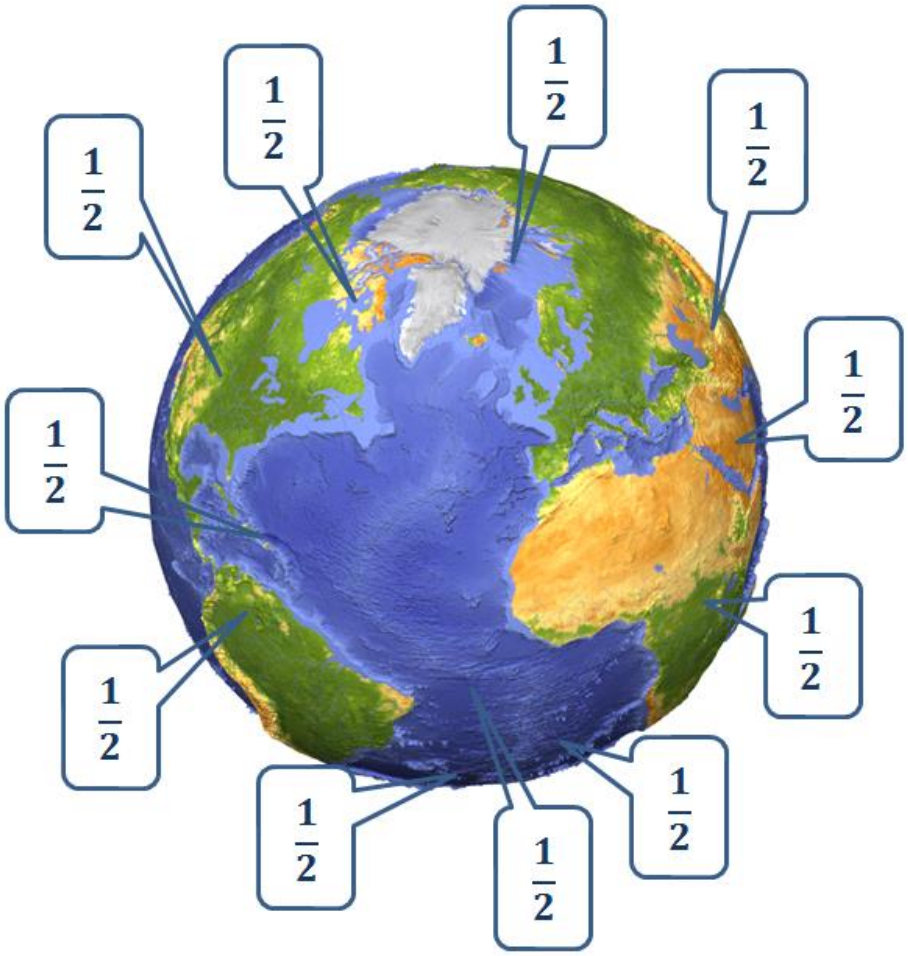
Do you see the pattern? In each case, the reduced fraction has the smallest denominator and numerator.

And that leads us to the next term, the **lowest form**.

$\frac{1}{2}$ is the lowest form of $\frac{1}{2}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{50}{100}$ etc. There is no way to write $\frac{1}{2}$ with any smaller numbers.

It is sort of amazing that all professional mathematicians, scientists, engineers, statisticians, and everybody else who does math have agreed to ALWAYS put their fractions into lowest terms when they give the answers. There is something amazing that everyone would agree to that. They do have a good reason to do that – it is for purposes of communication.

What if one person wrote the result as $\frac{50}{100}$, and another was talking about the same problem and wrote the answer as $\frac{20}{40}$ how would they know they actually had the same answer without having to think about it?! But $\frac{1}{2}$ is the lowest form of $\frac{50}{100}$ and $\frac{20}{40}$.



Practice 1: Reducing Fractions

a. Why is $\frac{1}{3}$ is a reduced version of $\frac{2}{6}$?

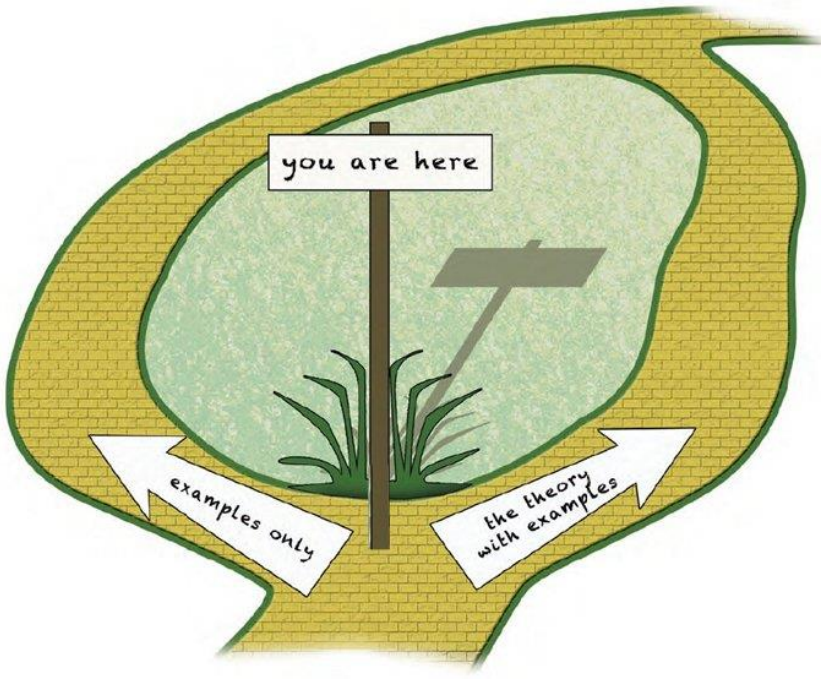
b. Why is $\frac{1}{3}$ is not a reduced version of $\frac{3}{6}$?

In these lists of equivalent fractions, which is lowest form? Explain why.

c. $\frac{7}{14}$ $\frac{1}{2}$ $\frac{3}{6}$ $\frac{5}{10}$ $\frac{2}{4}$

d. $\frac{4}{6}$ $\frac{40}{60}$ $\frac{10}{15}$ $\frac{2}{3}$ $\frac{20}{30}$

e. $\frac{2}{1}$ $\frac{4}{2}$ $\frac{10}{5}$ $\frac{8}{4}$ $\frac{20}{30}$



At this point, you have two choices to learn how to reduce fractions.
Choice one is to follow the pages into discovery learning.
The second choice is to read page 11 first, and then go back to page 9 and work through the activities.

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Practice 2: Equivalent Fractions – The Numbers get Bigger

We already know that fractions that are equivalent to 1 are $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$ etc.

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{1}{2}$	$\frac{1*3}{2*3}$	$\frac{3}{6}$
$\frac{1}{2}$	$\frac{1*8}{2*8}$	
$\frac{1}{3}$	$\frac{1*2}{3*2}$	
$\frac{2}{3}$	$\frac{2*2}{3*2}$	
$\frac{5}{2}$	$\frac{5*3}{2*3}$	
$\frac{1}{4}$	$\frac{1*3}{4*3}$	
$\frac{1}{4}$	$\frac{1*8}{4*8}$	

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{1}{2}$	$\frac{1*5}{2*5}$	
$\frac{1}{2}$	$\frac{1*10}{2*10}$	
$\frac{1}{3}$	$\frac{1*4}{3*4}$	
$\frac{2}{3}$	$\frac{2*5}{3*5}$	
$\frac{1}{5}$	$\frac{1*2}{5*2}$	
$\frac{7}{3}$	$\frac{7*2}{3*2}$	
$\frac{1}{4}$	$\frac{1*2}{4*2}$	

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Practice 3: Reducing Fractions – The Numbers get Smaller

We already know that fractions that are equivalent to 1 are $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$ etc.

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{9}{12}$	$\frac{9 \div 3}{12 \div 3}$	$\frac{3}{4}$
$\frac{8}{16}$	$\frac{8 \div 8}{16 \div 8}$	
$\frac{4}{6}$	$\frac{4 \div 2}{6 \div 2}$	
$\frac{12}{30}$	$\frac{12 \div 6}{30 \div 6}$	
$\frac{7}{14}$	$\frac{7 \div 7}{14 \div 7}$	
$\frac{21}{28}$	$\frac{21 \div 7}{28 \div 7}$	
$\frac{12}{10}$	$\frac{12 \div 2}{10 \div 2}$	

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{5}{10}$	$\frac{5 \div 5}{10 \div 5}$	
$\frac{10}{20}$	$\frac{10 \div 10}{20 \div 10}$	
$\frac{4}{12}$	$\frac{4 \div 4}{12 \div 4}$	
$\frac{10}{15}$	$\frac{10 \div 5}{15 \div 5}$	
$\frac{11}{55}$	$\frac{11 \div 11}{55 \div 11}$	
$\frac{12}{18}$	$\frac{12 \div 6}{18 \div 6}$	
$\frac{15}{9}$	$\frac{15 \div 3}{9 \div 3}$	

Reducing fractions is based on multiplication of fractions, which you just learned about.

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This explanation is pure algebra. You may or may not be able to really follow this. Please try! The bottom line here is that reducing fractions uses two big ideas — how to multiply fractions and $\frac{x}{x} = 1$. It's easier to do the problems with numbers than to understand the algebra.

Reducing fractions is based on multiplication of fractions, which you just learned about.

We have seen that to multiply fractions, the procedure is: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$

This procedure in the reverse direction is the algebraic way to find equivalent fractions. $\frac{a * c}{b * d} = \frac{a}{b} * \frac{c}{d}$

In the special case of c = d $\frac{a * c}{b * d} = \frac{a}{b} * \frac{c}{d} = \frac{a}{b} * \frac{c}{c}$, since c + d

and here is what is so fantastic: $\frac{c}{c} = 1$!!!

I will show you two examples, and then it's your turn. So pay close attention!

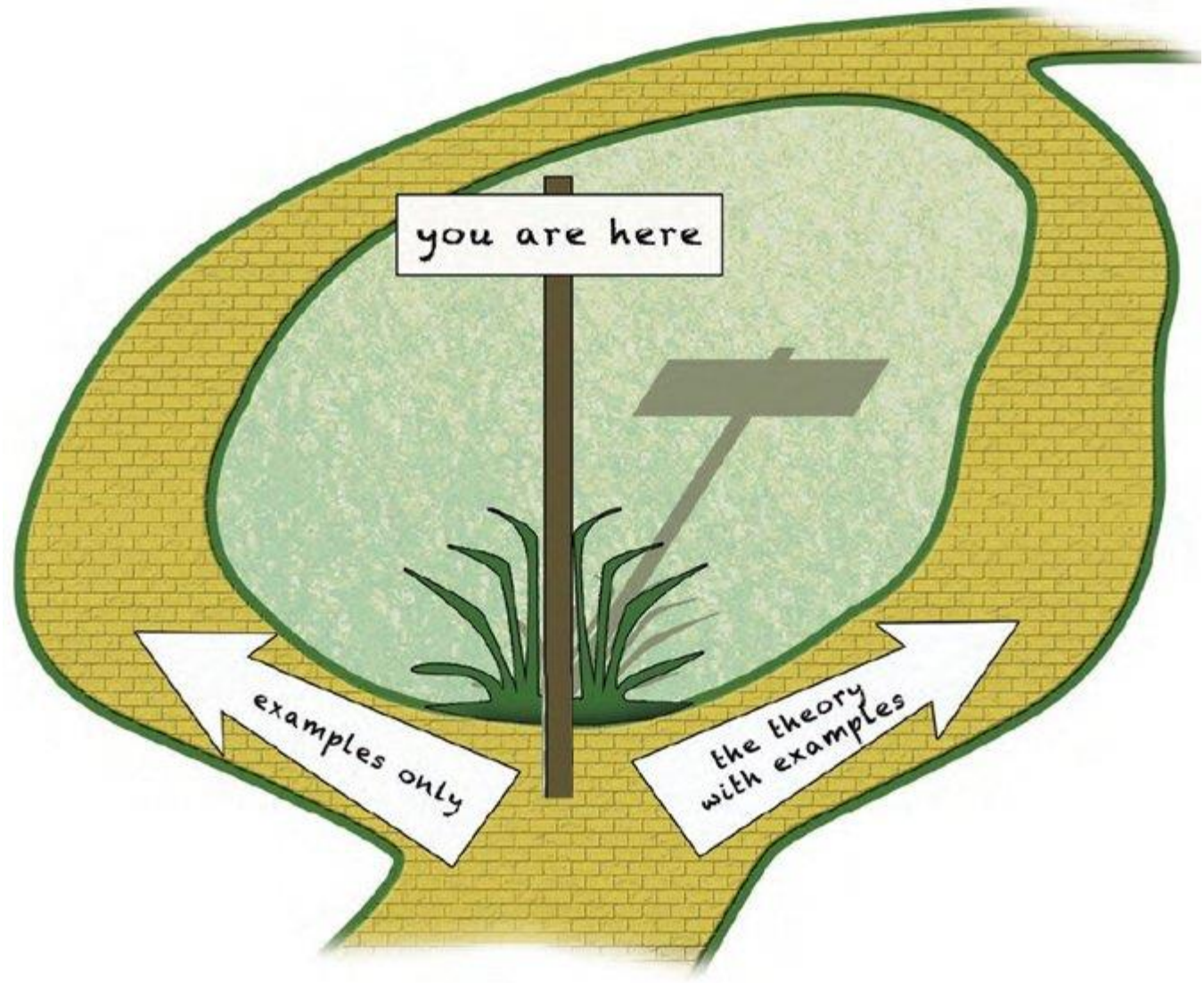
Example 1: Reduce $\frac{12}{33}$ $\frac{12}{33} = \frac{2 * 2 * 3}{3 * 11} = \frac{2 * 2 * \cancel{3}}{\cancel{3} * 11} = \frac{2 * 2}{11} = \frac{4}{11}$

Factor into Primes — Look for x / x = 1 — Reduced fraction guaranteed to be in lowest terms!!

Example 2: Reduce $\frac{12}{36}$

$\frac{12}{36} = \frac{2 * 2 * 3}{2 * 2 * 3 * 3} = \frac{\cancel{2} * \cancel{2} * \cancel{3}}{\cancel{2} * \cancel{2} * \cancel{3} * 3} = \frac{\cancel{2} * \cancel{2} * \cancel{3} * 1}{\cancel{2} * \cancel{2} * \cancel{3} * 3} = \frac{1}{3}$

Factor into Primes — Look for $\frac{x}{x} = 1$ — No factors left. Use a magic 1 (1*x = x) — Reduced fraction guaranteed to be in lowest terms!!



This is a great place to learn about yourself as a learner. Which path did you take? Did you prefer the examples and then the theory made more sense, or did you prefer to start with the theory and then the examples had more meaning? Engineers, lawyers, and doctors generally prefer the examples first. Mathematicians, physicists and university professors generally prefer the theory first.

Practice 4: Explain Each Step in Reducing This Fraction

$$\frac{44}{99}$$

$$= \frac{2 * 2 * 11}{3 * 3 * 11}$$

$$= \frac{2 * 2 * \cancel{11}}{3 * 3 * \cancel{11}}$$

$$= \frac{2 * 2}{3 * 3}$$

$$= \frac{4}{9}$$

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Practice 5: Reducing Fractions to Lowest Form

Reduce each fraction to simplest form. Please show every step. It is the process I want you to practice. "Seeing" the answer is not the goal. The goal is to practice the process so that you can apply it in cases when you can't see the answer. (For example, 111/111300 and more!)

a. $\frac{12}{24}$
b. $\frac{4xy}{8y}$
c. $\frac{15}{50}$
d. $\frac{7abc}{14bcw}$
e. $\frac{42}{33}$
f. $\frac{102}{3}$
g. $\frac{24}{12}$
h. $\frac{18ghw}{72ghw}$

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i. $\frac{30}{24}$
j. $\frac{6ab}{8abc}$
k. $\frac{60q}{50}$
l. $\frac{21acw}{14bcw}$
m. $\frac{121}{33}$
n. $\frac{60}{30}$
o. $\frac{55ab}{10}$
p. $\frac{24f}{72}$
q. $\frac{25uvw}{20ghw}$

Practice 6: Reduced Form or Not?

Decide which fractions are in reduced form and which are not. If a fraction is in reduced form, write "RF". If it is not in reduced form, show the magic ones that divide into it and put a box around the final reduced form.

$\frac{9}{12}$	$\frac{6}{13}$	$\frac{5}{10}$
$\frac{5}{15}$	4	$\frac{5}{8}$
$\frac{8}{15}$	$\frac{3}{9}$	$\frac{6}{18}$
$\frac{24}{48}$	$\frac{6}{24}$	$\frac{8}{24}$
$\frac{7}{21}$	$\frac{12}{21}$	$\frac{13}{13}$

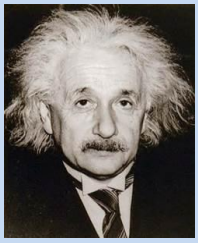
Write four fractions not already on this paper that are in reduced form:

Write four fractions not already on this paper that are not in reduced form:

Answers page 66

3. Multiplying Fractions: Part 2

You are now going to learn how to multiply fractions the way Einstein did. And you are ready! It's just one more step to what you already know. Make sure you have completed #5, Equivalent fractions part 2.



The basic idea of this most excellent way to multiply fractions is that you keep the numbers small until you absolutely have to make them larger by multiplication.

We figured out earlier that to multiply fractions, you do this:

$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

This is absolutely true. But that is not the entire story. Today's lesson is about multiplying more complicated numbers than we have used before.

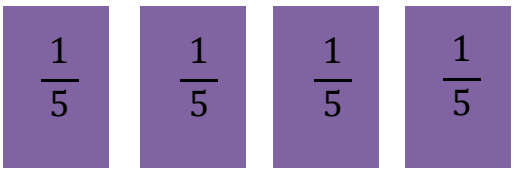
For a calculation like $\frac{1}{2} * \frac{1}{4} = \frac{1 * 1}{2 * 4} = \frac{1}{8}$ it works like a charm.

This next problem has a twist, in that the answer is not in reduced form. Everyone around the world has agreed to express all answers in reduced form, so we have to reduce.

$$\frac{1}{2} * \frac{4}{5} = \frac{1 * 4}{2 * 5} = \frac{4}{10}$$

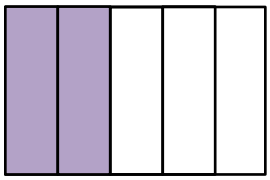
Using the meaning of multiplication $\frac{1}{2} * \frac{4}{5}$ *means* $\frac{1}{2}$ of $\frac{4}{5}$

If each of these fraction pieces is $\frac{1}{5}$, we can see that $\frac{1}{2}$ of them is $\frac{2}{5}$.

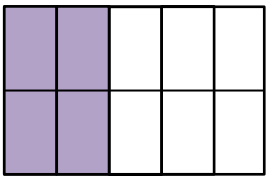


So, $\frac{1}{2} * \frac{4}{5} = \frac{2}{5}$ *and also* $\frac{1}{2} * \frac{4}{5} = \frac{4}{10}$ Two answers but $\frac{4}{10}$ is not in lowest terms.

So we know the answer is correct.

$\frac{4}{10} = \frac{2}{5}$ If this is one  this is $\frac{2}{5}$ 

There are two algebraic ways to show this

Method A: $\frac{1}{2} * \frac{4}{5} = \frac{1*4}{2*5} = \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$ 

Multiply here

Divide here

Method B: $\frac{1}{2} * \frac{4}{5} = \frac{1*4}{2*5} = \frac{1*2*2}{2*5} = \frac{1*2*2}{2*5} = \frac{1*2}{5} = \frac{2}{5}$

Multiply here

Factor here (using $\frac{x}{x} = 1$)

I want to summarize why **Method B works so much better than Method A.**

In Method A, you multiply the numerator and denominator right away. Then you are stuck with having to reduce the fraction. You reduce by dividing out common factors. You have large numbers and you keep dividing. It's easy to make a mistake. Method B is easier. It used the fact that multiplication and division are related.

Here is an example of a multiplication and division fact family:
$\mathbf{2 * 3 = 6,\ 3 * 2 = 6, 6 \div 2 = 3 \text{ and } 6 \div 3 = 2.}$

Multiplication and division undo each other. In Method A, you have multiplied and then you divide. (It's like throwing mud on your car before you wash it—extra work.)

The advantage of factoring, and not immediately multiplying, is more clear with this next example:

$$\frac{12}{17} * \frac{34}{33} = \frac{12 * 34}{17 * 33}$$

I'm depressed already. I am not going to multiply these numbers—too many chances to make a calculation error. I'll factor first.

$$= \frac{2 * 2 * 3 * 2 * 17}{17 * 3 * 11}$$

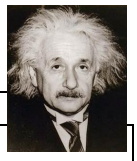
$$= \frac{2 * 2 * \cancel{3} * 2 * \cancel{17}}{\cancel{17} * \cancel{3} * 11}$$ Now I'll look for $\frac{x}{x} = 1$ $\frac{3}{3} = 1$ and $\frac{17}{17} = 1$

$$= \frac{2 * 2 * 2}{11}$$

$$= \frac{8}{11}$$ The perfect answer—reduced.

The way to really learn this method is to practice it. Here goes:

Practice 7: Multiply Fractions the Way Einstein Did



a. $\frac{12}{55} * \frac{34}{33} = \frac{2 * 2 * 3 * 3 * 11}{5 * 11 * 2 * 2} = \frac{3 * 3}{5} = \frac{9}{5}$

b. $\frac{17}{3} * \frac{3}{11} =$

c. $\frac{8}{25} * \frac{30}{7} =$

d. $\frac{9}{10} * \frac{5}{3} =$

e. $\frac{9}{10} * \frac{3}{5} =$

f. $\frac{1}{7} * \frac{14}{15} =$

g. $\frac{22}{23} * \frac{46}{11} =$

h. $\frac{1}{3} * \frac{6}{5} =$

i. $\frac{12}{13} * \frac{39}{4} =$

Answers page 67

j. $\frac{2}{15} * \frac{3}{5} =$
k. $\frac{2}{15} * \frac{5}{3} =$
l. $\frac{1}{3} * \frac{9}{8} =$
m. $\frac{2}{3} * \frac{9}{8} =$
n. $\frac{4}{3} * \frac{9}{8} =$
o. $\frac{1}{5} * \frac{10}{3} =$
p. $\frac{2}{5} * \frac{3}{10} =$
q. $\frac{2}{5} * \frac{10}{3} =$
r. $\frac{2}{5} * \frac{30}{11} =$

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s. $\frac{14}{15} * \frac{18}{7} =$
t. $\frac{17}{18} * \frac{9}{34} =$
u. $\frac{12}{13} * \frac{39}{4} =$
v. $\frac{12}{7} * \frac{28}{3} =$
w. $\frac{20}{21} * \frac{7}{4} =$
x. $\frac{20}{21} * \frac{7}{10} =$
y. $\frac{5}{12} * \frac{3}{10} =$
z. $\frac{6}{7} * \frac{14}{15} =$
aa. $\frac{11}{15} * \frac{30}{44} =$

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bb. $\frac{6}{55} * \frac{11}{10} =$
cc. $\frac{21}{23} * \frac{46}{7} =$
dd. $\frac{91}{12} * \frac{1}{3} =$
ee. $\frac{12}{91} * \frac{1}{3} =$
ff. $\frac{21}{23} * \frac{36}{7} =$
gg. $\frac{242}{5} * \frac{100}{121} =$
hh. $\frac{77}{5} * \frac{15}{22} =$
ii. $\frac{60}{11} * \frac{33}{40} =$
jj. $\frac{55}{2} * \frac{6}{11} =$

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Magic Ones – Very Useful for Work with Fractions

Magic Ones #1: $\frac{x}{x} = 1$

Ones are truly magic in math. They help us all over the place, including multiplying fractions.

$\frac{x}{x} = 1$ This is like putting a one into your pocket.

$$\frac{3}{2} * \frac{2}{5} = \frac{3 * 2}{2 * 5}$$

1

$$= \frac{3}{\cancel{2}} * \frac{\cancel{2}}{5} = \frac{3}{5} * \frac{\mathbf{2}}{\mathbf{2}} = \frac{3}{5} * 1$$ Using $\frac{\mathbf{2}}{\mathbf{2}} = 1$

Magic Ones #2: $x * 1 = x$

$x * 1 = x$ This is like pulling a one out of your pocket when you need it.

$$\frac{2}{3} * \frac{3}{10} = \frac{2 * 3}{3 * 10}$$

$$= \frac{\cancel{2} * \cancel{3}}{\cancel{3} * 2 * 5}$$

$$= \frac{\cancel{2} * \cancel{3} * 1}{\cancel{3} * \cancel{2} * 5}$$

There is nothing left in the numerator.
So we pull a "1" out of our pocket
using $x = x * 1$

Now we use $\frac{x}{x} = 1$

$= \frac{1}{5}$ Woo hoo!

Magic Ones #3: $\boldsymbol{x = \frac{x}{1}}$

Sometimes it's useful to pull a 1 out of your pocket.

4 is the same as $\frac{4}{1}$.

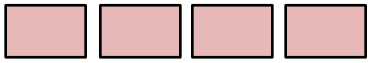
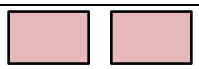
Do you believe this?

$\frac{4}{1}$ in word is 4 one tenths.

4 one tenths is 4.

Practice 8: Magic Ones $x = \frac{x}{1}$

Fill in the blanks. This is one. ☐

$\frac{4}{1}$		4
$\frac{3}{1}$		
$\frac{6}{1}$		
		2
		5
		7
		
		

Let's look at what you found.

$\frac{2}{1} = 2$

$\frac{3}{1} = 3$

$\frac{4}{1} = 4$

$\frac{5}{1} = 5$

And so on.

Does $\frac{x}{1} = x$?

Why or why not?

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Practice 9: Magic Ones - use $x * 1 = x$

a. $\frac{3}{28} * \frac{4}{3} =$	$\frac{3}{28} * \frac{4}{3} = \frac{3 * 2 * 2 * 1}{2 * 2 * 7 * 3} = \frac{1}{7}$
b. $\frac{2}{45} * \frac{15}{2} =$	
c. $\frac{2}{7} * \frac{7}{6} =$	
d. $\frac{2}{7} * \frac{7}{8} =$	
e. $\frac{5}{14} * \frac{7}{15} =$	
f. $\frac{22}{39} * \frac{13}{44} =$	
g. $\frac{3}{14} * \frac{7}{6} =$	
h. $\frac{3}{22} * \frac{11}{6} =$	

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i. $\frac{8}{45} * \frac{5}{24} =$
j. $\frac{3}{11} * \frac{11}{6} =$
k. $\frac{12}{55} * \frac{5}{24} =$
l . $\frac{12}{55} * \frac{11}{24} =$
m. $\frac{3}{55} * \frac{11}{3} =$
n. $\frac{3}{55} * \frac{11}{6} =$
o. $\frac{4}{15} * \frac{5}{12} =$
p. $\frac{4}{27} * \frac{3}{8} =$
q. $\frac{5}{21} * \frac{3}{20} =$

Answers page 73

Practice 10: Magic Ones - use $\frac{x}{1} = x$

This is another example of pulling one out of your pocket.

a. $6 * \frac{1}{2} = \frac{6}{1} * \frac{1}{2} = \frac{6*1}{1*2} = \frac{6}{2} = \frac{2*3}{2*1} = \frac{2*3}{2*1} = \frac{2*3}{2*1} = \frac{3}{1} = 3$

b. $3 * \frac{1}{2} =$

c. $3 * \frac{1}{3} =$

d. $5 * \frac{1}{5} =$

e. $3 * \frac{1}{5} =$

f. $5 * \frac{1}{3} =$

g. $6 * \frac{2}{3} =$

Answers page 74

h. $6 * \frac{2}{5} =$
i. $5 * \frac{2}{5} =$
j. $3 * \frac{x}{2} =$
k. $3 * \frac{x}{3} =$
l. $3 * \frac{x}{5} =$
m. $15 * \frac{x}{5} =$
n. $15 * \frac{x}{3} =$
o. $15 * \frac{x}{2} =$
p. $15 * \frac{x}{7} =$

q. $14 * \frac{x}{28} =$
r. $12 * \frac{3}{4} =$
s. $14 * \frac{4}{3} =$
t. $\frac{3}{5} * 6 =$
u. $\frac{3}{5} * 10 =$
v. $\frac{2}{5} * 4 =$
w. $\frac{2}{5} * 20 =$
x. $\frac{7}{8} * 16 =$
y. $\frac{7}{8} * 24 =$

z. $\frac{7}{8} * 3 =$
aa. $5 * \frac{3}{7} =$
bb. $21 * \frac{3}{7} =$
cc. $21 * \frac{2}{7} =$
dd. $17 * \frac{1}{34} =$
ee. $11 * \frac{1}{121} =$
ff. $11 * \frac{1}{242} =$
gg. $w * \frac{4}{w} =$
hh. $q * \frac{5}{q} =$

4. Addition and Subtraction of Fractions

Adding whole numbers (like 1, 2, 3 and so on) is easier than multiplying them. But with fractions, it is easier to multiply than to add. In fact, to add fractions in general, you have to use multiplication of fractions.

These examples show why adding fractions can be so difficult. The good news is that after this lesson, you will understand completely how to add fractions. Ready to start?

2 apples + 3 apples = _____

2 apples + 3 oranges = _____

2 apples + 3 washing machines = _____

What does it mean to add?

Here is what I hope you were able to see from these examples:

Adding means to take things that are the same and count them.

2 apples + 3 apples = 5 apples

2 apples + 3 oranges = 5 pieces of fruit
Note we had to make the units or labels (the oranges and apples) the same (pieces of fruit).

2 apples + 3 washing machines can't really be added.
They don't have anything in common.

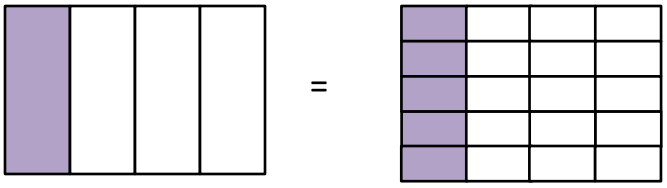
(We could say 5 things, but that isn't really descriptive.)
Let's take what you just learned and apply it to fractions.

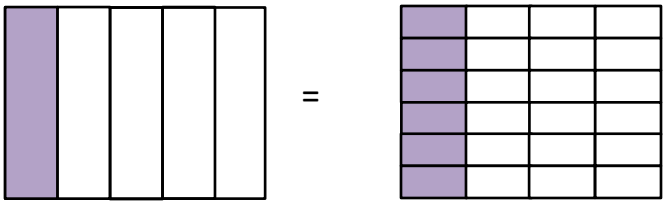
Example 1:
1 fourth + 2 fourths = 3 fourths. This can be added as the units/labels are the same.

Example 2:
1 fourth + 1 fifth cannot be added as is. The units/labels are different.

To add them we have to make the units/labels the same.

This is like the apples and oranges problem.

1 fourth = 5 twentieths 

1 fifth = 4 twentieths 

So, 1 fourth + 1 fifth = 5 twentieths and 4 twentieths = 9 twentieths.

Practice 11: Adding Fractions, Using our Fraction Pieces

In the activity, ☐ = one.

Symbols	Names	Pictures	Sum
$\frac{1}{4}+\frac{2}{4}$	one fourth + two fourths		three fourths
$\frac{1}{2}+\frac{3}{2}$			
	one sixth + two sixths		
	five sixths + one sixth		
$\frac{1}{3}+\frac{1}{3}$			
$\frac{1}{4}+\frac{1}{4}$			
$\frac{1}{5}+\frac{2}{5}$			
	one seventh + three sevenths		
$\frac{3}{8}+\frac{2}{8}$			
	one seventh + five sevenths		
	three tenths + seven tenths		
$\frac{1}{3}+\frac{2}{3}$			

Here is the important thing to notice in these problems:

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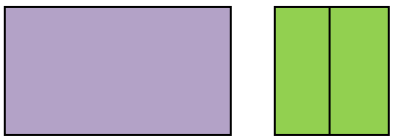
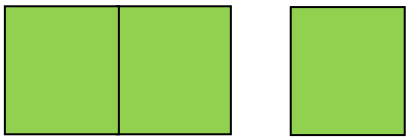
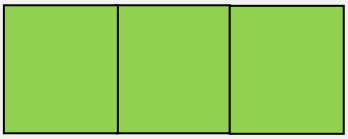
All of them had units or labels that were the same. We can add one fourth and two fourths, because they have the same unit/label.

But what about this problem? $\frac{1}{2} + \frac{1}{4} = ?$

One half + one fourth can't be added like this, because the labels/units (half, fourths) are different.

This is the same problem as one apple + one orange. We add them using the same idea—find a label or unit that is the same:

Can't add as is:	
1 half + 2 eighths 	1 daisy + 2 roses 
Can add as is:	
2 fourths + 1 fourth 	1 flower + 2 flowers 
Change labels:	
3 fourths 	3 flowers 

Practice 12: Addition of Fractions with Different Units/Labels

In Symbols	Labels in Words	Labels/Units for Sum
$\frac{1}{2} + \frac{3}{2} =$	1 half + 3 halves	halves
$\frac{1}{4} + \frac{3}{4} =$		
$\frac{1}{5} + \frac{3}{5} =$		
$\frac{1}{2} + \frac{1}{4} =$		
$\frac{1}{3} + \frac{1}{6} =$		
$\frac{2}{3} + \frac{2}{6} =$		

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In Symbols	Labels in Words	Labels/Units for Sum
$\frac{1}{2} + \frac{1}{5} =$	1 half + 3 halves	halves
$\frac{1}{6} + \frac{1}{2} =$		
$\frac{3}{4} + \frac{1}{2} =$		
$\frac{2}{3} + \frac{1}{6} =$		
$\frac{1}{6} + \frac{1}{6} =$		
$\frac{3}{3} + \frac{1}{6} =$		
$\frac{2}{5} + \frac{2}{5} =$		
$\frac{5}{5} + \frac{1}{5} =$		

Practice 13: First Addition of Fractions - Explain

a. Explain how to add this problem: $\frac{2}{3} + \frac{2}{3} =$

b. Explain how to add this problem: $\frac{1}{4} + \frac{1}{8} =$

c. Explain how to add this problem: $\frac{1}{3} + \frac{1}{2} =$

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The Algebraic Form of Addition of Fractions

We have seen that:

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

Etc.
In algebraic form, we have seen that:

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

In words, to add two fractions, they have to have the same denominator (b) as well as the same unit/labels.

Practice 14: Using Algebra to Add Fractions

Use: $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$	Find equivalent fraction if necessary	a = ?	b = ?	c = ?	Sum (show equation) $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
$\frac{1}{2}+\frac{3}{2}=$	none necessary	1	2	3	$\frac{1}{2}+\frac{3}{2}=\frac{1+3}{2}=\frac{4}{2}$
$\frac{1}{4}+\frac{3}{4}=$					
$\frac{1}{5}+\frac{3}{5}=$					
$\frac{1}{2}+\frac{1}{4}=$	$\frac{1}{2}=\frac{2}{4}$ $\frac{2}{4}+\frac{1}{4}$	2	4	1	$\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$
$\frac{1}{3}+\frac{1}{6}=$					
$\frac{2}{3}+\frac{2}{6}=$					
$\frac{1}{2}+\frac{1}{5}=$					

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Use: $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b}$	Find equivalent fraction if necessary	a = ?	b = ?	c = ?	Sum (show equation) $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
$\frac{1}{6}+\frac{1}{2}=$					
$\frac{3}{4}+\frac{1}{2}=$					
$\frac{2}{3}+\frac{1}{6}=$					
$\frac{1}{6}+\frac{1}{6}=$					
$\frac{3}{3}+\frac{1}{6}=$					
$\frac{2}{5}+\frac{2}{5}=$					
$\frac{5}{5}+\frac{1}{5}=$					
$\frac{4}{15}+\frac{3}{5}=$					

Finding Equivalent Fractions with Algebra

The only way to add fractions is to make sure they have the same denominator.

You may want to review Equivalent Fractions, part 2.

You can ALWAYS find a common denominator using equivalent fractions.

From equivalent fractions, part 2

Recall that:

$$\frac{a}{b} * \frac{x}{x} = \frac{a * x}{b * x}$$

The trick is to find the right $\frac{x}{x}$ to get the denominator you want.

How to find Common Denominators

There are **lots** of possible same denominators. Some are smaller than others. As long as you get the correct answer, your choice of denominator doesn't matter.

Example 1.

$$\frac{1}{2}+\frac{1}{4}=\left(\frac{1}{2}*\frac{2}{2}\right)+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$$

Example 2.

$$\frac{1}{2}+\frac{1}{4}=\left(\frac{1}{2}*\frac{4}{4}\right)+\left(\frac{1}{4}*\frac{2}{2}\right)=\frac{4}{8}+\frac{2}{8}=\frac{6}{8}=\frac{3*2}{4*2}=\frac{3}{4}$$

Same Answer!

Practice 15: Adding and Subtracting Fractions

Use algebra to make the fractions equivalent. Then add or subtract them as indicated.
Note: There are **lots** of possible same denominators. Some are smaller than others. As long as you get the correct answer, your choice of denominator doesn't matter.

a. $\frac{1}{2} + \frac{1}{4} =$
b. $\frac{1}{2} + \frac{1}{3} =$
c. $\frac{1}{2} - \frac{1}{4} =$
d. $\frac{1}{2} - \frac{1}{3} =$
e. $\frac{1}{2} + \frac{1}{5} =$
f. $\frac{1}{2} - \frac{1}{5} =$
g. $\frac{1}{3} + \frac{1}{4} =$

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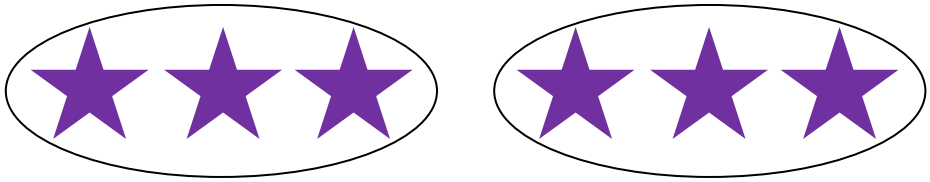
h. $\frac{1}{3} - \frac{1}{4} =$
i. $\frac{2}{3} + \frac{1}{4} =$
j. $\frac{2}{3} - \frac{1}{3} =$
k. $\frac{2}{3} - \frac{1}{5} =$
l. $\frac{1}{8} + \frac{1}{4} =$
m. $\frac{1}{4} - \frac{1}{8} =$
n. $\frac{1}{4} - \frac{1}{5} =$
o. $\frac{3}{4} - \frac{1}{5} =$

p. $\frac{3}{2} + \frac{1}{4} =$
q. $\frac{5}{8} - \frac{1}{4} =$
r. $\frac{5}{8} - \frac{1}{4} =$
s. $\frac{3}{7} + \frac{1}{3} =$
t. $\frac{3}{7} + \frac{1}{5} =$
u. $\frac{3}{7} - \frac{1}{5} =$
v. $\frac{1}{3} + \frac{1}{4} =$
w. $\frac{1}{2} - \frac{1}{7} =$

5. Division of Fractions

There are (at least) three ways to look at any division problem. They all lead to the same answer, of course. One of them leads to an understanding of division of fractions, and that is the one we will look at here.
For an example, we will look at 6 ÷ 3

Six divided by three means how many groups of three are in six:



There are two groups of three in six.

6 ÷ 3 answers the question:
How many groups of three are in six?
Note, this is reading right to left.

 Read left to right: 6 divided by 3 equals?

6 ÷ 3

 Read right to left: How many 3's in 6?

$3\overline{)6}$

Practice 16: One Meaning of Division

Problem	This Means	Picture	Answer
8 ÷ 4	How many 4's are in 8?		There are 2 groups of 4. 8 ÷ 4 = 2
10 ÷ 2			
15 ÷ 3			
16 ÷ 2			
16 ÷ 4			
12 ÷ 4			
12 ÷ 3			
12 ÷ 6			

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Problem	This Means	Picture	Answer
8 ÷ 2			
14 ÷ 2			
18 ÷ 9			
18 ÷ 3			
18 ÷ 6			
14 ÷ 7			
20 ÷ 4			
20 ÷ 5			

Activity 1: One Meaning of Division

In case you are one of those students who thinks it's ok to skip these hands-on activities, please think again. Your goal is to learn how to do this and then be able to go on in math and your life. Do this activity, and you will probably never need to learn how again!

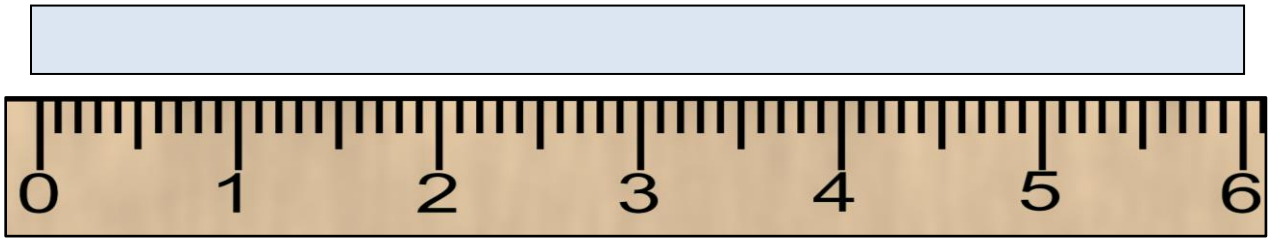
Materials:

- Ribbons, several feet
- Scissors
- Ruler

Example 1:
Cutting ribbons to prove the answer we already know – let's check the process.

6 divided by 2 = ? means **How many 2's in 6?**

First cut a six inch ribbon.



Then cut the ribbon into 2 inch lengths.

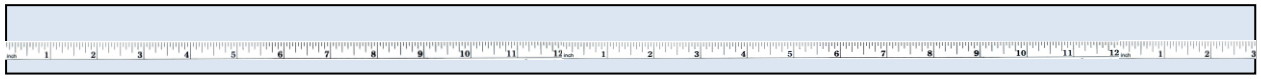
How many two inch segments are there in 6 inches?

6 ÷ 2 = 3 So this process works!!

Example 2: Where you don't know the answer—but you know the process.

$2\frac{1}{4}$ divided by $\frac{3}{4}$ = ? means **How many $\frac{3}{4}$'s in $2\frac{1}{4}$?**

First cut a piece of ribbon $2\frac{1}{4}$ feet long.
(Hint, if you need it: 1 foot is 12 inches. So, 2 feet is 24 inches long. Plus 3 inches 12 divided by 4 = 3)



Now we need to find out how many $\frac{3}{4}$ foot segments fit into this $2\frac{1}{4}$ foot ribbon.
(Hint if you need it: $\frac{3}{4}$ foot is 9 inches.)

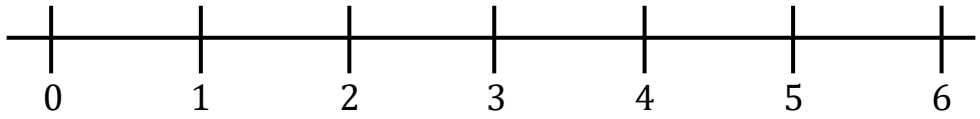
There are 3. So **$2\frac{1}{4} \div \frac{3}{4} = 3$**

Example 3: 6 divided by 2 using the number line

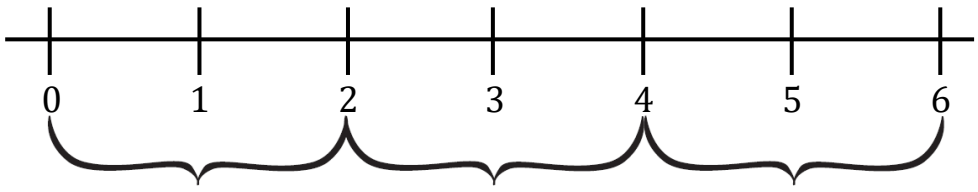
As in Example 1, rephrasing **6 divided by 2** into **How many 2's in 6?** leads to division of fractions.

This time we will use a number line model.
Here is the number line:

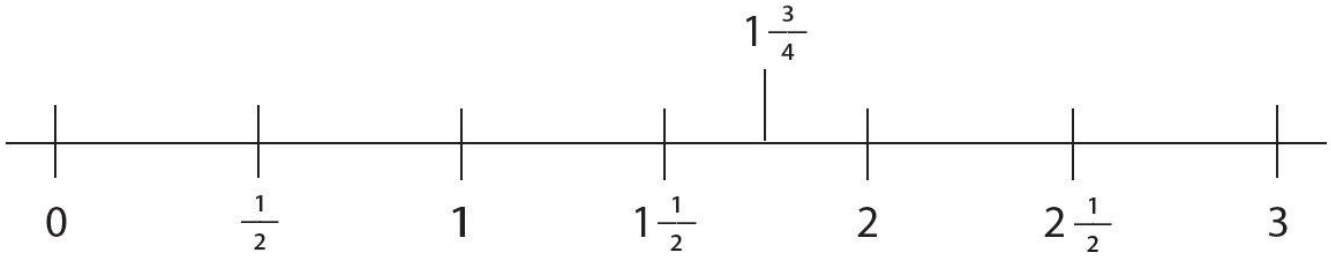
Example 4: Using the number line

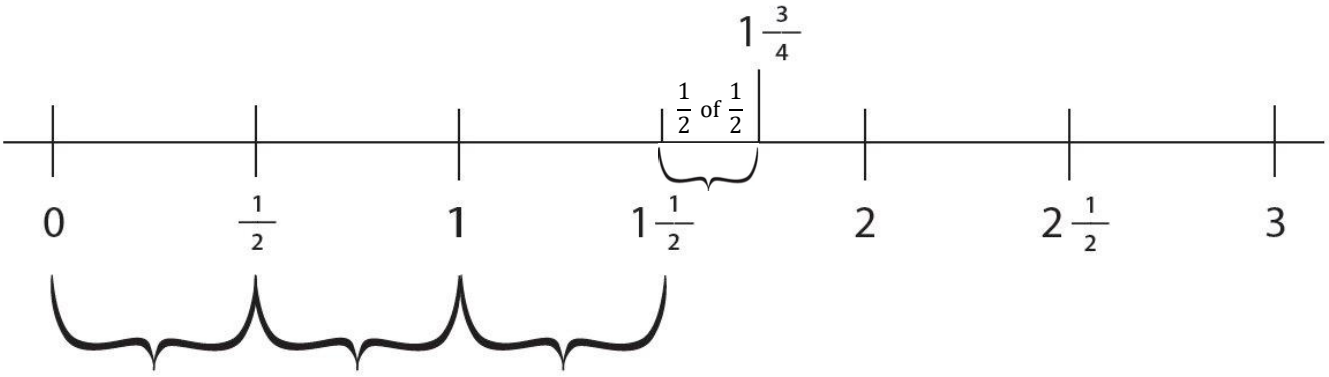


There are three 2's in 6.



$1\frac{3}{4}$ divided by $\frac{1}{2}$ = ? means **How many $\frac{1}{2}$'s in $1\frac{3}{4}$?**

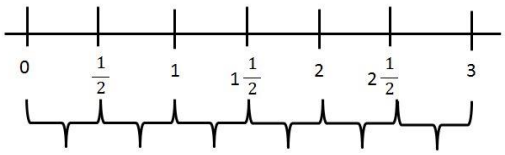




There are three and one-half $\frac{1}{2}$'s in $1\frac{3}{4}$.

Now it's your turn.

Practice 17: Division and Number Lines

$a \div b$	How many b's are in a	Draw a Number Line Picture	Answer
$3 \div \frac{1}{2}$	How many $\frac{1}{2}$'s in 3 ?		$3 \div \frac{1}{2} = 6$
$4 \div \frac{1}{2}$			
$5 \div \frac{1}{2}$			
$1\frac{1}{2} \div \frac{1}{2}$			
$1\frac{1}{2} \div \frac{1}{4}$			
$1\frac{1}{4} \div \frac{1}{4}$			
$1 \div \frac{1}{3}$			
$\frac{2}{3} \div \frac{1}{3}$			

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The Algebraic Way to Divide Fractions

$$\frac{a}{b} \div \frac{c}{d} = ?$$

Now we know the answer to several fraction division problems. We can figure out how to divide fractions from these.
Let's take: 6 ÷ 3 = 2.

After all, if it works for fractions, it had better work for whole numbers with $\frac{x}{1}$ = x.

To get this into the form of , we use our magic ones. 6 = $\frac{6}{1}$, and 3 = $\frac{3}{1}$.

$\frac{6}{1} \div \frac{3}{1} = \frac{2}{1}$ with **a = 6**, **b = 1**, **c = 3**, and **d = 1**

It looks like $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \div \frac{d}{c}$, so $\frac{6}{1} \div \frac{3}{1} = \frac{6}{1} * \frac{1}{3} = \frac{6 * 1}{1 * 3} = 2$

We will check with a more complicated problem.

$1\frac{3}{4} \div \frac{1}{2} = 3\frac{1}{2}$

$1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \div \frac{1}{2} = \frac{7}{4} * \frac{2}{1} = \frac{7 * \cancel{2}}{\cancel{2} * 2 * 1} = \frac{7}{2} = 3\frac{1}{2}$

So this works!

The way to divide fractions is:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c}$$

Practice 18: Division of Fractions Practice

a. $\frac{1}{2} \div \frac{1}{3} =$	b. $\frac{1}{3} \div \frac{1}{2} =$
c. $\frac{2}{5} \div \frac{1}{5} =$	d. $\frac{2}{5} \div \frac{2}{5} =$
e. $\frac{3}{4} \div \frac{1}{2} =$	f. $\frac{3}{4} \div \frac{1}{4} =$
g. $\frac{3}{4} \div \frac{1}{5} =$	h. $\frac{3}{4} \div \frac{1}{6} =$
i. $\frac{3}{4} \div \frac{1}{8} =$	j. $\frac{10}{11} \div \frac{12}{11} =$
k. $\frac{10}{7} \div \frac{15}{14} =$	l. $\frac{15}{16} \div \frac{1}{8} =$
m. $\frac{15}{16} \div \frac{8}{7} =$	n. $\frac{12}{17} \div \frac{6}{5} =$
o. $\frac{3}{17} \div \frac{5}{6} =$	p. $\frac{4}{9} \div \frac{8}{27} =$
q. $\frac{4}{9} \div \frac{1}{3} =$	r. $\frac{9}{7} \div \frac{9}{7} =$

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s. $\frac{9}{7} \div \frac{7}{9} =$	t. $\frac{14}{15} \div \frac{7}{3} =$
u. $\frac{9}{11} \div \frac{27}{22} =$	v. $\frac{5}{33} \div \frac{10}{3} =$
w. $\frac{5}{33} \div \frac{3}{22} =$	x. $\frac{12}{7} \div \frac{8}{21} =$
y. $\frac{12}{25} \div \frac{3}{5} =$	z. $\frac{3}{2} \div \frac{12}{25} =$
aa. $\frac{3}{5} \div \frac{3}{5} =$	bb. $3 \div \frac{1}{4} =$
cc. $5 \div \frac{1}{6} =$	dd. $\frac{1}{4} \div 2 =$
ee. $\frac{1}{9} \div \frac{1}{9} =$	ff. $\frac{1}{9} \div 9 =$
gg. $9 \div \frac{1}{9} =$	hh. $\frac{3}{8} \div \frac{6}{7} =$
ii. $\frac{17}{18} \div \frac{17}{9} =$	jj. $\frac{17}{9} \div \frac{17}{18} =$

6. Solutions / Answers

MATH WHISPERER

Where math makes sense

Practice 1: Reducing Fractions

a. Why $\frac{1}{3}$ is a reduced version of $\frac{2}{6}$?

The numerator and denominator have the smallest numbers possible to represent the quantity $\frac{1}{3}$

b. Why $\frac{1}{3}$ is not a reduced version of $\frac{3}{6}$?

Because $\frac{1}{3}$ is not equivalent to $\frac{3}{6}$. $\frac{3}{6}$ is equivalent to $\frac{1}{2}$, not $\frac{1}{3}$.

In these lists of equivalent fractions, which is lowest form? Explain why.

c. $\frac{7}{14}$ $\left(\frac{1}{2}\right)$ $\frac{3}{6}$ $\frac{5}{10}$ $\frac{2}{4}$

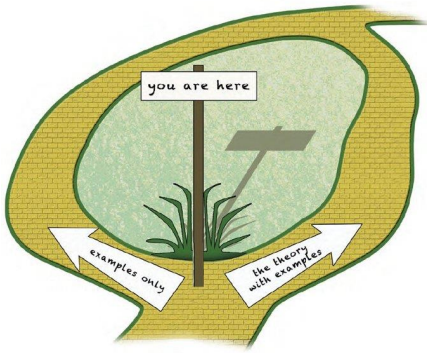
$\frac{1}{2}$ has the lowest form. The other fractions are equivalent, but their numerators and denominators have larger numbers.

d. $\frac{4}{6}$ $\frac{40}{60}$ $\frac{10}{15}$ $\left(\frac{2}{3}\right)$ $\frac{20}{30}$

$\frac{2}{3}$ has the lowest form. The other fractions are equivalent, but their numerators and denominators have larger numbers.

e. $\left(\frac{2}{1}\right)$ $\frac{4}{2}$ $\frac{10}{5}$ $\frac{8}{4}$ $\frac{20}{30}$

$\frac{2}{1}$ has the lowest form. The other fractions are equivalent, but their numerators and denominators have larger numbers.



Practice 2: Reducing Fractions – The Numbers get Bigger

We already know that fractions that are equivalent to 1 are $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$ etc.

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{1}{2}$	$\frac{1*3}{2*3}$	$\frac{3}{6}$
$\frac{1}{2}$	$\frac{1*8}{2*8}$	$\frac{8}{16}$
$\frac{1}{3}$	$\frac{1*2}{3*2}$	$\frac{2}{6}$
$\frac{2}{3}$	$\frac{2*2}{3*2}$	$\frac{4}{6}$
$\frac{5}{2}$	$\frac{5*3}{2*3}$	$\frac{15}{6}$
$\frac{1}{4}$	$\frac{1*3}{4*3}$	$\frac{3}{12}$
$\frac{1}{4}$	$\frac{1*8}{4*8}$	$\frac{8}{32}$

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{1}{2}$	$\frac{1*5}{2*5}$	$\frac{5}{10}$
$\frac{1}{2}$	$\frac{1*10}{2*10}$	$\frac{10}{20}$
$\frac{1}{3}$	$\frac{1*4}{3*4}$	$\frac{4}{12}$
$\frac{2}{3}$	$\frac{2*5}{3*5}$	$\frac{10}{15}$
$\frac{1}{5}$	$\frac{1*2}{5*2}$	$\frac{2}{10}$
$\frac{7}{3}$	$\frac{7*2}{3*2}$	$\frac{14}{6}$
$\frac{1}{4}$	$\frac{1*2}{4*2}$	$\frac{2}{8}$

Practice 3: Reducing Fractions – The Numbers get Smaller

We already know that fractions that are equivalent to 1 are $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and $\frac{5}{5}$ etc.

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{9}{12}$	$\frac{9 \div 3}{12 \div 3}$	$\frac{3}{4}$
$\frac{8}{16}$	$\frac{8 \div 8}{16 \div 8}$	$\frac{1}{2}$
$\frac{4}{6}$	$\frac{4 \div 2}{6 \div 2}$	$\frac{2}{3}$
$\frac{12}{30}$	$\frac{12 \div 6}{30 \div 6}$	$\frac{2}{5}$
$\frac{7}{14}$	$\frac{7 \div 7}{14 \div 7}$	$\frac{1}{2}$
$\frac{21}{28}$	$\frac{21 \div 7}{28 \div 7}$	$\frac{3}{4}$
$\frac{12}{10}$	$\frac{12 \div 2}{10 \div 2}$	$\frac{6}{5}$

Original Fraction	Multiply Fractions	Equivalent Fraction
$\frac{5}{10}$	$\frac{5 \div 5}{10 \div 5}$	$\frac{1}{2}$
$\frac{10}{20}$	$\frac{10 \div 10}{20 \div 10}$	$\frac{1}{2}$
$\frac{4}{12}$	$\frac{4 \div 4}{12 \div 4}$	$\frac{1}{3}$
$\frac{10}{15}$	$\frac{10 \div 5}{15 \div 5}$	$\frac{2}{3}$
$\frac{11}{55}$	$\frac{11 \div 11}{55 \div 11}$	$\frac{1}{5}$
$\frac{12}{18}$	$\frac{12 \div 6}{18 \div 6}$	$\frac{2}{3}$
$\frac{15}{9}$	$\frac{15 \div 3}{9 \div 3}$	$\frac{5}{3}$

Reducing fractions is based on multiplication of fractions, which you just learned in #4.

Practice 4: Explain each Step in Reducing This Fraction

$$\frac{44}{99}$$

$$= \frac{2 * 2 * 11}{3 * 3 * 11}$$

I factored 44 into prime factors, as well as factoring 99.

$$= \frac{2 * 2 * \cancel{11}}{3 * 3 * \cancel{11}}$$

I looked for $\frac{x}{x}$

$$= \frac{2 * 2}{3 * 3}$$

Then I multiplied.

$$= \frac{4}{9}$$

$\frac{4}{9}$ is the answer.

Practice 5: Reduce Fraction to Simplest Form

Reduce each fraction to simplest form. Please show every step. It is the process I want you to practice. "Seeing" the answer is not the goal. The goal is to practice the process so that you can apply it in cases when you can't see the answer. (For example, 111/111300 and more!)

a. $\frac{12}{24} = \frac{2*2*3}{2*2*2*3} = \frac{\cancel{2}*\cancel{2}*\cancel{3}*1}{2*\cancel{2}*\cancel{2}*\cancel{3}} = \frac{1}{2}$

b. $\frac{4xy}{8y} = \frac{2*2*x*y}{2*2*2*y} = \frac{\cancel{2}*\cancel{2}*x*\cancel{y}}{\cancel{2}*\cancel{2}*2*\cancel{y}} = \frac{x}{2}$

c. $\frac{15}{50} = \frac{3*5}{2*5*5} = \frac{3*\cancel{5}}{2*\cancel{5}*5} = \frac{3}{2*5} = \frac{3}{10}$

d. $\frac{7abc}{14bcw} = \frac{7*a*b*c}{2*7*b*c*w} = \frac{\cancel{7}*a*\cancel{b}*\cancel{c}}{2*\cancel{7}*\cancel{b}*\cancel{c}*w} = \frac{a}{2w}$

e. $\frac{42}{33} = \frac{2*3*7}{3*11} = \frac{2*\cancel{3}*7}{\cancel{3}*11} = \frac{2*7}{11} = \frac{14}{11}$

f. $\frac{102}{3} = \frac{2*3*17}{3} = \frac{2*\cancel{3}*17}{\cancel{3}*1} = \frac{2*17}{1} = \frac{34}{1} = 34$

g. $\frac{24}{12} = \frac{2*2*2*3}{2*2*3} = \frac{\cancel{2}*\cancel{2}*2*\cancel{3}}{\cancel{2}*\cancel{2}*\cancel{3}*1} = \frac{2}{1} = 2$

h. $\frac{18ghw}{72ghw} = \frac{2*3*3*g*h*w}{2*2*2*3*3*g*h*w} = \frac{\cancel{2}*\cancel{3}*\cancel{3}*\cancel{g}*\cancel{h}*\cancel{w}*1}{\cancel{2}*2*2*\cancel{3}*\cancel{3}*\cancel{g}*\cancel{h}*\cancel{w}} = \frac{1}{2*2} = \frac{1}{4}$

i. $\frac{30}{24} = \frac{2*3*5}{2*2*2*3} = \frac{\cancel{2}*\cancel{3}*5}{\cancel{2}*2*2*\cancel{3}} = \frac{5}{2*2} = \frac{5}{4}$

j. $\frac{6ab}{8abc} = \frac{2*3*a*b}{2*2*2*a*b*c} = \frac{\cancel{2}*3*\cancel{a}*\cancel{b}}{\cancel{2}*2*2*\cancel{a}*\cancel{b}*c} = \frac{3}{4c}$

k. $\frac{60q}{50} = \frac{2*2*3*5*q}{2*5*5} = \frac{\cancel{2}*2*3*\cancel{5}*q}{\cancel{2}*5*\cancel{5}} = \frac{6q}{5}$

l. $\frac{21acw}{14bcw} = \frac{3*7*a*c*w}{2*7*b*c*w} = \frac{3*\cancel{7}*a*\cancel{c}*\cancel{w}}{2*\cancel{7}*b*\cancel{c}*\cancel{w}} = \frac{3a}{2b}$

m. $\frac{121}{33} = \frac{11*11}{3*11} = \frac{\cancel{11}*11}{3*\cancel{11}} = \frac{11}{3}$

n. $\frac{60}{30} = \frac{2*2*3*5}{3*2*5} = \frac{\cancel{2}*2*\cancel{3}*\cancel{5}}{\cancel{3}*\cancel{2}*\cancel{5}} = \frac{2}{1} = 2$

o. $\frac{55ab}{10} = \frac{5*11*a*b}{2*5} = \frac{\cancel{5}*11*a*b}{2*\cancel{5}} = \frac{11ab}{2}$

p. $\frac{24f}{72} = \frac{2*2*2*3*f}{2*3*2*3*2} = \frac{\cancel{2}*\cancel{2}*\cancel{2}*\cancel{3}*f}{\cancel{2}*3*\cancel{2}*\cancel{3}*\cancel{2}} = \frac{f}{3}$

q. $\frac{25uvw}{20ghw} = \frac{5*5*u*v*w}{2*2*5*g*h*w} = \frac{\cancel{5}*5*u*v*\cancel{w}}{2*2*\cancel{5}*g*h*\cancel{w}} = \frac{5uv}{4gh}$

Practice 6: Reduced Form or Not?

Decide which fractions are in reduced form and which are not. If a fraction is in reduced form, write "RF". If it is not in reduced form, show the magic ones that divide into it and put a box around the final reduced form.

$\frac{9}{12} \frac{\div 3}{\div 3} = \frac{3}{4}$	$\frac{6}{13}$ RF	$\frac{5}{10} \frac{\div 5}{\div 5} = \frac{1}{2}$
$\frac{5}{15} \frac{\div 5}{\div 5} = \frac{1}{3}$	4 RF	$\frac{5}{8}$ RF
$\frac{8}{15}$ RF	$\frac{3}{9} \frac{\div 3}{\div 3} = \frac{1}{3}$	$\frac{6}{18} \frac{\div 6}{\div 6} = \frac{1}{3}$
$\frac{24}{48} \frac{\div 24}{\div 24} = \frac{1}{2}$	$\frac{6}{24} \frac{\div 6}{\div 6} = \frac{1}{4}$	$\frac{8}{24} \frac{\div 8}{\div 8} = \frac{1}{3}$
$\frac{7}{21} \frac{\div 7}{\div 7} = \frac{1}{3}$	$\frac{12}{21} \frac{\div 3}{\div 3} = \frac{4}{7}$	$\frac{13}{13} \frac{\div 13}{\div 13} = \frac{1}{1} = 1$

Write four fractions not already on this paper that are in reduced form:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ as an example

Write four fractions not already on this paper that are not in reduced form:

$\frac{2}{4}$ $\frac{2}{6}$ $\frac{2}{8}$ $\frac{2}{10}$ as an example

Practice 7: Multiply Fractions the Way Einstein Did

a. $\frac{12}{55} * \frac{34}{33} = \frac{12 * 34}{55 * 33} = \frac{2 * 2 * 3 * 3 * 11}{5 * 11 * 2 * 2} = \frac{3 * 3}{5} = \frac{9}{5}$

b. $\frac{17}{3} * \frac{3}{11} = \frac{17 * 3}{3 * 11} = \frac{17}{11}$

c. $\frac{8}{25} * \frac{30}{7} = \frac{8 * 30}{25 * 7} = \frac{2 * 2 * 2 * 2 * 3 * 5}{5 * 5 * 7} = \frac{48}{25}$

d. $\frac{9}{10} * \frac{5}{3} = \frac{9 * 5}{10 * 3} = \frac{3 * 3 * 5}{2 * 5 * 3} = \frac{3}{2}$

e. $\frac{9}{10} * \frac{3}{5} = \frac{9 * 3}{10 * 5} = \frac{3 * 3 * 3}{2 * 5 * 5} = \frac{27}{50}$

f. $\frac{1}{7} * \frac{14}{15} = \frac{1 * 14}{7 * 15} = \frac{1 * 2 * 7}{7 * 3 * 5} = \frac{2}{15}$

g. $\frac{22}{23} * \frac{46}{11} = \frac{22 * 46}{23 * 11} = \frac{2 * 11 * 2 * 23}{23 * 11 * 1} = \frac{4}{1} = 4$

h. $\frac{1}{3} * \frac{6}{5} = \frac{1 * 6}{3 * 5} = \frac{1 * 2 * 3}{3 * 5} = \frac{2}{5}$

i. $\frac{12}{13} * \frac{39}{4} = \frac{12 * 39}{13 * 4} = \frac{2 * 2 * 3 * 3 * 13}{13 * 2 * 2 * 1} = \frac{9}{1} = 9$

j. $\frac{2}{15} * \frac{3}{5} = \frac{2*3}{15*5} = \frac{2*\cancel{3}}{\cancel{3}*5*5} = \frac{2}{25}$

k. $\frac{2}{15} * \frac{5}{3} = \frac{2*5}{15*3} = \frac{2*\cancel{5}}{3*5*\cancel{5}} = \frac{2}{15}$

l. $\frac{1}{3} * \frac{9}{8} = \frac{1*9}{3*8} = \frac{1*3*\cancel{3}}{\cancel{3}*2*2*2} = \frac{3}{8}$

m. $\frac{2}{3} * \frac{9}{8} = \frac{2*9}{3*8} = \frac{\cancel{2}*\cancel{3}*3}{\cancel{3}*\cancel{2}*2*2} = \frac{3}{4}$

n. $\frac{4}{3} * \frac{9}{8} = \frac{4*9}{3*8} = \frac{\cancel{2}*\cancel{2}*\cancel{3}*3}{\cancel{3}*\cancel{2}*\cancel{2}*2} = \frac{3}{2}$

o. $\frac{1}{5} * \frac{10}{3} = \frac{1*10}{5*3} = \frac{1*2*\cancel{5}}{\cancel{5}*3} = \frac{2}{3}$

p. $\frac{2}{5} * \frac{3}{10} = \frac{2*3}{5*10} = \frac{\cancel{2}*3}{5*\cancel{2}*5} = \frac{3}{25}$

q. $\frac{2}{5} * \frac{10}{3} = \frac{2*10}{5*3} = \frac{2*2*\cancel{5}}{\cancel{5}*3} = \frac{4}{3}$

r. $\frac{2}{5} * \frac{30}{11} = \frac{2*30}{5*11} = \frac{2*2*3*\cancel{5}}{\cancel{5}*11} = \frac{12}{11}$

s. $\frac{14}{15} * \frac{18}{7} = \frac{14 * 18}{15 * 7} = \frac{2 * 7 * 2 * 3 * 3}{3 * 5 * 7} = \frac{12}{5}$

t. $\frac{17}{18} * \frac{9}{34} = \frac{17 * 9}{18 * 34} = \frac{17 * 3 * 3 * 1}{2 * 3 * 3 * 2 * 17} = \frac{1}{4}$

u. $\frac{12}{13} * \frac{39}{4} = \frac{12 * 39}{13 * 4} = \frac{2 * 2 * 3 * 3 * 13}{13 * 2 * 2 * 1} = \frac{9}{1} = 9$

v. $\frac{12}{7} * \frac{28}{3} = \frac{12 * 28}{7 * 3} = \frac{2 * 2 * 3 * 2 * 2 * 7}{7 * 3 * 1} = \frac{16}{1} = 16$

w. $\frac{20}{21} * \frac{7}{4} = \frac{20 * 7}{21 * 4} = \frac{2 * 2 * 5 * 7}{3 * 7 * 2 * 2} = \frac{5}{3}$

x. $\frac{20}{21} * \frac{7}{10} = \frac{20 * 7}{21 * 10} = \frac{2 * 2 * 5 * 7}{3 * 7 * 2 * 5} = \frac{2}{3}$

y. $\frac{5}{12} * \frac{3}{10} = \frac{5 * 3}{12 * 10} = \frac{5 * 3 * 1}{2 * 2 * 3 * 2 * 5} = \frac{1}{8}$

z. $\frac{6}{7} * \frac{14}{15} = \frac{6 * 14}{7 * 15} = \frac{2 * 3 * 2 * 7}{7 * 3 * 5} = \frac{4}{5}$

aa. $\frac{11}{15} * \frac{30}{44} = \frac{11 * 30}{15 * 44} = \frac{11 * 2 * 3 * 5 * 1}{3 * 5 * 2 * 2 * 11} = \frac{1}{2}$

bb. $\frac{6}{55} * \frac{11}{10} = \frac{6 * 11}{55 * 10} = \frac{2 * 3 * 11}{5 * 11 * 2 * 5} = \frac{3}{25}$

cc. $\frac{21}{23} * \frac{46}{7} = \frac{21 * 46}{23 * 7} = \frac{3 * 7 * 2 * 23}{23 * 7 * 1} = \frac{6}{1} = 6$

dd. $\frac{91}{12} * \frac{1}{3} = \frac{91 * 1}{12 * 3} = \frac{7 * 13 * 1}{2 * 2 * 3 * 13} = \frac{7}{12}$

ee. $\frac{12}{91} * \frac{1}{3} = \frac{12 * 1}{91 * 3} = \frac{2 * 2 * 3 * 1}{7 * 13 * 3} = \frac{4}{91}$

ff. $\frac{21}{23} * \frac{36}{7} = \frac{21 * 36}{23 * 7} = \frac{3 * 7 * 2 * 3 * 2 * 3}{23 * 7} = \frac{108}{23}$

gg. $\frac{242}{5} * \frac{100}{121} = \frac{242 * 100}{5 * 121} = \frac{2 * 11 * 11 * 2 * 5 * 2 * 5}{5 * 11 * 11 * 1} = \frac{40}{1} = 40$

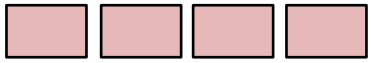
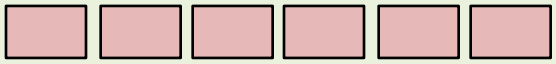
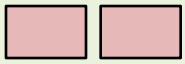
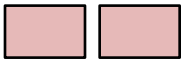
hh. $\frac{77}{5} * \frac{15}{22} = \frac{77 * 15}{5 * 22} = \frac{7 * 11 * 3 * 5}{5 * 11 * 1} = \frac{21}{1} = 21$

ii. $\frac{60}{11} * \frac{33}{40} = \frac{60 * 33}{11 * 40} = \frac{2 * 3 * 2 * 5 * 3 * 11}{11 * 2 * 2 * 2 * 5} = \frac{9}{2}$

jj. $\frac{55}{2} * \frac{6}{11} = \frac{55 * 6}{2 * 11} = \frac{5 * 11 * 2 * 3}{2 * 11 * 1} = \frac{15}{1} = 15$

Practice 8: Magic Ones $x = \frac{x}{1}$

Fill in the blanks. This is one. ☐

$\frac{4}{1}$		4
$\frac{3}{1}$		3
$\frac{6}{1}$		6
$\frac{2}{1}$		2
$\frac{5}{1}$		5
$\frac{7}{1}$		7
$\frac{2}{1}$		2
$\frac{5}{1}$		5

Let's look at what you found.

$\frac{2}{1} = 2$

$\frac{3}{1} = 3$

$\frac{4}{1} = 4$

$\frac{5}{1} = 5$

And so on.

Does $\frac{x}{1} = x$ **?**

Why or why not? The pattern is that $\frac{x}{1} = x$**.** This is not a vigorous mathematic proof, but it looks like $\frac{x}{1} = x$ every time.

Practice 9: Magic Ones $x * 1 = x$

a. $\frac{3}{28} * \frac{4}{3} = \frac{3*4}{28*3} = \frac{\cancel{3}*\cancel{2}*\cancel{2}*1}{\cancel{2}*\cancel{2}*7*\cancel{3}} = \frac{1}{7}$

b. $\frac{2}{45} * \frac{15}{2} = \frac{2*15}{45*2} = \frac{\cancel{2}*\cancel{3}*\cancel{5}*1}{\cancel{3}*3*\cancel{5}*\cancel{2}} = \frac{1}{3}$

c. $\frac{2}{7} * \frac{7}{6} = \frac{2*7}{7*6} = \frac{\cancel{2}*\cancel{7}*1}{\cancel{7}*\cancel{2}*3} = \frac{1}{3}$

d. $\frac{2}{7} * \frac{7}{8} = \frac{2*7}{7*8} = \frac{\cancel{2}*\cancel{7}*1}{\cancel{7}*\cancel{2}*2*2} = \frac{1}{4}$

e. $\frac{5}{14} * \frac{7}{15} = \frac{5*7}{14*15} = \frac{\cancel{5}*\cancel{7}*1}{2*\cancel{7}*3*\cancel{5}} = \frac{1}{6}$

f. $\frac{22}{39} * \frac{13}{44} = \frac{22*13}{39*44} = \frac{\cancel{2}*\cancel{11}*\cancel{13}*1}{3*\cancel{13}*\cancel{11}*\cancel{2}*2} = \frac{1}{6}$

g. $\frac{3}{14} * \frac{7}{6} = \frac{3*7}{14*6} = \frac{\cancel{3}*\cancel{7}*1}{2*\cancel{7}*2*\cancel{3}} = \frac{1}{4}$

h. $\frac{3}{22} * \frac{11}{6} = \frac{3*11}{22*6} = \frac{\cancel{3}*\cancel{11}*1}{\cancel{11}*2*2*\cancel{3}} = \frac{1}{4}$

i. $\frac{8}{45} * \frac{5}{24} = \frac{8 * 5}{45 * 24} = \frac{2 * 2 * 2 * 5 * 1}{3 * 3 * 5 * 2 * 2 * 2 * 3} = \frac{1}{27}$

j. $\frac{3}{11} * \frac{11}{6} = \frac{3 * 11}{11 * 6} = \frac{3 * 11 * 1}{11 * 2 * 3} = \frac{1}{2}$

k. $\frac{12}{55} * \frac{5}{24} = \frac{12 * 5}{55 * 24} = \frac{2 * 2 * 3 * 5 * 1}{5 * 11 * 2 * 2 * 2 * 3} = \frac{1}{22}$

l. $\frac{12}{55} * \frac{11}{24} = \frac{12 * 11}{55 * 24} = \frac{2 * 2 * 3 * 11 * 1}{5 * 11 * 2 * 2 * 2 * 3} = \frac{1}{10}$

m. $\frac{3}{55} * \frac{11}{3} = \frac{3 * 11}{55 * 3} = \frac{3 * 11 * 1}{5 * 11 * 3} = \frac{1}{5}$

n. $\frac{3}{55} * \frac{11}{6} = \frac{3 * 11}{55 * 6} = \frac{3 * 11 * 1}{5 * 11 * 2 * 3} = \frac{1}{10}$

o. $\frac{4}{15} * \frac{5}{12} = \frac{4 * 5}{15 * 12} = \frac{2 * 2 * 5 * 1}{3 * 5 * 2 * 2 * 3} = \frac{1}{9}$

p. $\frac{4}{27} * \frac{3}{8} = \frac{4 * 3}{27 * 8} = \frac{2 * 2 * 3 * 1}{3 * 3 * 3 * 2 * 2 * 2} = \frac{1}{18}$

q. $\frac{5}{21} * \frac{3}{20} = \frac{5 * 3}{21 * 20} = \frac{5 * 3 * 1}{3 * 7 * 2 * 2 * 5} = \frac{1}{28}$

Practice 10: Magic Ones $\frac{x}{1} = x$

Magic Ones three: $\frac{x}{1} = x$

This is another example of pulling one out of your pocket.

a. $6 * \frac{1}{2} = \frac{6}{1} * \frac{1}{2} = \frac{6 * 1}{1 * 2} = \frac{\cancel{2} * 3 * 1}{1 * \cancel{2}} = \frac{3}{1} = 3$

b. $3 * \frac{1}{2} = \frac{3}{1} * \frac{1}{2} = \frac{3 * 1}{1 * 2} = \frac{3}{2}$

c. $3 * \frac{1}{3} = \frac{3}{1} * \frac{1}{3} = \frac{\cancel{3} * 1}{1 * \cancel{3}} = \frac{1}{1} = 1$

d. $5 * \frac{1}{5} = \frac{5}{1} * \frac{1}{5} = \frac{\cancel{5} * 1}{1 * \cancel{5}} = \frac{1}{1} = 1$

e. $3 * \frac{1}{5} = \frac{3}{1} * \frac{1}{5} = \frac{3 * 1}{1 * 5} = \frac{3}{5}$

f. $5 * \frac{1}{3} = \frac{5}{1} * \frac{1}{3} = \frac{5 * 1}{1 * 3} = \frac{5}{3}$

g. $6 * \frac{2}{3} = \frac{6}{1} * \frac{2}{3} = \frac{6 * 2}{1 * 3} = \frac{2 * \cancel{3} * 2}{1 * \cancel{3}} = \frac{4}{1} = 4$

h. $6 * \frac{2}{5} = \frac{6}{1} * \frac{2}{5} = \frac{6*2}{1*5} = \frac{2*3*2}{1*5} = \frac{12}{5}$

i. $5 * \frac{2}{5} = \frac{5}{1} * \frac{2}{5} = \frac{5*2}{1*5} = \frac{\cancel{5}*2}{1*\cancel{5}} = \frac{2}{1} = 2$

j. $3 * \frac{x}{2} = \frac{3}{1} * \frac{x}{2} = \frac{3*x}{1*2} = \frac{3x}{2}$

k. $3 * \frac{x}{3} = \frac{3}{1} * \frac{x}{3} = \frac{3*x}{1*3} = \frac{3x}{1} = 3x$

l. $3 * \frac{x}{5} = \frac{3}{1} * \frac{x}{5} = \frac{3*x}{1*5} = \frac{3x}{5}$

m. $15 * \frac{x}{5} = \frac{15}{1} * \frac{x}{5} = \frac{15*x}{1*5} = \frac{3*5*x}{1*5} = \frac{3x}{1} = 3x$

n. $15 * \frac{x}{3} = \frac{15}{1} * \frac{x}{3} = \frac{15}{1} * \frac{x}{3} = \frac{\cancel{3}*5*x}{1*\cancel{3}} = \frac{5x}{1} = 5x$

o. $15 * \frac{x}{2} = \frac{15}{1} * \frac{x}{2} = \frac{15*x}{1*2} = \frac{3*5*x}{1*2} = \frac{15x}{2}$

p. $15 * \frac{x}{7} = \frac{15}{1} * \frac{x}{7} = \frac{15*x}{1*7} = \frac{3*5*x}{1*7} = \frac{15x}{7}$

q. $14 * \frac{x}{28} = \frac{14}{1} * \frac{x}{28} = \frac{14 * x}{1 * 28} = \frac{2 * 7 * x}{7 * 2 * 2} = \frac{x}{2}$

r. $12 * \frac{3}{4} = \frac{12}{1} * \frac{3}{4} = \frac{12 * 3}{1 * 4} = \frac{2 * 2 * 3 * 3}{1 * 2 * 2} = \frac{9}{1} = 9$

s. $14 * \frac{4}{3} = \frac{14}{1} * \frac{4}{3} = \frac{14 * 4}{1 * 3} = \frac{2 * 7 * 2 * 2}{1 * 3} = \frac{56}{3}$

t. $\frac{3}{5} * 6 = \frac{3}{5} * \frac{6}{1} = \frac{3 * 6}{5 * 1} = \frac{2 * 3 * 3}{5 * 1} = \frac{18}{5}$

u. $\frac{3}{5} * 10 = \frac{3}{5} * \frac{10}{1} = \frac{3 * 10}{5 * 1} = \frac{3 * 2 * 5}{1 * 5} = \frac{6}{1} = 6$

v. $\frac{2}{5} * 4 = \frac{15}{1} * \frac{x}{7} = \frac{15 * x}{1 * 7} = \frac{3 * 5 * x}{1 * 7} = \frac{15x}{7}$

w. $\frac{2}{5} * 20 = \frac{2}{5} * \frac{20}{1} = \frac{2 * 20}{5 * 1} = \frac{2 * 2 * 2 * 5}{5 * 1} = \frac{8}{1} = 8$

x. $\frac{7}{8} * 16 = \frac{7}{8} * \frac{16}{1} = \frac{7 * 16}{8 * 1} = \frac{7 * 2 * 2 * 2 * 2}{2 * 2 * 2 * 1} = \frac{14}{1} = 14$

y. $\frac{7}{8} * 24 = \frac{7}{8} * \frac{24}{1} = \frac{7 * 2 * 2 * 2 * 3}{2 * 2 * 2 * 1} = \frac{21}{1} = 21$

z. $\frac{7}{8} * 3 = \frac{7}{8} * \frac{3}{1} = \frac{7*3}{8*1} = \frac{7*3}{2*2*2*1} = \frac{21}{8}$

aa. $5 * \frac{3}{7} = \frac{5}{1} * \frac{3}{7} = \frac{5*3}{1*7} = \frac{15}{7}$

bb. $21 * \frac{3}{7} = \frac{21}{1} * \frac{3}{7} = \frac{21*3}{1*7} = \frac{\cancel{7}*3*3}{1*\cancel{7}} = \frac{9}{1} = 9$

cc. $21 * \frac{2}{7} = \frac{21}{1} * \frac{2}{7} = \frac{21*2}{1*7} = \frac{\cancel{7}*3*2}{1*\cancel{7}} = \frac{6}{1} = 6$

dd. $17 * \frac{1}{34} = \frac{17}{1} * \frac{1}{34} = \frac{17*1}{1*34} = \frac{\cancel{17}*1}{1*\cancel{17}*2} = \frac{1}{2}$

ee. $11 * \frac{1}{121} = \frac{11}{1} * \frac{1}{121} = \frac{11*1}{1*121} = \frac{\cancel{11}*1}{1*\cancel{11}*11} = \frac{1}{11}$

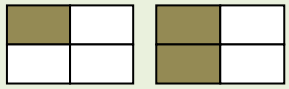
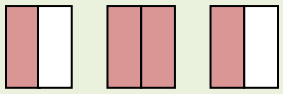
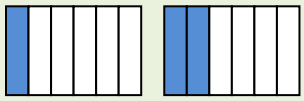
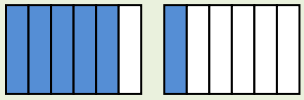
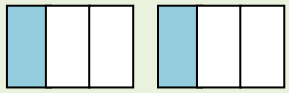
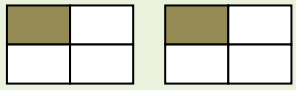
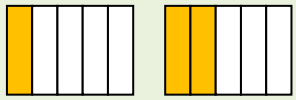
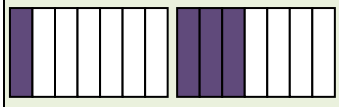
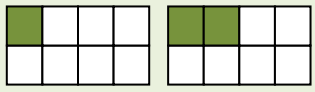
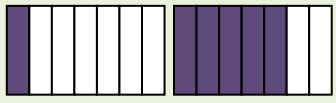
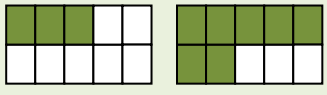
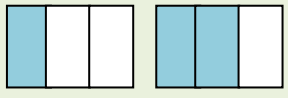
ff. $11 * \frac{1}{242} = \frac{11}{1} * \frac{1}{242} = \frac{11*1}{1*242} = \frac{\cancel{11}*1}{1*2*\cancel{11}*11} = \frac{1}{22}$

gg. $w * \frac{4}{w} = \frac{w}{1} * \frac{4}{w} = \frac{\cancel{w}*4}{1*\cancel{w}} = \frac{4}{1} = 4$

hh. $q * \frac{5}{q} = \frac{q}{1} * \frac{5}{q} = \frac{\cancel{q}*5}{1*\cancel{q}} = \frac{5}{1} = 5$

Practice 11: Adding Fractions, Using our Fraction Pieces

In the activity, ☐ = one.

Symbols	Names	Pictures	Sum
$\frac{1}{4}+\frac{2}{4}$	one fourth + two fourths		three fourths
$\frac{1}{2}+\frac{3}{2}$	one half + three halves		four halves
$\frac{1}{6}+\frac{2}{6}$	one sixth + two sixths		three sixths
$\frac{5}{6}+\frac{1}{6}$	five sixths + one sixth		six sixths
$\frac{1}{3}+\frac{1}{3}$	one third + one third		two thirds
$\frac{1}{4}+\frac{1}{4}$	one fourth + one fourth		two fourths
$\frac{1}{5}+\frac{2}{5}$	one fifth + two fifths		three fifths
$\frac{1}{7}+\frac{3}{7}$	one seventh + three sevenths		four sevenths
$\frac{3}{8}+\frac{2}{8}$	one eighth + two eighths		three eighths
$\frac{1}{7}+\frac{5}{7}$	one seventh + five sevenths		six sevenths
$\frac{3}{10}+\frac{7}{10}$	three tenths + seven tenths		ten tenths
$\frac{1}{3}+\frac{2}{3}$	one third + two thirds		three thirds

Practice 12: First Addition of Fractions

In Symbols	Labels in Words	Labels/Units for Sum
$\frac{1}{2} + \frac{3}{2} =$	1 half + 3 halves	halves
$\frac{1}{4} + \frac{3}{4} =$	1 fourth + 3 fourths	fourths
$\frac{1}{5} + \frac{3}{5} =$	1 fifths + 3 fifths	fifths
$\frac{1}{2} + \frac{1}{4} =$	1 half + 1 fourth	fourths (1 half = 2 fourths)
$\frac{1}{3} + \frac{1}{6} =$	1 third + 1 sixth	sixths (1 third = 2 sixths)
$\frac{2}{3} + \frac{2}{6} =$	2 thirds + 2 sixths	sixths (2 thirds = 4 sixths)

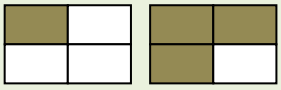
In Symbols	Labels in Words	Labels/Units for Sum
$\frac{1}{2} + \frac{1}{5} =$	1 half + 3 halves	halves
$\frac{1}{6} + \frac{1}{2} =$	1 sixth + 1 half	sixths (1 half = 3 sixths)
$\frac{3}{4} + \frac{1}{2} =$	3 fourths + 1 half	fourths (1 half = 2 fourths)
$\frac{2}{3} + \frac{1}{6} =$	2 thirds + 1 sixth	sixths (2 thirds = 4 sixths)
$\frac{1}{6} + \frac{1}{6} =$	1 sixth + 1 sixths	sixths
$\frac{3}{3} + \frac{1}{6} =$	3 thirds + 1 sixths	sixths (3 thirds = 6 sixths)
$\frac{2}{5} + \frac{2}{5} =$	2 fifths + 2 fifths	fifths
$\frac{5}{5} + \frac{1}{5} =$	5 fifths + 1 fifth	fifths

Practice 13: First Addition of Fractions - Explain

a. Explain how to add this problem: $\frac{1}{4} + \frac{3}{4} =$

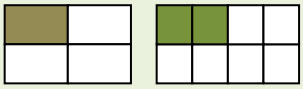
b.

This is 1 fourth + 3 fourths. The labels/units are the same so I can count them. There are 4 fourths. So $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$.



c. Explain how to add this problem: $\frac{1}{4} + \frac{1}{8} =$

The units/labels are different, so I can't add them like this. $\frac{1}{4} = \frac{2}{8}$ so, I can change this to 2 eighths + 1 eighth = 3 eighths.



d. Explain how to add this problem: $\frac{1}{3} + \frac{1}{2} =$

The units/labels are different, so I can't add them as is. I can change 1 third to 2 sixths. I can change 1 half to 3 sixths. So $\frac{1}{3} + \frac{1}{2}$ = 1 third + 1 half

= 2 sixths + 3 sixths $\frac{2}{6} + \frac{3}{6}$

= 5 sixths $= \frac{5}{6}$

Practice 14: Using Algebra to Add Fractions

Use: $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$	Find equivalent fraction if necessary	a = ?	b = ?	c = ?	Sum (show equation) $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
$\frac{1}{2}+\frac{3}{2}=$	none necessary	1	2	3	$\frac{1}{2}+\frac{3}{2}=\frac{1+3}{2}=\frac{4}{2}$
$\frac{1}{4}+\frac{3}{4}=$	none necessary	1	4	3	$\frac{1}{4}+\frac{3}{4}=\frac{1+3}{4}=\frac{4}{4}$
$\frac{1}{5}+\frac{3}{5}=$	none necessary	1	5	3	$\frac{1}{5}+\frac{3}{5}=\frac{1+3}{5}=\frac{4}{5}$
$\frac{1}{2}+\frac{1}{4}=$	$\frac{1}{2}=\frac{2}{4}$ $\frac{2}{4}+\frac{1}{4}$	2	4	1	$\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$
$\frac{1}{3}+\frac{1}{6}=$	$\frac{1}{3}=\frac{2}{6}$ $\frac{2}{6}+\frac{1}{6}$	2	6	1	$\frac{1}{3}+\frac{1}{6}=\frac{2+1}{6}=\frac{3}{6}$
$\frac{2}{3}+\frac{2}{6}=$	$\frac{2}{3}=\frac{4}{6}$ $\frac{4}{6}+\frac{2}{6}$	4	6	2	$\frac{4}{6}+\frac{2}{6}=\frac{4+2}{6}=\frac{6}{6}$
$\frac{1}{2}+\frac{1}{5}=$	$\frac{1}{2}=\frac{5}{10}$ $\frac{1}{5}=\frac{2}{10}$ $\frac{5}{10}+\frac{2}{10}$	5	10	2	$\frac{5}{10}+\frac{2}{10}=\frac{5+2}{10}=\frac{7}{10}$

Use: $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b}$	Find equivalent fraction if necessary	a = ?	b = ?	c = ?	Sum (show equation) $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b}$
$\frac{1}{6}+\frac{1}{2}=$	$\frac{1}{2}=\frac{3}{6}$ $\frac{1}{6}+\frac{3}{6}$	1	6	3	$\frac{1}{6}+\frac{3}{6}=\frac{1+3}{6}=\frac{4}{6}$
$\frac{3}{4}+\frac{1}{2}=$	$\frac{1}{2}=\frac{2}{4}$ $\frac{3}{4}+\frac{2}{4}$	3	4	2	$\frac{3}{4}+\frac{2}{4}=\frac{3+2}{4}=\frac{5}{4}$
$\frac{2}{3}+\frac{1}{6}=$	$\frac{2}{3}=\frac{4}{6}$ $\frac{4}{6}+\frac{1}{6}$	4	6	1	$\frac{4}{6}+\frac{1}{6}=\frac{4+1}{6}=\frac{5}{6}$
$\frac{1}{6}+\frac{1}{6}=$	none necessary	1	6	1	$\frac{1}{6}+\frac{1}{6}=\frac{1+1}{6}=\frac{2}{6}$
$\frac{3}{3}+\frac{1}{6}=$	$\frac{3}{3}=\frac{6}{6}$ $\frac{6}{6}+\frac{1}{6}$	6	6	1	$\frac{6}{6}+\frac{1}{6}=\frac{6+1}{6}=\frac{7}{6}$
$\frac{2}{5}+\frac{2}{5}=$	none necessary	2	5	2	$\frac{2}{5}+\frac{2}{5}=\frac{2+2}{5}=\frac{4}{5}$
$\frac{5}{5}+\frac{1}{5}=$	none necessary	5	5	1	$\frac{5}{5}+\frac{1}{5}=\frac{5+1}{5}=\frac{6}{5}$
$\frac{4}{15}+\frac{3}{5}=$	$\frac{3}{5}=\frac{9}{15}$ $\frac{4}{15}+\frac{9}{15}$	4	15	9	$\frac{4}{15}+\frac{9}{15}=\frac{4+9}{15}=\frac{13}{15}$

Practice 15: Adding and Subtracting Fractions

Use algebra to make the fractions equivalent. Then add or subtract them as indicated.
Note: There are **lots** of possible same denominators. Some are smaller than others. As long as you get the correct answer, your choice of denominator doesn't matter.

a. $\frac{1}{2} + \frac{1}{4} = \frac{1}{2} * \left(\frac{2}{2}\right) + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$

b. $\frac{1}{2} + \frac{1}{3} = \frac{1}{2} * \left(\frac{3}{3}\right) + \frac{1}{3} * \left(\frac{2}{2}\right) = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$

c. $\frac{1}{2} - \frac{1}{4} = \frac{1}{2} * \left(\frac{2}{2}\right) - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$

d. $\frac{1}{2} - \frac{1}{3} = \frac{1}{2} * \left(\frac{3}{3}\right) - \frac{1}{3} * \left(\frac{2}{2}\right) = \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$

e. $\frac{1}{2} + \frac{1}{5} = \frac{1}{2} * \left(\frac{5}{5}\right) + \frac{1}{5} * \left(\frac{2}{2}\right) = \frac{5}{10} + \frac{2}{10} = \frac{5+2}{10} = \frac{7}{10}$

f. $\frac{1}{2} - \frac{1}{5} = \frac{1}{2} * \left(\frac{5}{5}\right) - \frac{1}{5} * \left(\frac{2}{2}\right) = \frac{5}{10} - \frac{2}{10} = \frac{5-2}{10} = \frac{3}{10}$

g. $\frac{1}{3} + \frac{1}{4} = \frac{1}{3} * \left(\frac{4}{4}\right) + \frac{1}{4} * \left(\frac{3}{3}\right) = \frac{4}{12} + \frac{3}{12} = \frac{4+3}{12} = \frac{7}{12}$

h. $\frac{1}{3} - \frac{1}{4} = \frac{1}{3} * \left(\frac{4}{4}\right) - \frac{1}{4} * \left(\frac{3}{3}\right) = \frac{4}{12} - \frac{3}{12} = \frac{4-3}{12} = \frac{1}{12}$

i. $\frac{2}{3} + \frac{1}{4} = \frac{2}{3} * \left(\frac{4}{4}\right) + \frac{1}{4} * \left(\frac{3}{3}\right) = \frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$

j. $\frac{2}{3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

k. $\frac{2}{3} - \frac{1}{5} = \frac{2}{3} * \left(\frac{5}{5}\right) - \frac{1}{5} * \left(\frac{3}{3}\right) = \frac{10}{15} - \frac{3}{15} = \frac{10-3}{15} = \frac{7}{15}$

l. $\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{1}{4} * \left(\frac{2}{2}\right) = \frac{1}{8} + \frac{2}{8} = \frac{1+2}{8} = \frac{3}{8}$

m. $\frac{1}{4} - \frac{1}{8} = \frac{1}{4} * \left(\frac{2}{2}\right) - \frac{1}{8} = \frac{2}{8} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$

n. $\frac{1}{4} - \frac{1}{5} = \frac{1}{4} * \left(\frac{5}{5}\right) - \frac{1}{5} * \left(\frac{4}{4}\right) = \frac{5}{20} - \frac{4}{20} = \frac{5-4}{20} = \frac{1}{20}$

o. $\frac{3}{4} - \frac{1}{5} = \frac{3}{4} * \left(\frac{5}{5}\right) - \frac{1}{5} * \left(\frac{4}{4}\right) = \frac{15}{20} - \frac{4}{20} = \frac{15-4}{20} = \frac{11}{20}$

p. $\frac{3}{2}+\frac{1}{4}=\frac{3}{2}*\left(\frac{2}{2}\right)+\frac{1}{4}=\frac{6}{4}+\frac{1}{4}=\frac{6+1}{4}=\frac{7}{4}$

q. $\frac{5}{8}-\frac{1}{4}=\frac{5}{8}*\left(\frac{3}{3}\right)+\frac{1}{3}*\left(\frac{8}{8}\right)=\frac{15}{24}+\frac{8}{24}=\frac{15+8}{24}=\frac{23}{24}$

r. $\frac{5}{8}-\frac{1}{4}=\frac{5}{8}-\frac{1}{4}*\left(\frac{2}{2}\right)=\frac{5}{8}-\frac{2}{8}=\frac{5-2}{8}=\frac{3}{8}$

s. $\frac{3}{7}+\frac{1}{3}=\frac{3}{7}*\left(\frac{3}{3}\right)+\frac{1}{3}*\left(\frac{7}{7}\right)=\frac{9}{21}+\frac{7}{21}=\frac{9+7}{21}=\frac{16}{21}$

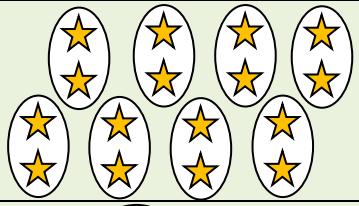
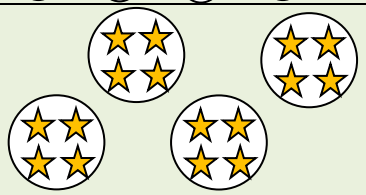
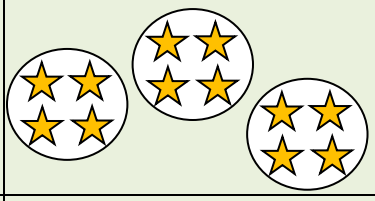
t. $\frac{3}{7}+\frac{1}{5}=\frac{3}{7}*\left(\frac{5}{5}\right)-\frac{1}{5}*\left(\frac{7}{7}\right)=\frac{15}{35}+\frac{7}{35}=\frac{15+7}{35}=\frac{22}{35}$

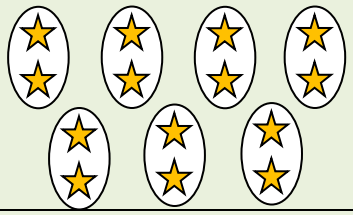
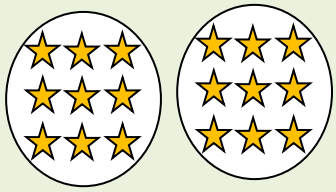
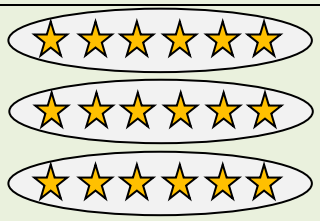
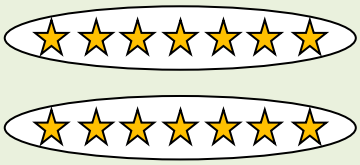
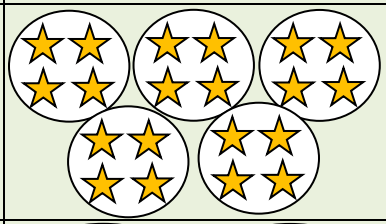
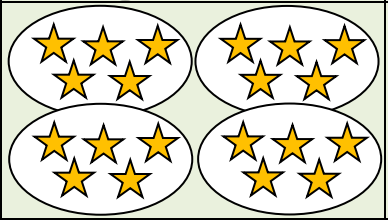
u. $\frac{3}{7}-\frac{1}{5}=\frac{3}{7}*\left(\frac{5}{5}\right)-\frac{1}{5}*\left(\frac{7}{7}\right)=\frac{15}{35}-\frac{7}{35}=\frac{15-7}{35}=\frac{8}{35}$

v. $\frac{1}{3}+\frac{1}{4}=\frac{1}{3}*\left(\frac{4}{4}\right)+\frac{3}{4}*\left(\frac{3}{3}\right)=\frac{4}{12}+\frac{9}{12}=\frac{4+9}{12}=\frac{13}{12}$

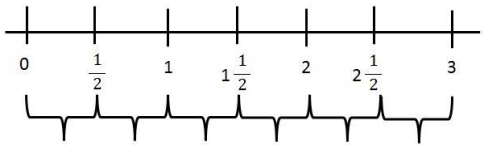
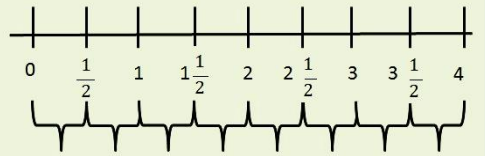
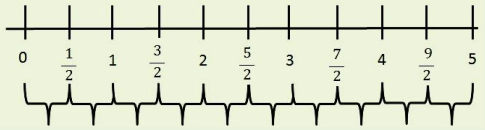
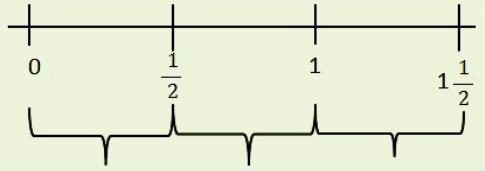
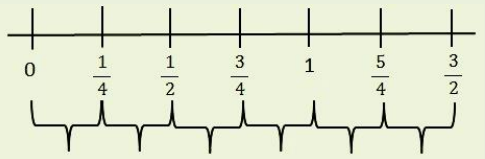
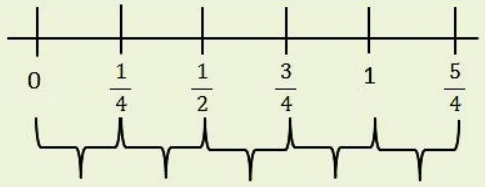
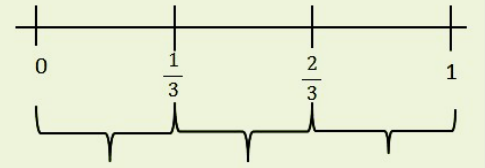
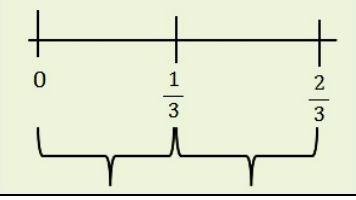
w. $\frac{1}{2}-\frac{1}{7}=\frac{1}{2}*\left(\frac{7}{7}\right)-\frac{1}{7}*\left(\frac{2}{2}\right)=\frac{7}{14}-\frac{2}{14}=\frac{7-2}{14}=\frac{5}{14}$

Practice 16: One Meaning of Division

Problem	This Means	Picture	Answer
8 ÷ 4	How many 4's are in 8		There are 2 groups of 4 in 8. 8 ÷ 4 = 2
10 ÷ 2	How many 2's are in 10		There are 5 groups of 2 in 10. 10 ÷ 2 = 5
15 ÷ 3	How many 3's are in 15		There are 5 groups of 3 in 15. 15 ÷ 3 = 5
16 ÷ 2	How many 2's are in 16		There are 8 groups of 2 in 16. 16 ÷ 2 = 8
16 ÷ 4	How many 4's are in 16		There are 4 groups of 4 in 16. 16 ÷ 4 = 4
12 ÷ 4	How many 4's are in 12		There are 3 groups of 4 in 12. 12 ÷ 4 = 3
12 ÷ 3	How many 3's are in 12		There are 4 groups of 3 in 12. 12 ÷ 3 = 4
12 ÷ 6	How many 6's are in 12		There are 2 groups of 6 in 12. 12 ÷ 6 = 2

Problem	This Means	Picture	Answer
8 ÷ 2	How many 2's are in 8		There are 4 groups of 2 in 8. 8 ÷ 2 = 4
14 ÷ 2	How many 2's are in 14		There are 7 groups of 2 in 14. 14 ÷ 2 = 7
18 ÷ 9	How many 9's are in 18		There are 2 groups of 9 in 18. 18 ÷ 9 = 2
18 ÷ 3	How many 3's are in 18		There are 6 groups of 3 in 18. 8 ÷ 3 = 6
18 ÷ 6	How many 6's are in 18		There are 3 groups of 6 in 18. 18 ÷ 6 = 3
14 ÷ 7	How many 7's are in 14		There are 2 groups of 7 in 14. 14 ÷ 7 = 2
20 ÷ 4	How many 4's are in 20		There are 5 groups of 4 in 20. 20 ÷ 4 = 5
20 ÷ 5	How many 5's are in 20		There are 4 groups of 5 in 20. 20 ÷ 5 = 4

Practice 17: Division and Number Lines

$a \div b$	How many b's are in a	Draw a Number Line Picture	Answer
$3 \div \frac{1}{2}$	How many $\frac{1}{2}$'s in 3 ?		$3 \div \frac{1}{2} = 6$
$4 \div \frac{1}{2}$	How many $\frac{1}{2}$'s in 4 ?		$4 \div \frac{1}{2} = 8$
$5 \div \frac{1}{2}$	How many $\frac{1}{2}$'s in 5 ?		$5 \div \frac{1}{2} = 10$
$1\frac{1}{2} \div \frac{1}{2}$	How many $\frac{1}{2}$'s in $1\frac{1}{2}$?		$1\frac{1}{2} \div \frac{1}{2} = 3$
$1\frac{1}{2} \div \frac{1}{4}$	How many $\frac{1}{4}$'s in $1\frac{1}{2}$?		$1\frac{1}{2} \div \frac{1}{4} = 6$
$1\frac{1}{4} \div \frac{1}{4}$	How many $\frac{1}{4}$'s in $1\frac{1}{4}$?		$1\frac{1}{4} \div \frac{1}{4} = 5$
$1 \div \frac{1}{3}$	How many $\frac{1}{3}$'s in 1 ?		$1 \div \frac{1}{3} = 3$
$\frac{2}{3} \div \frac{1}{3}$	How many $\frac{1}{3}$'s in $\frac{2}{3}$?		$\frac{1}{2} \div \frac{1}{3} = 2$

Practice 18: Division of Fractions Practice

a. $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} * \frac{3}{1} = \frac{1*3}{2*1} = \frac{3}{2}$	b. $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} * \frac{2}{1} = \frac{1*2}{3*1} = \frac{2}{3}$
c. $\frac{2}{5} \div \frac{1}{5} = \frac{2}{\cancel{5}} * \frac{\cancel{5}}{1} = \frac{2}{1} = 2$	d. $\frac{2}{5} \div \frac{2}{5} = \frac{2}{5} * \frac{5}{2} = \frac{\cancel{2}*5}{5*\cancel{2}} = 1$
e. $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} * \frac{2}{1} = \frac{3*\cancel{2}}{2*\cancel{2}*1} = \frac{3}{2}$	f. $\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} * \frac{4}{1} = \frac{3*\cancel{4}}{\cancel{4}*1} = \frac{3}{1} = 3$
g. $\frac{3}{4} \div \frac{1}{5} = \frac{3}{4} * \frac{5}{1} = \frac{3*5}{4*1} = \frac{15}{4}$	h. $\frac{3}{4} \div \frac{1}{6} = \frac{3}{4} * \frac{6}{1} = \frac{3*\cancel{2}*3}{\cancel{2}*2*1} = \frac{9}{2}$
i. $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} * \frac{8}{1} = \frac{3*\cancel{2}*\cancel{2}*2}{\cancel{2}*\cancel{2}*1} = \frac{6}{1} = 6$	j. $\frac{10}{11} \div \frac{12}{11} = \frac{10}{11} * \frac{11}{12} = \frac{\cancel{2}*5*\cancel{11}}{\cancel{11}*\cancel{2}*2*3} = \frac{5}{6}$
k. $\frac{10}{7} \div \frac{15}{14} = \frac{10}{7} * \frac{14}{15} = \frac{2*\cancel{5}*\cancel{7}*2}{\cancel{7}*3*\cancel{5}} = \frac{4}{3}$	l. $\frac{15}{16} \div \frac{1}{8} = \frac{15}{16} * \frac{8}{1} = \frac{3*5*\cancel{8}}{2*\cancel{8}*1} = \frac{15}{2}$
m. $\frac{15}{16} \div \frac{8}{7} = \frac{15}{16} * \frac{8}{7} = \frac{3*5*\cancel{2}*\cancel{2}*\cancel{2}}{2*\cancel{2}*\cancel{2}*\cancel{2}*7} = \frac{15}{14}$	n. $\frac{12}{17} \div \frac{6}{5} = \frac{12}{17} * \frac{5}{6} = \frac{2*\cancel{6}*5}{17*\cancel{6}} = \frac{10}{17}$
o. $\frac{3}{17} \div \frac{5}{6} = \frac{3}{17} * \frac{6}{5} = \frac{3*6}{17*5} = \frac{18}{85}$	p. $\frac{4}{9} \div \frac{8}{27} = \frac{4}{9} * \frac{27}{8} = \frac{\cancel{4}*3*\cancel{9}}{\cancel{9}*2*\cancel{4}} = \frac{3}{2}$
q. $\frac{4}{9} \div \frac{1}{3} = \frac{4}{9} * \frac{3}{1} = \frac{2*2*\cancel{3}}{\cancel{3}*3*1} = \frac{4}{3}$	r. $\frac{9}{7} \div \frac{9}{7} = \frac{9}{7} * \frac{7}{9} = \frac{9*7}{7*9} = 1$

s. $\frac{9}{7} \div \frac{7}{9} = \frac{9}{7} * \frac{9}{7} = \frac{9 * 9}{7 * 7} = \frac{81}{49}$	t. $\frac{14}{15} \div \frac{7}{3} = \frac{14}{15} * \frac{3}{7} = \frac{2 * \cancel{7} * \cancel{3}}{\cancel{3} * 5 * \cancel{7}} = \frac{2}{5}$
u. $\frac{9}{11} \div \frac{27}{22} = \frac{9}{11} * \frac{22}{27} = \frac{\cancel{9} * 2 * \cancel{11}}{\cancel{11} * 3 * \cancel{9}} = \frac{2}{3}$	v. $\frac{5}{33} \div \frac{10}{3} = \frac{5}{33} * \frac{3}{10} = \frac{\cancel{5} * \cancel{3} * 1}{\cancel{3} * 11 * 2 * \cancel{5}} = \frac{1}{22}$
w. $\frac{5}{33} \div \frac{3}{22} = \frac{5}{33} * \frac{22}{3} = \frac{5 * 2 * \cancel{11}}{3 * \cancel{11} * 3} = \frac{10}{9}$	x. $\frac{12}{7} \div \frac{8}{21} = \frac{12}{7} * \frac{21}{8} = \frac{\cancel{2} * \cancel{2} * 3 * 3 * \cancel{7}}{\cancel{7} * \cancel{2} * \cancel{2} * 2} = \frac{9}{2}$
y. $\frac{12}{25} \div \frac{3}{5} = \frac{12}{25} * \frac{5}{3} = \frac{\cancel{3} * 4 * \cancel{5}}{\cancel{5} * 5 * \cancel{3}} = \frac{4}{5}$	z. $\frac{3}{2} \div \frac{12}{25} = \frac{3}{2} * \frac{25}{12} = \frac{\cancel{3} * 5 * 5}{2 * 2 * 2 * \cancel{3}} = \frac{25}{8}$
aa. $\frac{3}{5} \div \frac{3}{5} = \frac{3}{5} * \frac{5}{3} = \frac{3 * 5}{5 * 3} = 1$	bb. $3 \div \frac{1}{4} = \frac{3}{1} * \frac{4}{1} = \frac{3 * 4}{1 * 1} = \frac{12}{1} = 12$
cc. $5 \div \frac{1}{6} = \frac{5}{1} * \frac{6}{1} = \frac{5 * 6}{1 * 1} = \frac{30}{1} = 30$	dd. $\frac{1}{4} \div 2 = \frac{1}{4} * \frac{1}{2} = \frac{1 * 1}{4 * 2} = \frac{1}{8}$
ee. $\frac{1}{9} \div \frac{1}{9} = \frac{1}{9} * \frac{9}{1} = \frac{1 * 9}{9 * 1} = \frac{9}{9} = 1$	ff. $\frac{1}{9} \div 9 = \frac{1}{9} * \frac{1}{9} = \frac{1 * 1}{9 * 9} = \frac{1}{81}$
gg. $9 \div \frac{1}{9} = \frac{9}{1} * \frac{9}{1} = \frac{9 * 9}{1 * 1} = \frac{81}{1} = 81$	hh. $\frac{3}{8} \div \frac{6}{7} = \frac{3}{8} * \frac{7}{6} = \frac{\cancel{3} * 7}{8 * 2 * \cancel{3}} = \frac{7}{16}$
ii. $\frac{17}{18} \div \frac{17}{9} = \frac{17}{18} * \frac{9}{17} = \frac{\cancel{17} * \cancel{9}}{2 * \cancel{9} * \cancel{17}} = \frac{1}{2}$	jj. $\frac{17}{9} \div \frac{17}{18} = \frac{17}{9} * \frac{18}{17} = \frac{\cancel{17} * 2 * \cancel{9}}{\cancel{9} * \cancel{17}} = \frac{2}{1} = 1$

Help for Helpers

I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as "I know you can do this. I believe in you." And then leave the student alone to do the

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn't going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don't generally want to ask our boss or friend: "Am I right? Am I right?" As an adult, we have to know the answer is right ourselves. The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student's life. You will gain confidence in your students as you watch them be successful on their own.

Fractions are a huge part of math. Also, they are useful in everyday life. But their major importance is that they are an absolutely critical part of algebra. All Algebra 1 consists of is: fraction operators and the distributive law, with positive and negative numbers and variables, like x. So understanding fractions and fraction operations is a key to Algebra success.

The Way to Factor

Factoring means to take a number and find the smaller whole numbers that multiplied together give you that number.

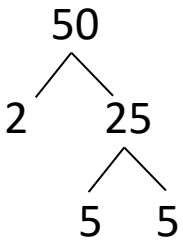
Example: $2 * 3 = 6$
2 and 3 are factors of 6

The method I use breaks numbers into their prime factors.

I start with the smallest prime factors and work my way up.

Here is a very useful secret. In my materials and all math books, math teachers write the problems. We use numbers with small factors, usually less than 11, so 2, 3, 5, 7, 11. On rare occasions we may use 13 or 17. This makes your life much easier.

Take a number like 50:



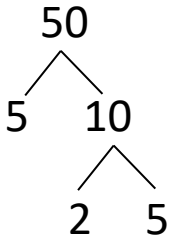
Note: $2 * 25 = 50$

2 is prime so we are done there

5 is prime

50 = 2 * 5 * 5

Another way:



5 is prime

50 = 2 * 5 * 5 Same answer