MATH WHISPERER

Where math makes sense

Research Edition
Number Sense Series

Fractions 1

Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information, please go to www.mathwhisperer.com

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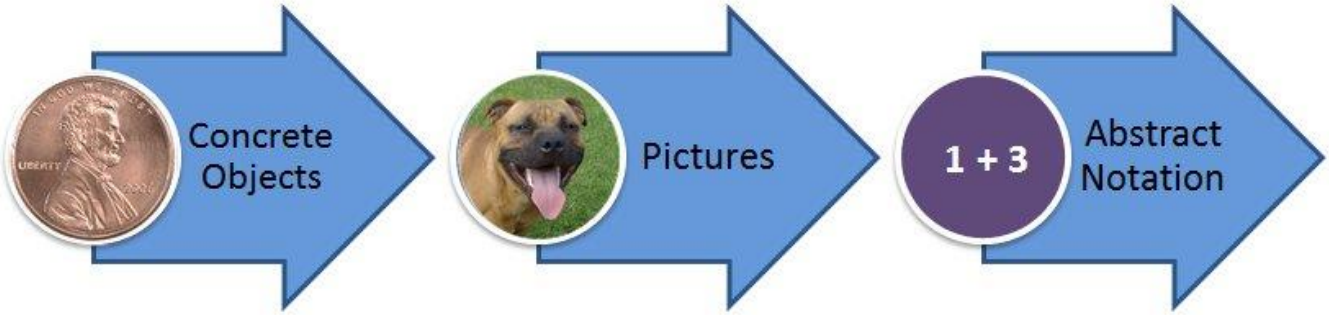
DEDICATION

Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

For help, watch videos at mathwhisperer.com/fraction-videos

1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is actually supposed to make sense. When you start with hands-on objects, math can make sense.



You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three-step progression works for everybody.

It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three-step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. **Wouldn't it feel great to never have to learn this again?** The math will stick with you with the three-step progression.

Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you.

Welcome to Fractions 1

You will need:

- Colored Fraction Pieces – page 9. While they will work in black and white you are encouraged to use a colored set. It will make things a little easier for you.
- Fraction order set – page 39
- Scissors

Welcome to fractions. This is where fractions will make sense to you and become easy. It is amazing how easy fractions can be. I am so happy you are giving yourself this gift of truly understanding fractions.

These lessons will look different from what you have done before. I guarantee you that they work. Your part is to trust me that the fraction pieces are necessary—no short cuts. And just watch yourself be fantastically successful in no time.

2. What is a Fraction?
Activity 1: What is a fraction really?

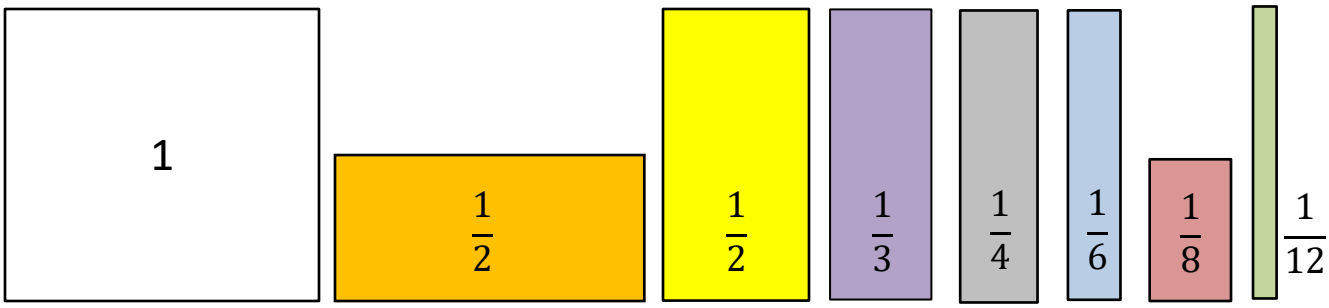
You will need

- Fraction Pieces – page 9

Your first step is to cut out the Fraction Pieces. Carefully cut on the black lines.

Now figure out the names of your fraction pieces. To begin with, let the big rectangle equal 1. How do you decide what the names of the pieces are?

Here are some answers:



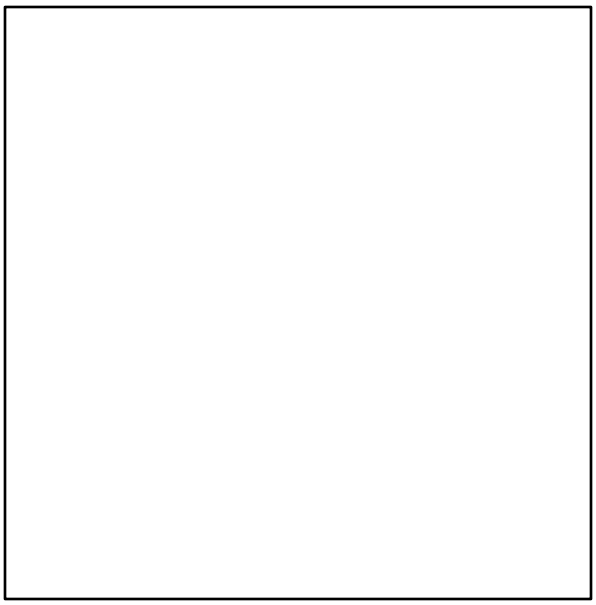
Here is how I decide how the pieces are named: I look at them as a division problem written up to down

$\frac{1}{x}$ ↓ is a division problem written up to down.

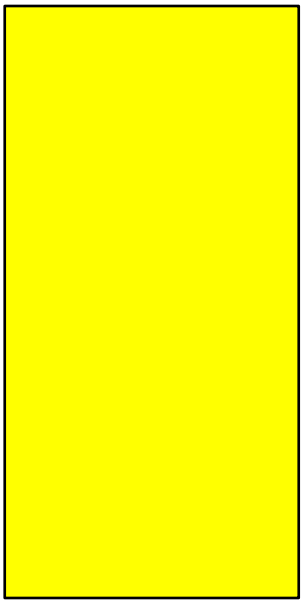
It is read as, "1 object divided into x pieces" going left to right. →

For example, $\frac{1}{2}$ means **one** object divided into **two** pieces.

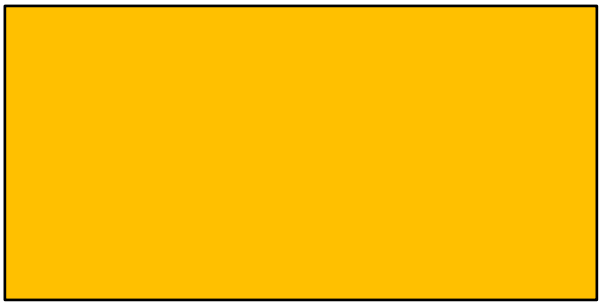
You may decide what the names of the pieces are in a different way.
If so, write down your way:



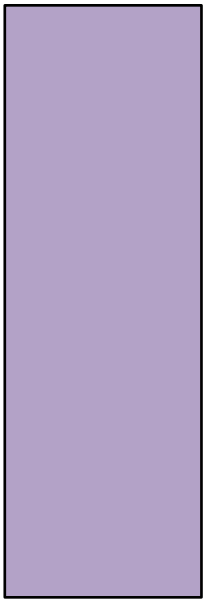
white



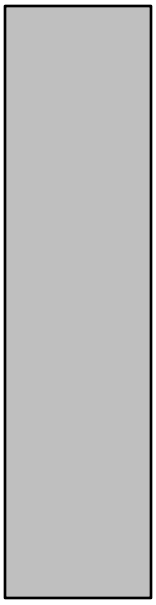
yellow



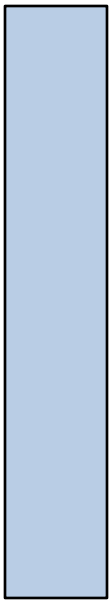
orange



purple

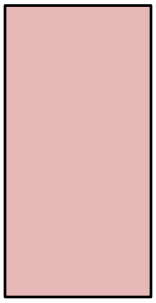


gray

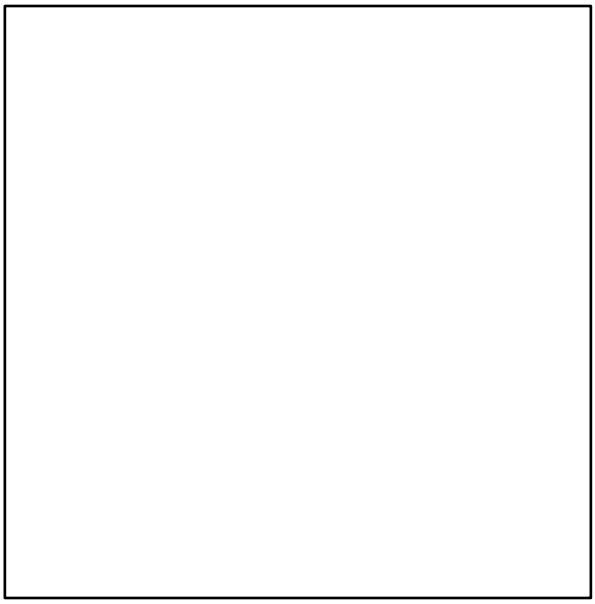


blue

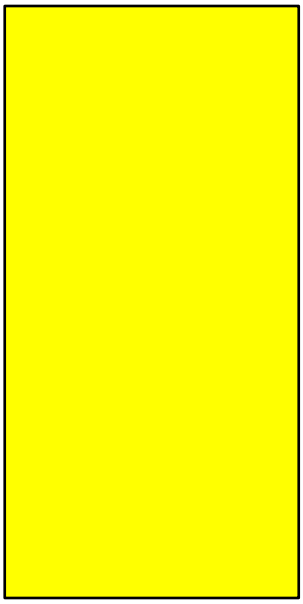
green



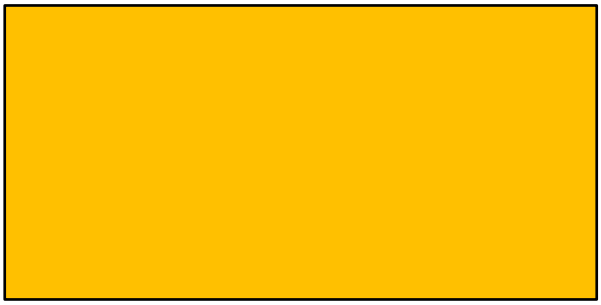
pink



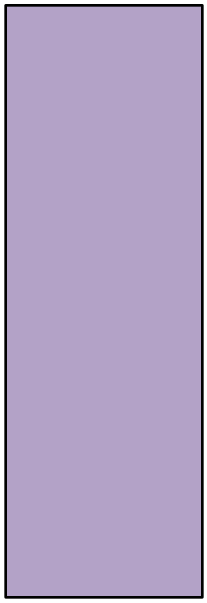
white



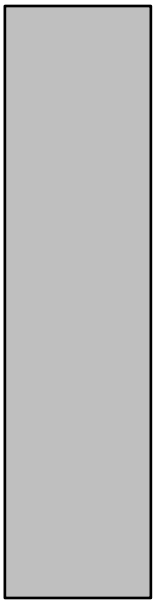
yellow



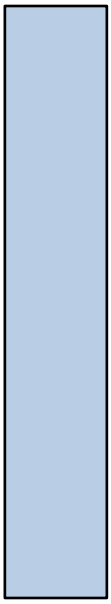
orange



purple

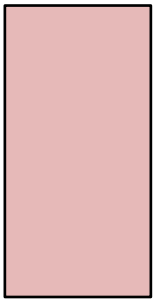


gray



blue

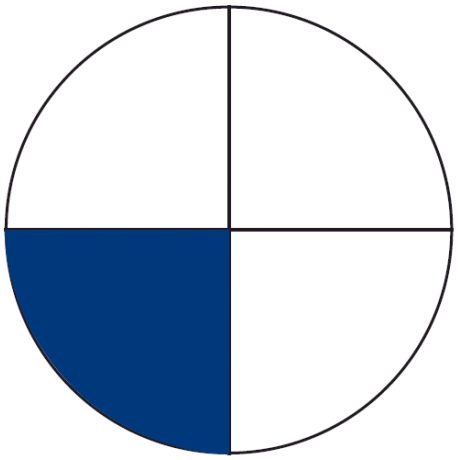
green

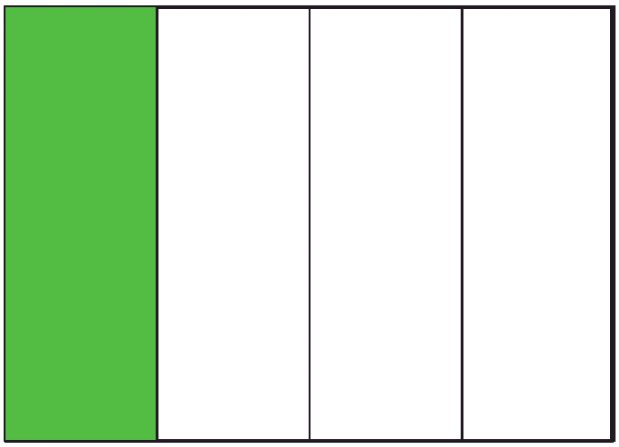


pink

Practice 1: Explaining one fourth

Explain why each of these can be called one fourth $\{\frac{1}{4}\}$ when they look so different.





Activity 2: Explaining one fourth again

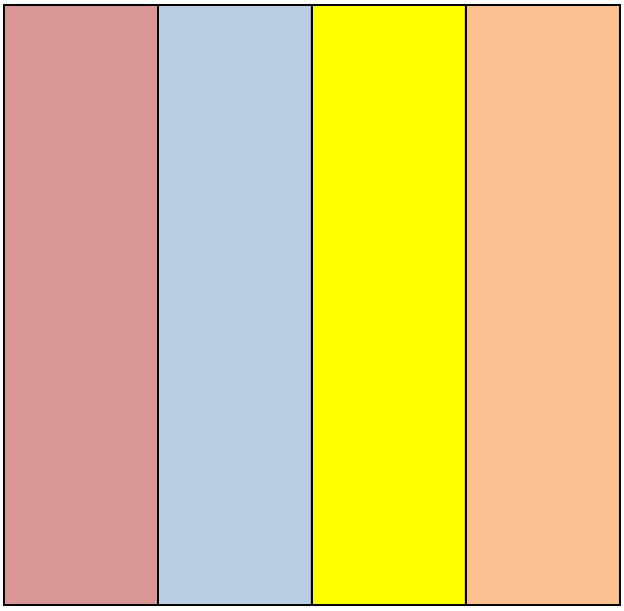
Materials

- Scissors

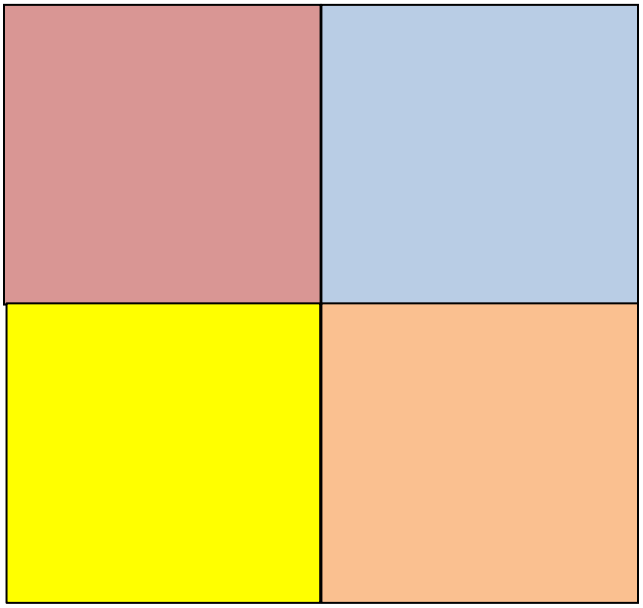
Cut each rectangle below into fourths.

Then compare $\frac{1}{4}$ of rectangle A with $\frac{1}{4}$ of rectangle B. Are they the same? _____

How are they different? How are they the same?



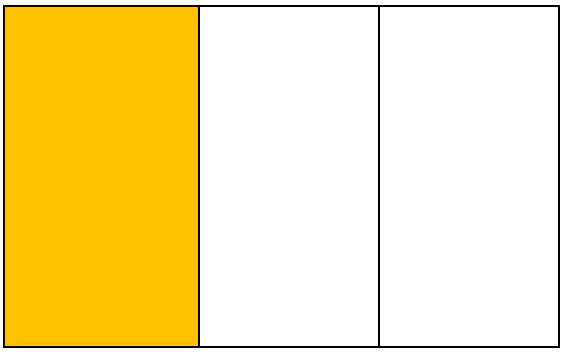
Rectangle A

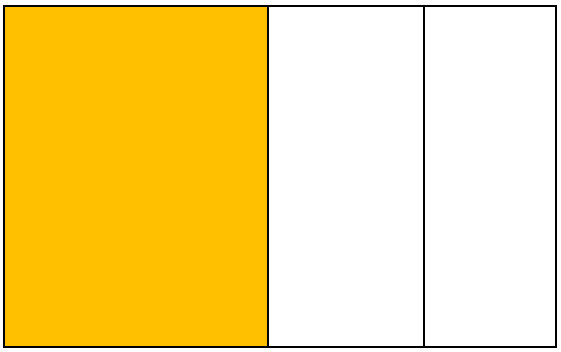


Rectangle B

Practice 2: Thirds

Does each picture show $\frac{1}{3}$? _____





Explain why or why not.

Sometimes math can be frustrating. What is important is to stick with the problem. Albert Einstein would work on the same problems for years. So plan on being frustrated sometimes. If need to yell or cry or run around the house, that is ok.

This next activity brings up emotions for many students. If that happens to you, stick with it. Practice 3 really helps you understand fractions. It is worth the frustration.

Answers – page 80

Practice 3: Can you name me now?

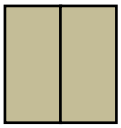
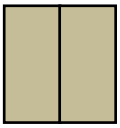
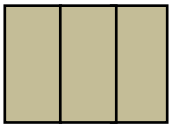
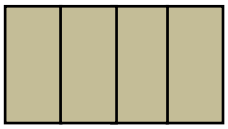
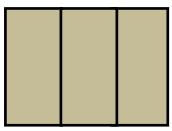
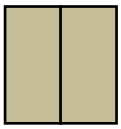
First, how did you determine the name of each fraction piece (in Activity 1)?

Second, use your cutouts and change the whole piece to the piece that is indicated, then tell the fraction of that whole for each description. Hint: Some of the answers will be greater than one.

If the whole is:	Tell what fraction of the whole each of these is:		
White = 1	Pink = ?	Green = ?	Orange = ?
Pink = 1	White = ?	Gray = ?	Yellow = ?
Green = 1	White = ?	Gray = ?	Purple = ?
Orange = 1	Blue = ?	Gray = ?	White =?
Purple = 1	White = ?	Blue = ?	Green = ?

Answers – page 81

Practice 4: What is "1"?

If this is $\frac{1}{3}$ 	draw 1
If this is $\frac{2}{5}$ 	draw 1
If this is $\frac{2}{3}$ 	draw 1
If this is $\frac{3}{4}$ 	draw 1
If this is $\frac{1}{6}$ 	draw 1
If this is 2 	draw 1
If this is 3 	draw 1
If this is $\frac{4}{3}$ 	draw 1
If this is $\frac{3}{2}$ 	draw 1
If this is 2 	draw 1

Practice 5: Quick summary

Look back over Practices 1 through 4, and Activities 1 and 2. Why is it so important to know what “one” is?

Answers – page 83

3. Naming the parts of a fraction

These are the parts of the fraction:

$$\frac{a}{b}$$

numerator
fraction bar
denominator

Names of denominators are:

Halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, tenths, etc.

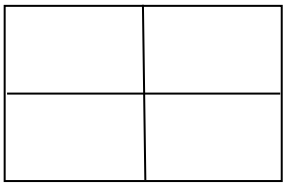
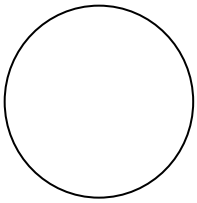
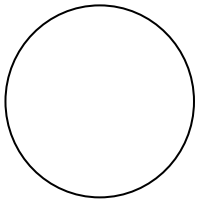
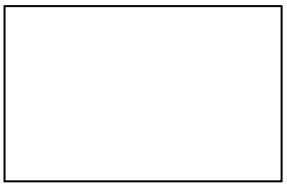
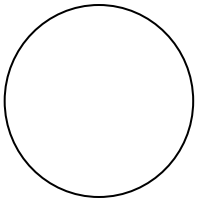
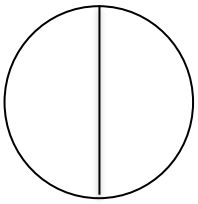
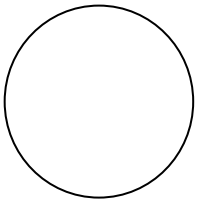
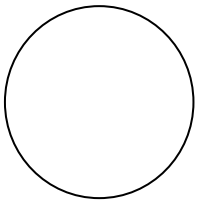
Notice that except for halves and thirds, all denominator names end in "ths." It's kinds of like the Mr., Mrs., or Ms. for fractions. When you hear Mr. Smith or Ms. Jones, you know the person's last name. When you hear "fourths" or "fifths" you know it's a fraction denominator name.

We will be looking at the meaning of the fraction bar in a few pages. It means "divide".

Personally, I do not like learning math vocabulary. Hopefully you are different. But regardless, it's useful to know the names of the parts and what they do. So I urge you to work through this. You may want to refer back to this later. The different parts of the fraction will have more meaning for you as you do the practices.

Practice 6: Denominator practice

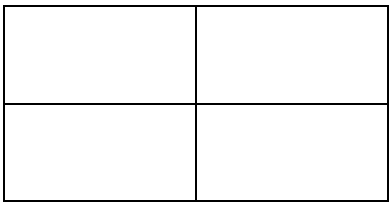
The first row is done for you.

Denominator	Fraction Name in Words	This is One	Fraction Picture
$\frac{}{4}$	fourths		
	thirds		
$\frac{}{5}$			
			
	fourths		
	sevenths		

The meaning of fraction symbols

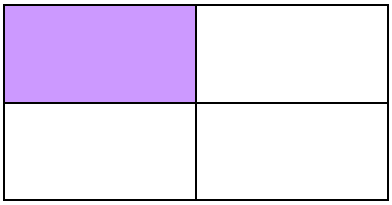
The **denominator** only tells us how many pieces we need to construct a whole and therefore what to call the pieces: their name.

For example, if we know the rectangle is "one," here is the picture. Denominator is 4.

$\frac{\quad}{4}$ 

But we don't know how many pieces to shade. For this we need the numerator.

The **numerator** tells us how many pieces we actually have. Let's look at our example. If we know that the rectangle is "one," here is the picture:

$\frac{1}{4}$ 

The fraction bar is a division sign. $\frac{a}{b}$ is the same as $a \div b$ or $b\overline{)\,a}$

For example, $\frac{1}{2}$ is the same as $2\overline{)\,1}$

or one object divided into 2 equal parts

4. Size of fractions

Fractions can be smaller than 1 or greater than 1. For fractions greater than 1, there are two names.

Following are two new vocabulary words you will read in math textbooks for fractions that represent numbers greater than one:

A **mixed number** is a whole number next to a fraction, such as $1\frac{1}{2}$, or $2\frac{3}{5}$.

I think this name makes some sense. A mixed number is a whole number with a fraction, mixed together. It is read like there is an "and" in between the numbers, like "one and a half," or "two and three fifths."

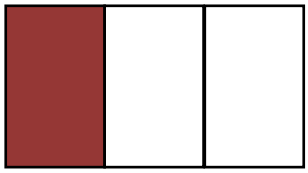
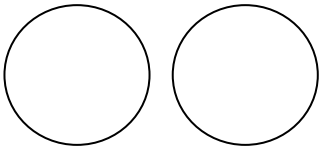
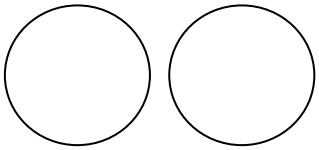
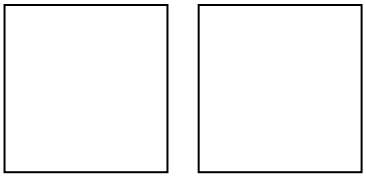
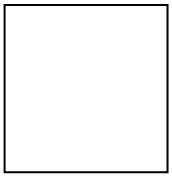
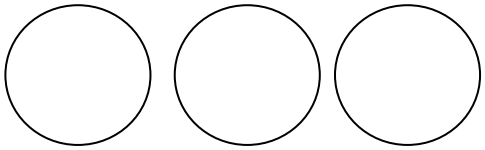
An **improper fraction** is the other name. When a fraction is written as $\frac{a}{b}$, if a in greater than b that fraction is called "improper."

An improper fraction is a very strange name. The main reason I think this is a strange name is that if you want to add, subtract, multiply, or divide fractions, they must be in the form of $\frac{a}{b}$.

$\frac{5}{8}$ is not an improper fraction, but $\frac{8}{5}$ is. Yet, $\frac{8}{5}$ is the form necessary in to add, subtract, multiply, or divide. So what's improper about that? I think that name is a mistake, but it was invented before my time.

Practice 7: Getting more familiar with numerators and denominators

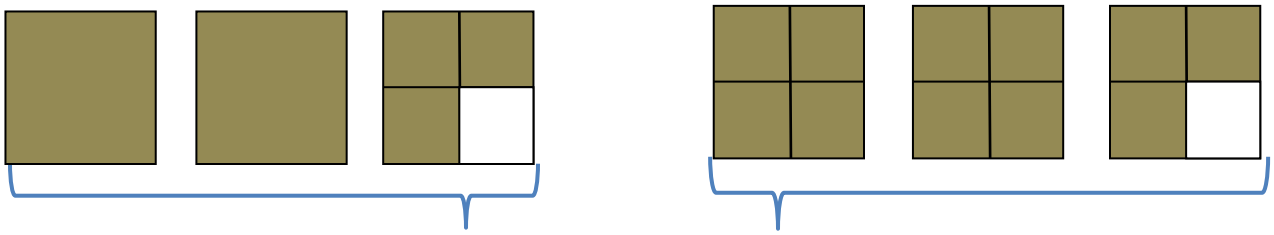
Specifically, the denominator tells us how many pieces the whole is cut into, and the numerator tells us how many of those pieces we have. This activity uses different shapes for "one whole." Shade the amount to show what the fraction says. Hint: Some of these will be greater than one.

Box 1: denominator 3, numerator 1	Box 2: denominator 4, numerator 5
	
Box 3: denominator 3, numerator 6	**Box 4: denominator 8, numerator 0**
	
Box 5: denominator 4, numerator 3	**Box 6: denominator 3, numerator 4**
	
Box 7: denominator 2, numerator 2	**Box 8: denominator 9, numerator 7**
	
Box 9: denominator 1, numerator 1	**Box 10: denominator 2, numerator 5**
	

Practice 8: Mixed numbers and "Improper" fractions

Here is a picture of a **mixed** number. It is a **mixed** number because it is a whole number with a fraction.

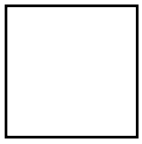
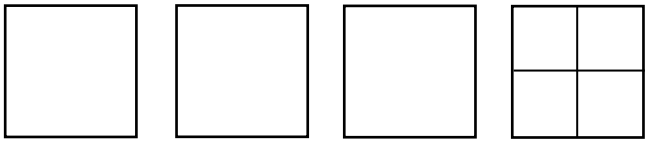
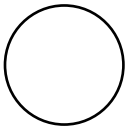
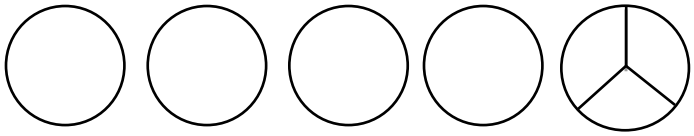
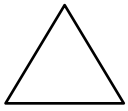
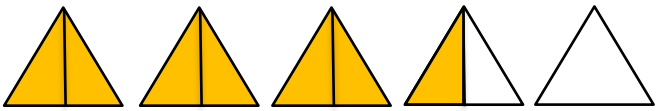
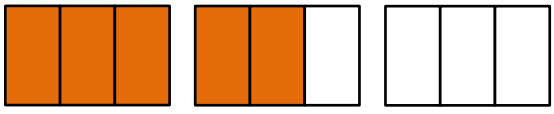
Also, here is a picture of its "**improper**" fraction form. It is in improper fraction form because it is made of only a numerator, fraction bar, and denominator.



The mixed number: $2\frac{3}{4}$ = $\frac{11}{4}$ improper fraction form

These are both ways of expressing an amount that is more than one whole.

Shade the pictures to match the given mixed number or "improper" fraction form. If the picture is already shaded, give both the mixed number and fraction form.

One	Picture	Mixed Number	"Improper" Fraction Form
		$3\frac{3}{4}$	
			$\frac{14}{3}$
			
			

One	Picture	Mixed Number	"Improper" Fraction Form
		$2\frac{4}{5}$	
			$\frac{16}{4}$

For the remaining problems, draw any shape you like to check your answers.			
		$2\frac{1}{3}$	
			$\frac{5}{2}$
		$3\frac{1}{4}$	
			$\frac{10}{3}$
			$\frac{10}{2}$

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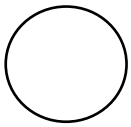
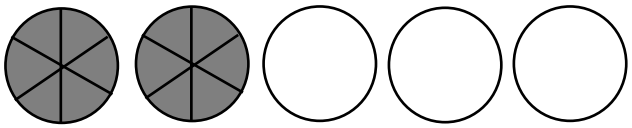
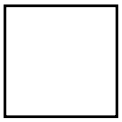
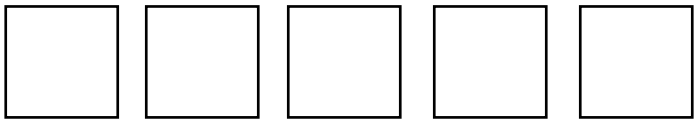
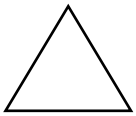
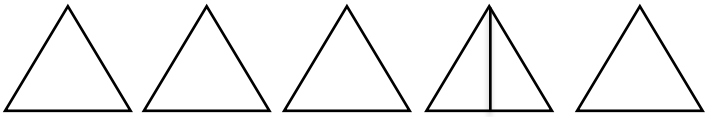
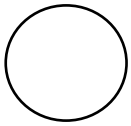
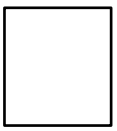
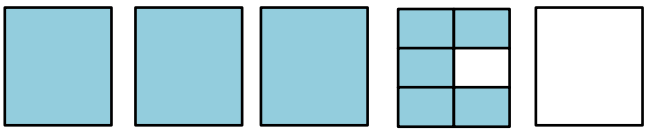
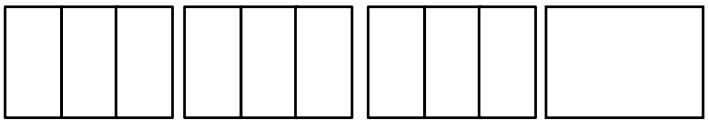
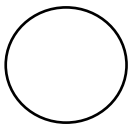
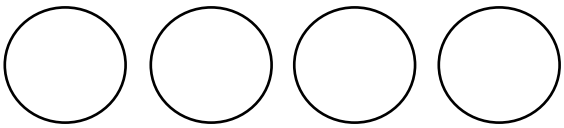
Practice 9: More mixed numbers and "Improper" fractions

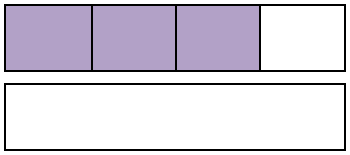
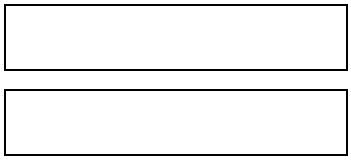
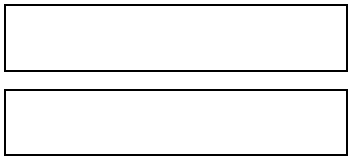
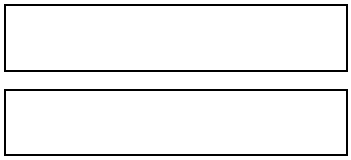
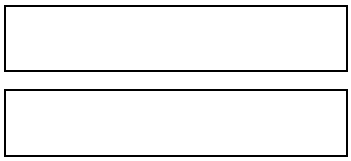
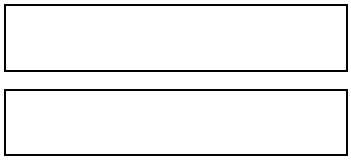
Give the missing forms of each amount.

"Improper" Fraction Form	One	Picture	Mixed Number
			
$\frac{18}{4}$			
			$3\frac{1}{2}$
			
			
$\frac{9}{3}$			
			$2\frac{3}{4}$

Practice 10: Practice with fractions greater and less than one

Show a picture for each fraction by shading in the rectangles. Each rectangle is one whole. Then write whether the given fraction is greater than, less than, or equal to 1. Then write as a mixed number, if possible.

The symbol for less than is **<**. The symbol for greater than is **>**.

Fraction	One	Picture	Compare to 1 Use >, <, or =	Mixed Number if > 1
$\frac{3}{4}$			$\frac{3}{4} < 1$	
$\frac{7}{4}$				
$\frac{6}{6}$				
$\frac{5}{4}$				
$\frac{5}{8}$				
$\frac{10}{8}$				

Fraction	One	Picture	Compare to 1 Use $>$, $<$, or $=$	Mixed Number if > 1
$\frac{9}{4}$				
$\frac{8}{6}$				
$\frac{12}{8}$				
$\frac{8}{3}$				
$\frac{5}{5}$				

Question: We have seen that $\frac{8}{3} > 1$, and $\frac{12}{8} > 1$. How can you tell just by looking at the fraction if it greater than one?

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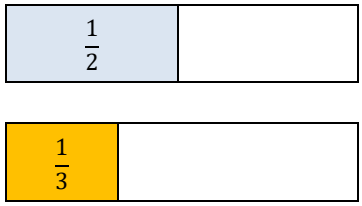
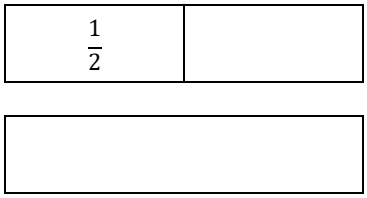
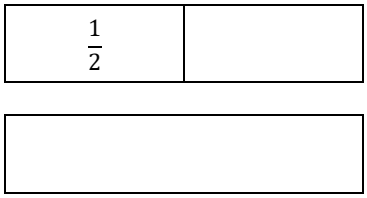
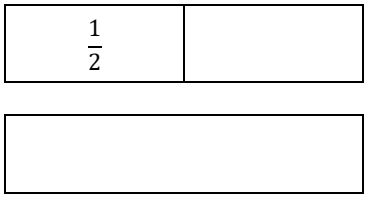
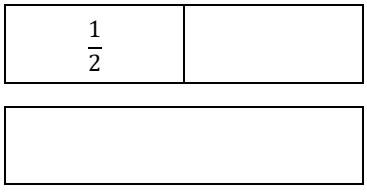
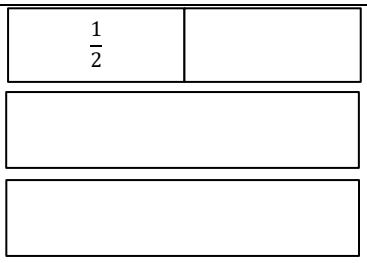
Practice 11: Comparing to $\frac{1}{2}$

One half can be used as a benchmark to determine the relative size of fractions. For each fraction, shade in the appropriate portion of the rectangle divided into halves. Then write whether the given fraction is greater than, less than, or equal to $\frac{1}{2}$. Remember the denominator tells how may pieces to divide the 1 whole into.

Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{1}{3}$		$\frac{1}{3} < \frac{1}{2}$
$\frac{2}{5}$		
$\frac{4}{6}$		
$\frac{1}{4}$		
$\frac{5}{8}$		
$\frac{8}{5}$		

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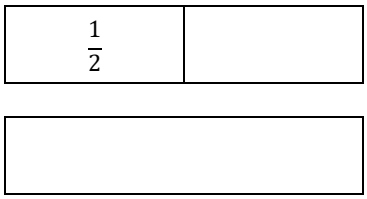
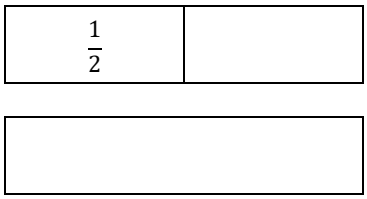
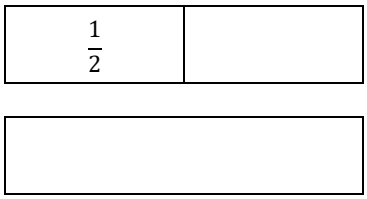
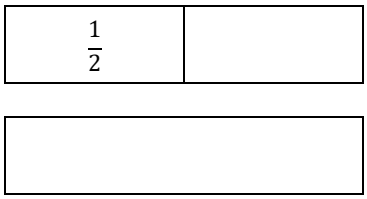
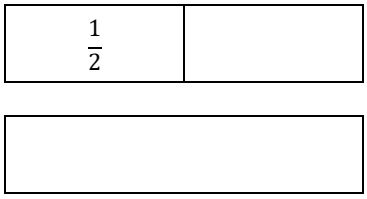
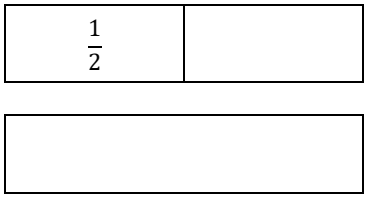
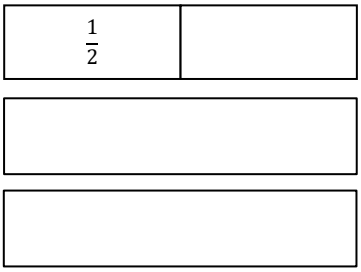
Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{3}{6}$		
$\frac{7}{12}$		
$\frac{8}{10}$		
$\frac{3}{7}$		
$\frac{5}{9}$		
$\frac{3}{12}$		
$\frac{9}{5}$		

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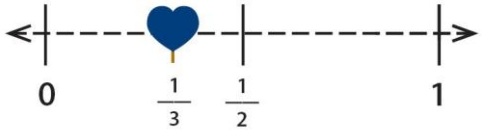
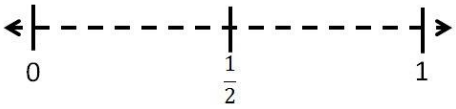
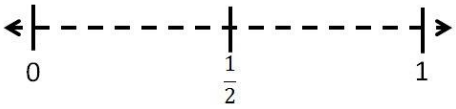
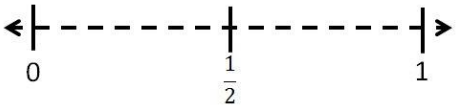
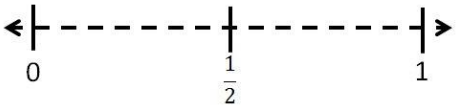
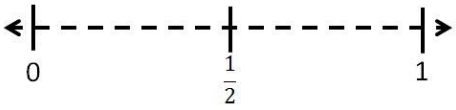
Practice 12: Continued - Using $\frac{1}{2}$ as a benchmark

One half can be used as a benchmark to determine the relative size of fractions. Put the fraction on the number line, then write $>$, $<$, or $=$ in the circle to make a true statement.

Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{1}{3}$		$\frac{1}{3}$ < $\frac{1}{2}$
$\frac{1}{4}$		$\frac{1}{4}$ ◯ $\frac{1}{2}$
$\frac{4}{8}$		$\frac{4}{8}$ ◯ $\frac{1}{2}$
$\frac{4}{5}$		$\frac{4}{5}$ ◯ $\frac{1}{2}$
$\frac{2}{3}$		$\frac{2}{3}$ ◯ $\frac{1}{2}$
$\frac{3}{4}$		$\frac{3}{4}$ ◯ $\frac{1}{2}$

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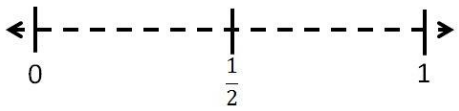
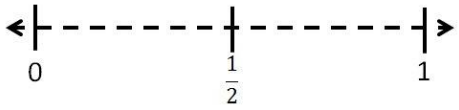
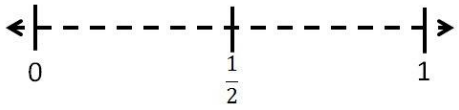
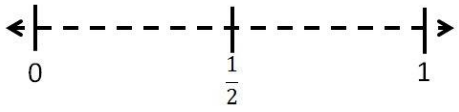
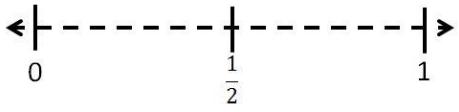
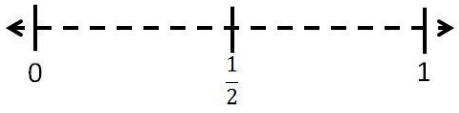
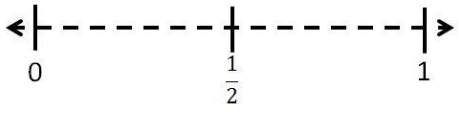
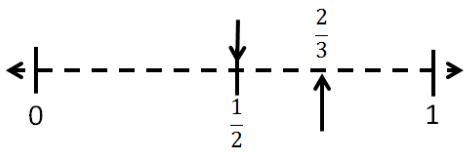
Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{3}{8}$		$\frac{3}{8}$ ◯ $\frac{1}{2}$
$\frac{6}{8}$		$\frac{6}{8}$ ◯ $\frac{1}{2}$
$\frac{4}{10}$		$\frac{4}{10}$ ◯ $\frac{1}{2}$
$\frac{2}{5}$		$\frac{4}{9}$ ◯ $\frac{1}{2}$
$\frac{5}{10}$		$\frac{5}{10}$ ◯ $\frac{1}{2}$
$\frac{9}{10}$		$\frac{9}{10}$ ◯ $\frac{1}{2}$
$\frac{7}{8}$		$\frac{7}{8}$ ◯ $\frac{1}{2}$

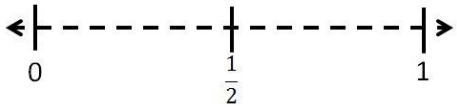
Answers – page 93

Practice 13: Which is larger?

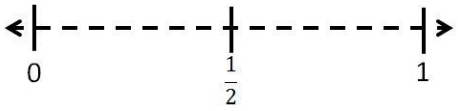
Show where the fractions are on the number line. Write >, <, or = in the circle between the fractions.



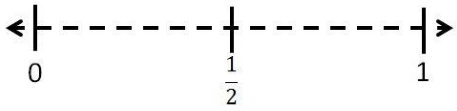
$\frac{2}{3}$ > $\frac{1}{2}$



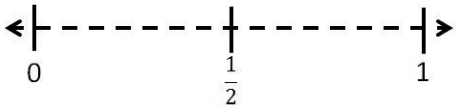
$\frac{2}{10}$ ◯ $\frac{1}{2}$



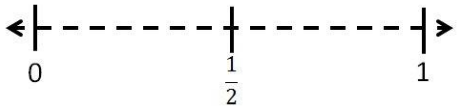
$\frac{3}{4}$ ◯ $\frac{2}{5}$



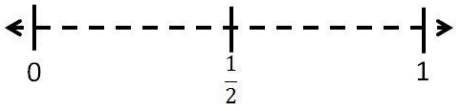
$\frac{1}{5}$ ◯ $\frac{2}{3}$



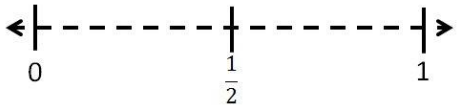
$\frac{3}{6}$ ◯ $\frac{7}{8}$



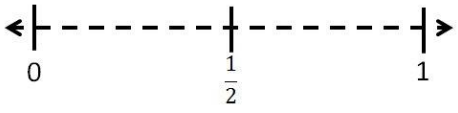
$\frac{6}{10}$ ◯ $\frac{3}{8}$



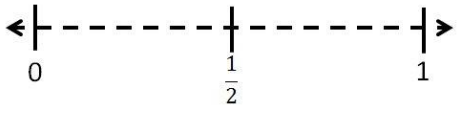
$\frac{5}{6}$ ◯ $\frac{4}{4}$



$\frac{2}{4}$ ◯ $\frac{2}{3}$



$\frac{3}{8}$ ◯ $\frac{3}{4}$



$\frac{2}{4}$ ◯ $\frac{5}{10}$

Answers – page 94

Practice 14: Who is right?

Mrs. Robben has a daughter in elementary school who is just learning about fractions. She thinks that because 3 is bigger than 2 then $\frac{1}{3}$ is bigger than $\frac{1}{2}$. Is Mrs. Robben's daughter correct? Explain why or why not, using words and pictures.

Activity 3: Order, please

Materials

- Fraction Order Set – page 39
- Scissors

Cut out each of the numbers from the Fraction Order Set. Then put them in order from smallest to largest. You may want to have your fraction pieces handy to do this activity.

When I do this practice, I write the number next to the words. It helps me.

Do all the ones you can. If there are some you are unable to use, that is okay. You will be able to do all of them after you learn about equivalent fractions.

If you want to do this with a partner, now you each have a page.

one half	three eighths
one third	four halves
one fourth	four thirds
one fifth	four fifths
one sixth	four tenths
one eighth	five halves
one tenth	five thirds
two thirds	five fourths
two fifths	six halves
three halves	seven tenths
three fourths	nine tenths
three fifths	You choose one

Answers – page 96

If you want to do this with a partner, now you each have a page.

one half	three eighths
one third	four halves
one fourth	four thirds
one fifth	four fifths
one sixth	four tenths
one eighth	five halves
one tenth	five thirds
two thirds	five fourths
two fifths	six halves
three halves	seven tenths
three fourths	nine tenths
three fifths	You choose one

Answers – page 96

5. Equivalent fractions

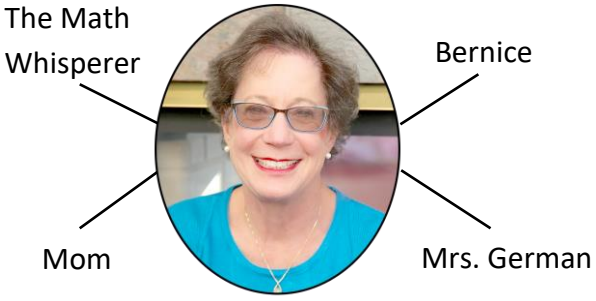
Great job! You have done a lot of work to really understand the concept of fractions. Now it's time to move on to another big topic with fractions, equivalent fractions.

The first step is to get out your fraction pieces. You will use your fraction pieces to create equivalent fractions. You may think you don't need to use your fraction pieces. It may be that you can get the correct answers without the fraction pieces.

I really urge you to use the fraction pieces. The fraction pieces give you the powerful effects of physical modeling, which enhances conceptual understanding. So – please trust me on this, and use your fraction pieces. It will take an extra five to ten minutes, not much time for the positive effects.

The word **equivalent** has the same sound as **equa**l in it.

My name is Bernice German. Some people call me the Math Whisperer, some call me Bernice, some call me Mrs. German, my children call me Mom. They are all names for the same person – me. The names are different, but I'm the same person. Math Whisperer is equivalent to Mrs. German.

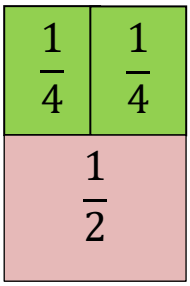


What names do you have?

Here is another example. Say you have a dollar bill. This is equivalent to four quarters.

Is equivalent to

Note that for a fraction to be equivalent, the new fraction must fit exactly over the original fraction pieces. Close matches don't count.



As an example, $\frac{2}{4}$ is equivalent to $\frac{1}{2}$.

Put them on top of each other, and see that they exactly cover each other.

I will show you 3 ways to look at equivalent fractions:

Here is the first way to look at equivalent fractions: Use your fraction pieces.

Equivalent fractions are a very, very important concept in fractions. Like all concepts, we begin with concrete objects. We will use fraction pieces for the concrete representation.

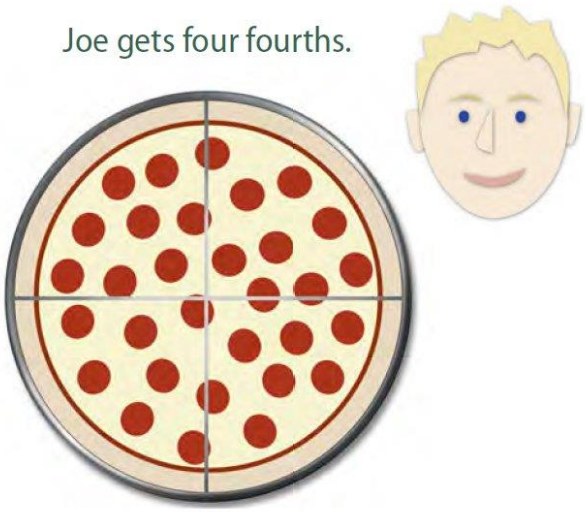
In the space below, use your fraction pieces to trace.

Trace a $\frac{1}{2}$ fraction piece. Then trace over it four $\frac{1}{8}$ fraction pieces.

Based on your tracing, $\frac{1}{2}$ covers the same area as $\frac{?}{8}$

So, $\frac{1}{2}$ is equivalent to $\frac{?}{8}$.

Practice 15: Equivalent fractions - Pizza



Who gets more pizza, Jo or Joe?

The correct answer is that they get the same amount of pizza. Jo gets one giant slice that is half of the pizza. Joe gets two big slices, each of which is one fourth of the pizza, for a total of two fourths of the pizza. They get the same amount of yummy pizza.

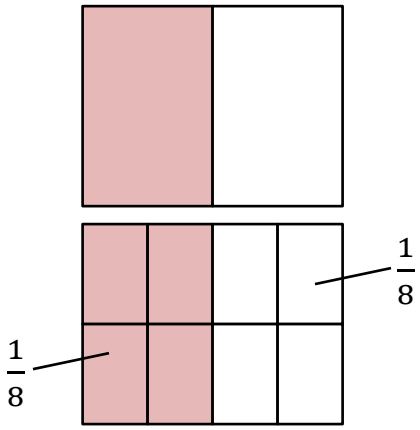
Equivalent Fractions: Using algebra

This is a second way to look at equivalent fractions.

In picture form:

The one has been divided into two equal parts.
Each is $\frac{1}{2}$.

Now I divide each of the one halves into four equal parts. Each of these parts is $\frac{1}{8}$.



There is an algebraic way to go from $\frac{1}{2}$ to $\frac{4}{8}$

$$\frac{1*4 = 4}{2*4 = 8}$$

Each of the halves was divided into four equal pieces.
(The pink half and the white half were each divided into four equal pieces.)

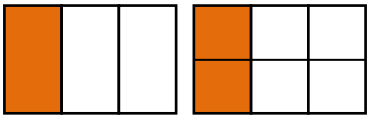
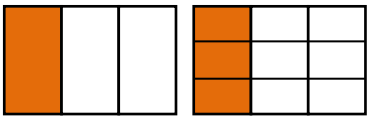
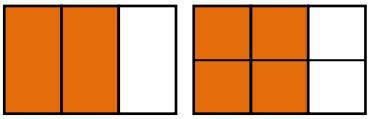
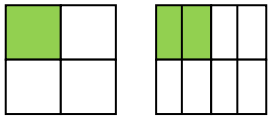
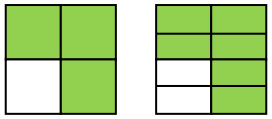
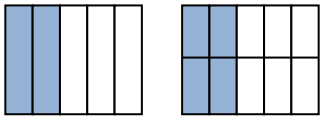
And we used a factor of 4 to go from $\frac{1}{2}$ to $\frac{4}{8}$

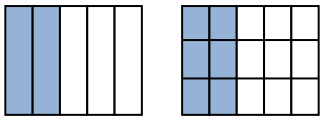
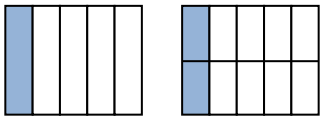
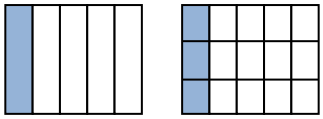
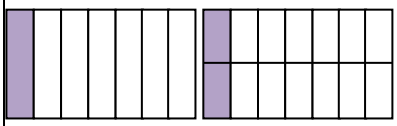
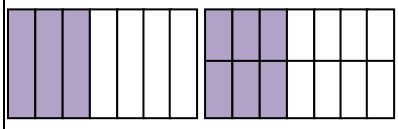
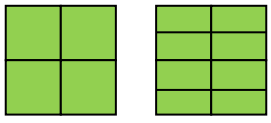
$$\frac{1*4 = 4}{2*4 = 8}$$

FACTOR OF 4

Is this a coincidence?! The original fraction piece (1/2) divided into four equal pieces, and a factor of 4?! Your mission, should you choose to accept it, is to find out!

Practice 16: Two looks at equivalent fractions

Equivalent Fractions	Picture	How many equal parts did you make from each original part?	Algebraic form:
$\frac{1}{3} = \frac{2}{6}$		Each of the one thirds is divided into **2** equal pieces	$\frac{1 * 2 = 2}{3 * 2 = 6}$
$\frac{1}{3} = \frac{3}{9}$			
$\frac{2}{3} = \frac{4}{6}$			
$\frac{1}{4} = \frac{2}{8}$			
$\frac{3}{4} = \frac{6}{8}$			
$\frac{2}{5} = \frac{4}{10}$			

Equivalent Fractions	Picture	How many equal parts did you make from each original part?	Algebraic form:
$\frac{2}{5} = \frac{6}{15}$			
$\frac{1}{5} = \frac{2}{10}$			
$\frac{1}{5} = \frac{3}{15}$			
$\frac{1}{7} = \frac{2}{14}$			
$\frac{3}{7} = \frac{6}{14}$			
$\frac{4}{4} = \frac{8}{8}$			

Look at your examples in this activity you just did.

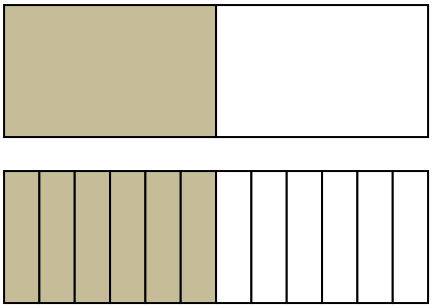
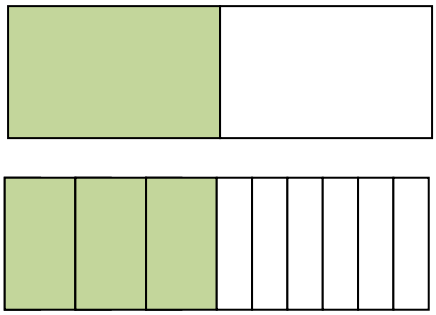
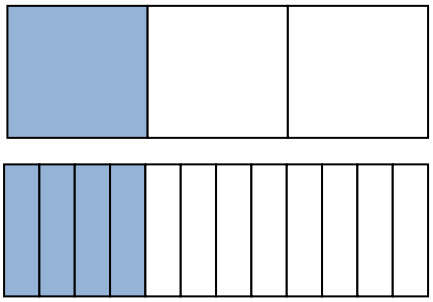
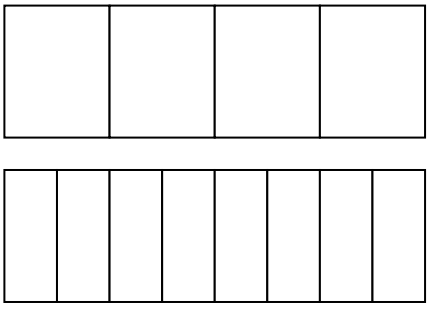
$$\frac{1 * 3 = 3}{3 * 3 = 6}$$ Here each of the two thirds was divided into 3 pieces. And so on.

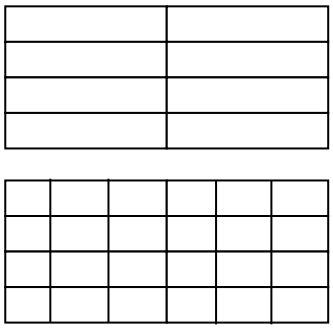
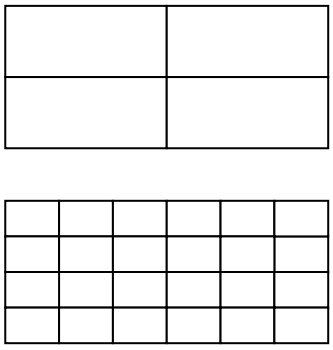
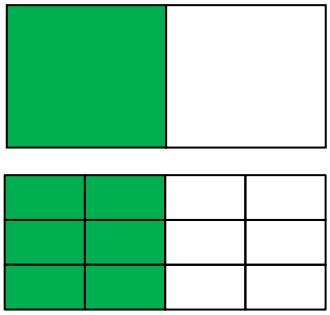
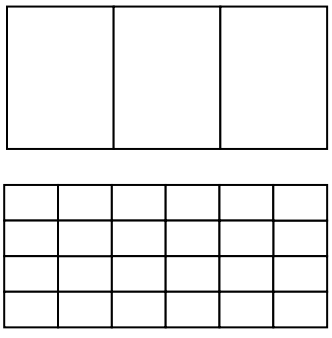
Try to make a general case here. Do your best!

$$\frac{1 * n =}{2 * n =}$$ means each of the halves is divided into _____(how many) pieces?

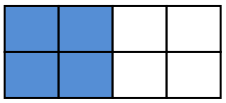
$$\frac{1 * n =}{3 * n =}$$ means each of the thirds is divided into _____(how many) pieces?

Practice 17: More work with equivalent fractions

Picture	Symbol	Relationship	Explanation
	$\frac{1}{2}$ $\frac{6}{12}$	$\frac{1 * 6}{2 * 6} = \frac{6}{12}$ $\frac{6 \div 6}{12 \div 6} = \frac{1}{2}$	$\frac{1}{2}$ is equivalent to $\frac{6}{12}$ because they cover the same area
			
			
	$\frac{1}{4}$ $\frac{2}{8}$	$\frac{1 *}{4 *} = \frac{2}{8}$ $\frac{2 \div}{8 \div} = \frac{1}{4}$	$\frac{1}{4}$ is equivalent to

Picture	Symbol	Relationship	Explanation
	$\frac{3}{8}$ $\frac{9}{24}$		
	$\frac{1}{4}$ $\frac{6}{24}$		
		$\frac{1 * 6}{2 * 6} = \frac{6}{12}$ $\frac{6 \div 6}{12 \div 6} = \frac{1}{2}$	
			$\frac{2}{3}$ is equivalent to $\frac{16}{24}$ because they cover the same area

Practice 18: Using pictures to find equivalent fractions – The Area Model

Original Fraction	Picture	Final Fraction	Picture	Factor for Numerator and Denominator
$\frac{1}{2}$		$\frac{4}{8}$		For every 1 piece of $\frac{1}{2}$, there are 4 pieces for $\frac{4}{8}$
$\frac{1}{2}$		$\frac{3}{6}$		
$\frac{1}{2}$		$\frac{5}{10}$		
$\frac{1}{4}$		$\frac{3}{12}$		
$\frac{1}{4}$		$\frac{2}{8}$		
$\frac{1}{3}$		$\frac{2}{6}$		
$\frac{2}{5}$		$\frac{4}{10}$		
$\frac{1}{3}$		$\frac{3}{9}$		

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Activity 4: Finding equivalent fractions with your fraction pieces

Use your fraction pieces for this activity.

$\frac{1}{2}$	$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{5}{10}$
$\frac{1}{3}$	
$\frac{1}{5}$	
$\frac{2}{3}$	
$\frac{2}{5}$	

Answers – page 103

$\frac{5}{10}$	
$\frac{3}{12}$	
$\frac{1}{4}$	

Thinking about it:

a. What is the same about the list of equivalent fractions for $\frac{1}{2}$ and $\frac{5}{10}$?

b. What is the same about the list of equivalent fractions for $\frac{3}{12}$ and $\frac{1}{4}$?

c. How can you tell without fraction pieces that two fractions are equivalent?
(Do your best – there will be more about this later)

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Practice 19: More equivalent fraction practice

Circle the fractions that are equivalent to the given fraction.

$\frac{1}{2}$	$\frac{4}{8}$ $\frac{120}{240}$ $\frac{2}{1}$ $\frac{50}{25}$ $\frac{3}{6}$ $\frac{75}{150}$ $\frac{16}{36}$ $\frac{5}{12}$ $\frac{11}{20}$ $\frac{82}{164}$ $\frac{4}{2}$ $\frac{14}{18}$ $\frac{350}{700}$
$\frac{1}{3}$	$\frac{9}{3}$ $\frac{120}{300}$ $\frac{3}{1}$ $\frac{50}{150}$ $\frac{3}{6}$ $\frac{2}{6}$ $\frac{16}{46}$ $\frac{4}{12}$ $\frac{11}{33}$ $\frac{30}{10}$ $\frac{6}{2}$ $\frac{14}{42}$ $\frac{300}{900}$
$\frac{1}{4}$	$\frac{4}{12}$ $\frac{120}{480}$ $\frac{4}{1}$ $\frac{50}{25}$ $\frac{4}{8}$ $\frac{25}{100}$ $\frac{75}{150}$ $\frac{3}{12}$ $\frac{11}{42}$ $\frac{80}{320}$ $\frac{8}{2}$ $\frac{4}{14}$ $\frac{100}{400}$
$\frac{2}{5}$	$\frac{4}{20}$ $\frac{200}{500}$ $\frac{5}{1}$ $\frac{50}{125}$ $\frac{10}{5}$ $\frac{20}{100}$ $\frac{12}{50}$ $\frac{5}{10}$ $\frac{22}{55}$ $\frac{8}{20}$ $\frac{5}{2}$ $\frac{12}{30}$ $\frac{20}{5}$
$\frac{2}{3}$	$\frac{4}{12}$ $\frac{100}{300}$ $\frac{12}{13}$ $\frac{50}{75}$ $\frac{9}{6}$ $\frac{30}{20}$ $\frac{16}{24}$ $\frac{4}{6}$ $\frac{22}{33}$ $\frac{80}{120}$ $\frac{3}{2}$ $\frac{9}{12}$ $\frac{250}{350}$

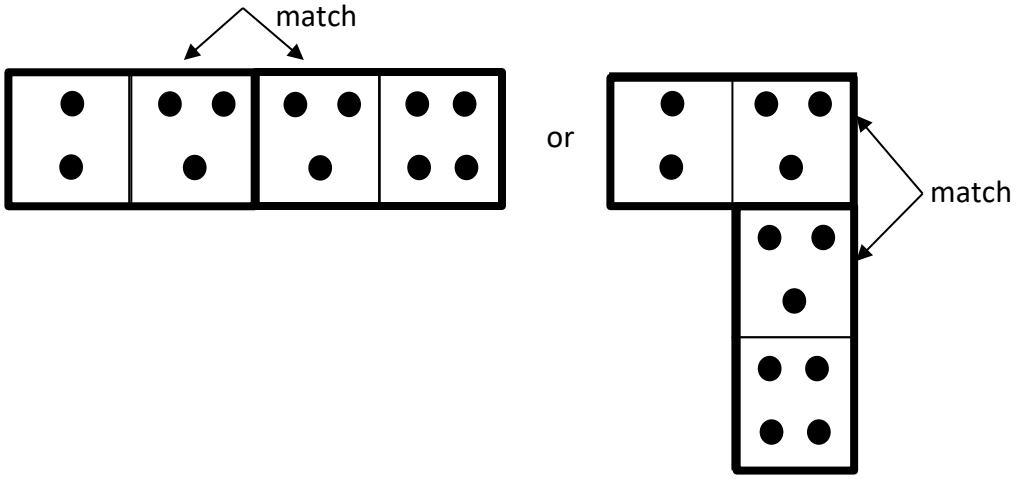
Answers – page 105

Activity 5: Equivalent fraction dominoes

Dominoes have been played for over 300 years. Maybe not with equivalent fractions...

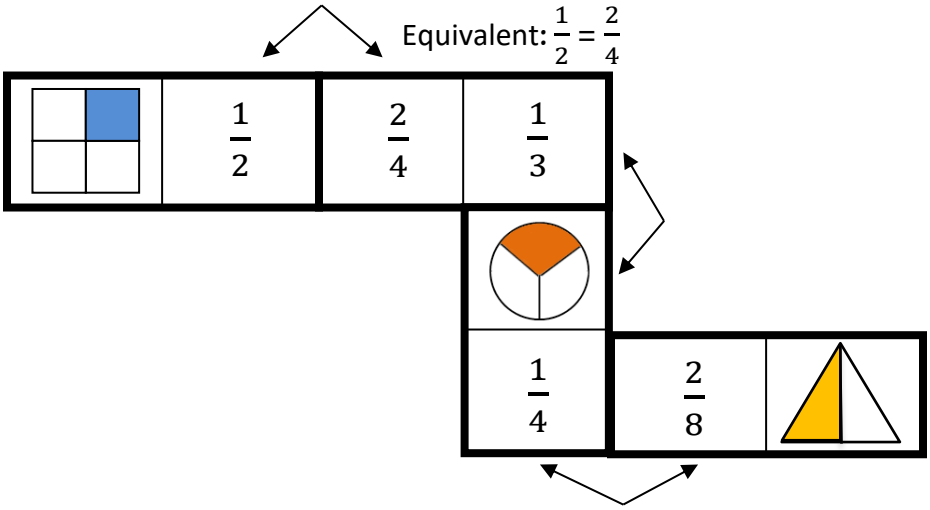
In case you don't know how to play, here is an explanation for playing dominoes:

Say you pick:

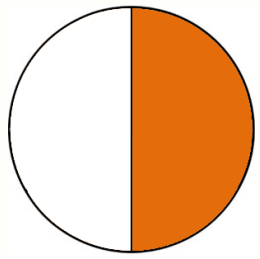
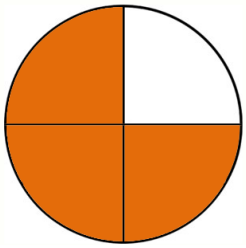
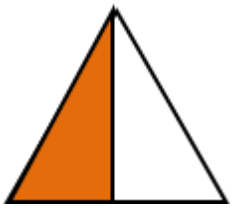
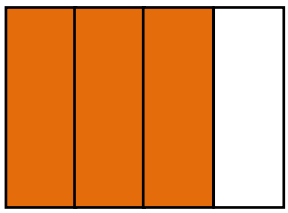
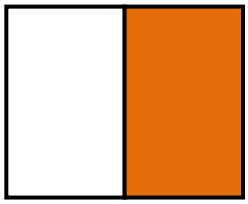
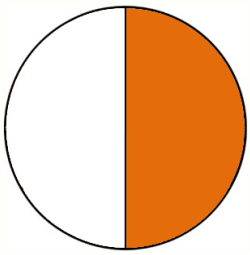
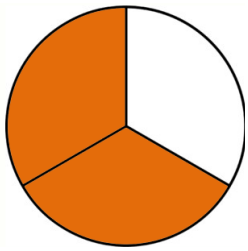
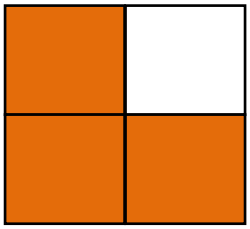
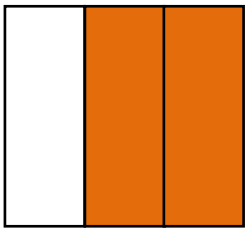
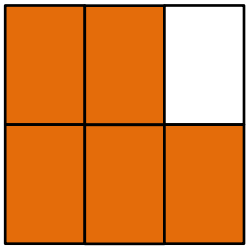
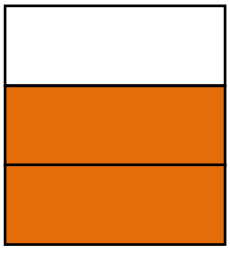


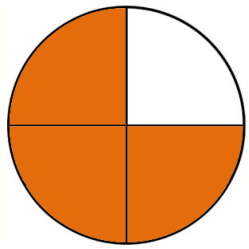
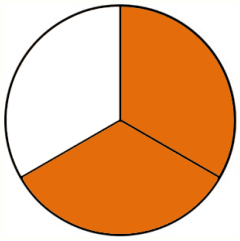
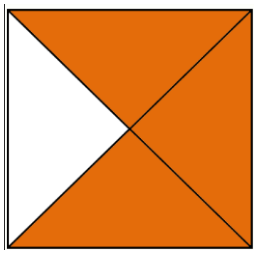
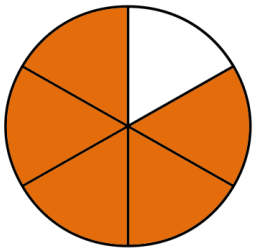
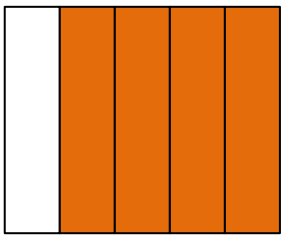
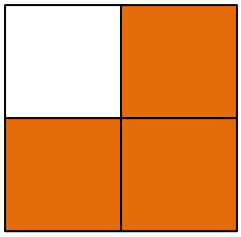
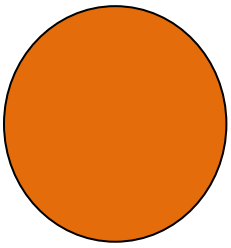
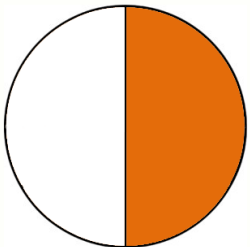
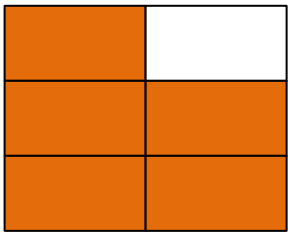
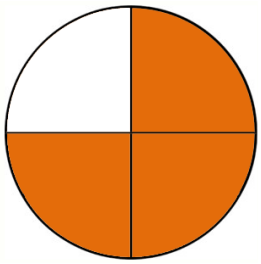
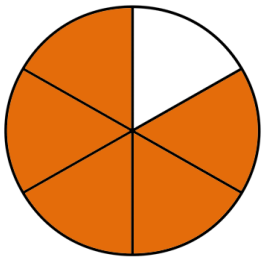
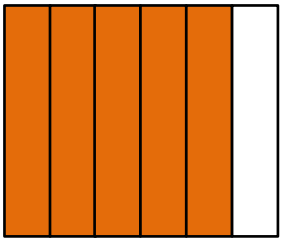
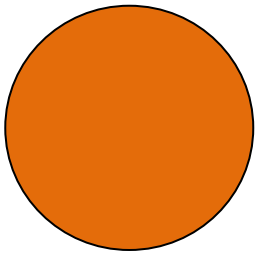
You can work in pairs on this, taking turns to add one domino each and check your partner's work. Or you can play by yourself. Either way, you practice and learn!

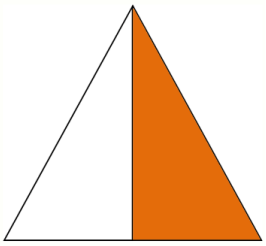
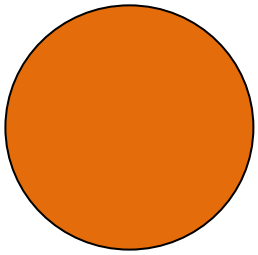
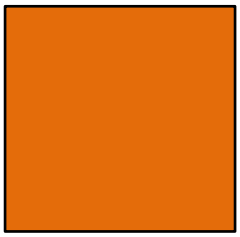
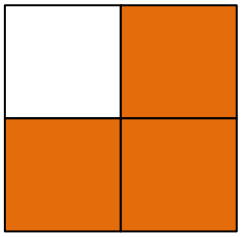
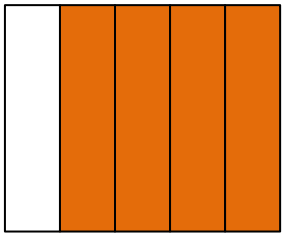
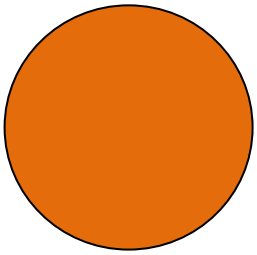
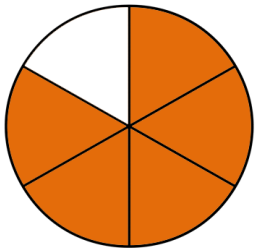
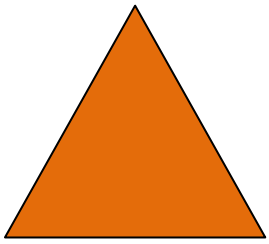
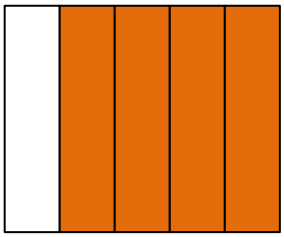
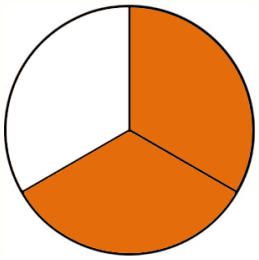
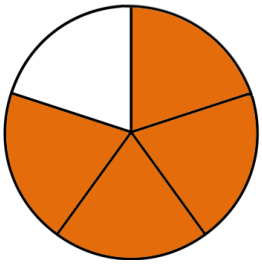
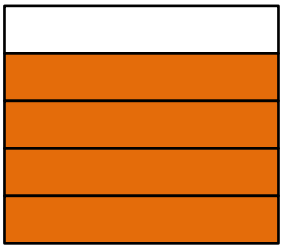
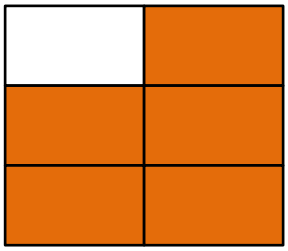
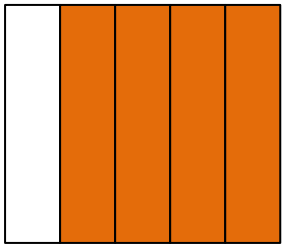
Matches can be either number or pictures



Cut out the dominoes (solid lines only) on pages 57 - 61.

$\frac{1}{2}$			$\frac{2}{3}$
$\frac{1}{2}$			$\frac{4}{5}$
$\frac{1}{2}$			1
$\frac{2}{3}$			$\frac{2}{3}$
$\frac{2}{3}$			$\frac{2}{3}$
$\frac{2}{3}$			1

	$\frac{1}{2}$	$\frac{1}{4}$	
	$\frac{3}{4}$	$\frac{3}{4}$	
$\frac{3}{4}$			
$\frac{5}{6}$			$\frac{2}{3}$
$\frac{5}{6}$			$\frac{4}{5}$
$\frac{5}{6}$		$\frac{5}{6}$	

1			$\frac{2}{3}$
		1	
		1	
	$\frac{1}{2}$	$\frac{4}{5}$	
	$\frac{3}{4}$		$\frac{4}{5}$
$\frac{4}{5}$			1

6. Multiplying fractions

Multiplying fractions is pretty much just like multiplying whole numbers.

Watch video on Multiplication Fractions 1 at mathwhisperer.com/fraction-videos

We begin with an analogy to multiplication in general, with a whole number example:

2 * 3 means **2 groups of 3**

← One group of 3

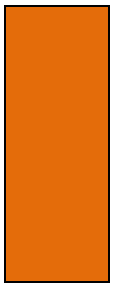
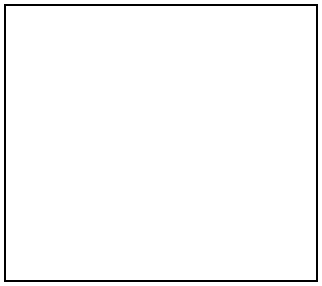
← A second group of 3

$\frac{1}{2} * \frac{1}{3}$ must mean $\frac{1}{2}$ group of $\frac{1}{3}$.

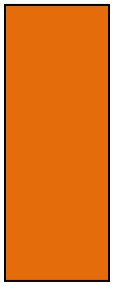
But $\frac{1}{2}$ group of $\frac{1}{3}$ doesn't make much sense.

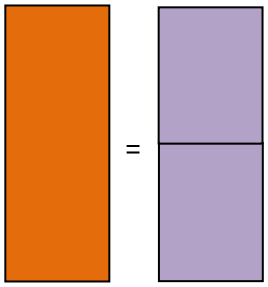
However, $\frac{1}{2}$ of $\frac{1}{3}$ makes a lot of sense.

So $\frac{1}{2} * \frac{1}{3}$ must mean $\frac{1}{2}$ of $\frac{1}{3}$.

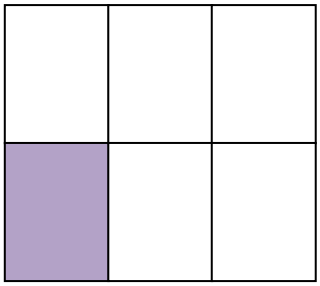
Here is $\frac{1}{3}$  And this is 1 for reference. 

So, what is the name of this fraction, $\frac{1}{2}$ of $\frac{1}{3}$? _____

Given that this  is $\frac{1}{3}$, then $\frac{1}{2}$ of $\frac{1}{3}$ is this 



Comparing to our "1" for reference, we can see that six of these cover the "1."



So, the answer must be $\frac{1}{6}$.

Now we know that $\frac{1}{2} * \frac{1}{3} = \frac{1}{6}$.

or

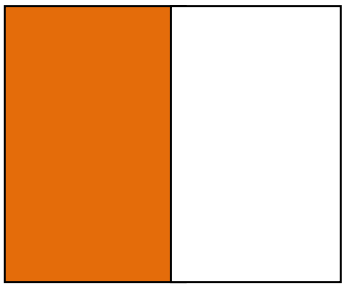
$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$

Activity 6: Multiply fractions – Tear Paper

Now you do another problem by tearing paper.

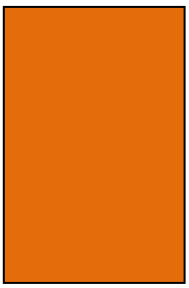
Hold out one sheet of paper.

To find $\frac{1}{2}$ of $\frac{1}{2}$, we must start with $\frac{1}{2}$.

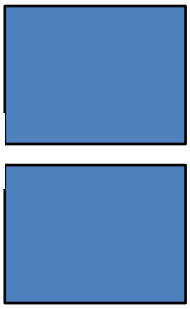


So, tear your paper so that you have $\frac{1}{2}$.

Now tear $\frac{1}{2}$ in half



And this is what you



Compare this result to an untouched 8 ½ by 11 piece of paper, and see for yourself that 4 of these cover that paper.

In other words, four of these

have the same area as the original 8 ½ by 11 piece of paper.

So, the name of this piece is $\frac{1}{4}$.

The general form of multiplication of any fraction is: $\frac{a}{b} * \frac{c}{d} = ?$

In our example: a = 1
b = 2
c = 1
d = 3

$\frac{1}{2} * \frac{1}{3} = ?$

In this specific case, how do we go from a, b, c, d to $\frac{1}{6}$?

$\frac{a}{b} * \frac{c}{d} = \frac{1}{2} * \frac{1}{3} = \frac{1}{6}$

Can you understand that the numerator is $\boldsymbol{a * c}$?

Can you understand that the denominator is $\boldsymbol{b * d}$?

$$\frac{1}{2} * \frac{1}{3} = \frac{1 * 1}{2 * 3} = \frac{1}{6}$$

It is looking like the way to multiply fractions is:

$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

In the video that goes with this lesson, I give another example. It gives the same result for multiplication of fractions, thank goodness!

The next step is for you to practice.

You may want to have this sheet in front of you for the problems.

The only way to multiply fractions is:

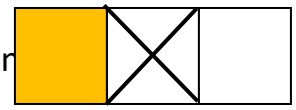
$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

This also means that a fraction not in the form of $\frac{a}{b}$ must be put into that form to multiply.

For **whole numbers**, all you need to do to turn it into a fraction is to put a 1 in the dominator.

$$1 * \frac{1}{3} \quad tuns\ into \quad \frac{1}{1} * \frac{1}{3}$$

Practice 20: Using pictures and algebra to multiply fractions

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a*c}{b*d}$
$\frac{1}{2} * \frac{2}{3} =$	Start with $\frac{2}{3}$  Do the multiplication for the "of"  Result $\frac{1}{3}$  $\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$	$\frac{1}{2} * \frac{2}{3} = \frac{1*2}{2*3} = \underbrace{\frac{2}{6} = \frac{1}{3}}_{\text{optional}}$
$\frac{1}{3} * \frac{3}{4} =$		
$\frac{1}{2} * \frac{4}{5} =$		

Answers – page 106

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{2} * \frac{6}{5} =$		
$\frac{1}{4} * \frac{4}{5} =$		
$\frac{1}{2} * \frac{2}{5} =$		

Answers – page 107

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{3} * \frac{1}{4} =$		
$\frac{1}{4} * \frac{1}{3} =$		
$\frac{1}{2} * 1 =$		

Answers – page 109

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{3} * 1 =$		
$\frac{2}{3} * \frac{1}{2} =$		
$\frac{2}{3} * \frac{3}{2} =$		

Answers – page 111

Practice 21: Multiplication using the algorithm

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a*c}{b*d}$
$\frac{1}{2} * \frac{2}{3} =$	 $\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$ or $\frac{1}{6}$	$\frac{1}{2} * \frac{2}{3} = \frac{1*2}{2*3} = \frac{2}{6} = \frac{1}{3}$
$\frac{1}{3} * \frac{3}{4} =$		
$\frac{1}{2} * \frac{4}{5} =$		
$\frac{1}{2} * \frac{6}{5} =$		
$\frac{1}{4} * \frac{4}{5} =$		
$\frac{1}{2} * \frac{2}{5} =$		

Practice 22: More multiplication of fractions practice

a. $\frac{1}{3} * \frac{2}{5} =$	b. $\frac{1}{5} * \frac{2}{7} =$
c. $\frac{1}{3} * \frac{11}{7} =$	d. $\frac{1}{3} * \frac{4}{5} =$
e. $\frac{1}{4} * \frac{1}{8} =$	f. $\frac{1}{3} * \frac{1}{3} =$
g. $\frac{2}{3} * \frac{2}{5} =$	h. $\frac{1}{4} * \frac{3}{4} =$
i. $\frac{1}{5} * \frac{2}{11} =$	j. $\frac{1}{10} * \frac{3}{7} =$
k. $\frac{10}{3} * \frac{2}{7} =$	l. $\frac{1}{7} * \frac{4}{5} =$
m. $\frac{1}{3} * \frac{2}{3} =$	n. $\frac{5}{3} * \frac{2}{3} =$
o. $\frac{1}{17} * \frac{2}{3} =$	p. $\frac{10}{7} * \frac{3}{1} =$

Answers – page 115

Practice 23: And even more practice multiplying fractions

a. $\frac{1}{3} * \frac{w}{5} = \frac{1 * w}{3 * 5} = \frac{1w}{15} = \frac{w}{15}$	b. $\frac{1}{y} * \frac{2}{7} =$
c. $\frac{1}{3} * \frac{11}{x} =$	d. $\frac{r}{3} * \frac{4}{5} =$
e. $\frac{x}{4} * \frac{1}{8} =$	f. $\frac{1}{s} * \frac{2}{3} =$
g. $\frac{2}{3} * \frac{2}{5} =$	h. $\frac{x}{y} * \frac{3}{4} =$
i. $\frac{1}{5} * \frac{r}{11} =$	j. $\frac{t}{10} * \frac{3}{7} =$
k. $\frac{10}{3} * \frac{k}{7} =$	l. $\frac{1}{7} * \frac{x}{5} =$
m. $\frac{1}{3} * \frac{m}{3} =$	n. $\frac{5}{q} * \frac{2}{3} =$
o. $\frac{1}{7} * \frac{2}{3} =$	p. $\frac{1}{7} * \frac{3}{1} =$

Answers – page 116

7. Optional Activities

Making a recipe in an excellent way to practice fractions.

Recipe 1: German Apple Pancakes

Ingredients

4 eggs

½ cup milk (or coconut milk)

½ cup all-purpose flour (or oat flour)

½ tablespoon baking powder

1 tablespoon sugar

Pinch of salt

¼ teaspoon ground nutmeg

2 large apples or 4 small apples peeled and sliced thin

3 tablespoons butter

2 tablespoons sugar (this in addition to the sugar above)

Maple syrup if desired

Directions

Preheat a 10-inch cast iron skillet in a 450° oven.

Whisk together the eggs, flour, salt, and nutmeg. Set aside for up to half an hour. The flour may lump. That's the reason for whisking and waiting—after the eggs and milk warm to room temperature the lumps will whisk away.

Take the skillet out of the oven, being careful not to burn yourself. Heat the butter in the skillet until it bubbles. Then lay the apple slices on top. Sprinkle with the sugar and cinnamon. Pour the batter into the skillet, on top of the apples and return to the oven. Lower the heat to 350°. Bake, uncovered, for 20 minutes. The pancake will puff up and the edges may turn brown and crisp.

Serve with maple syrup or your favorite topping.

Recipe 2: Cherry Clafouti

Ingredients

1 pound pitted cherries (may be defrosted from frozen or fresh)

1 ¼ cups milk or coconut milk

5 tablespoons honey

3 large eggs

1 tablespoon vanilla

Pinch of salt

$\frac{2}{3}$ cup all purpose flour (or oat flour)

Directions

Preheat the oven to 350°. Butter an oven-proof skillet or other baking dish.

Drain any juice from the cherries and save it.

Put the cherry juice, milk, honey, eggs, vanilla and salt into a bowl or mixer. Add the flour and mix until the batter is smooth.

Add the batter to the cherries, mix, and pour into the buttered skillet or baking dish. Bake for 45 minutes, or until the top is browned and a knife that is inserted comes out clean. This can be served either hot or cold.

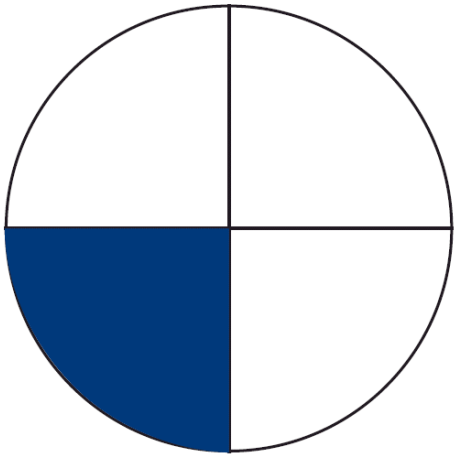
8. Solutions / Answers

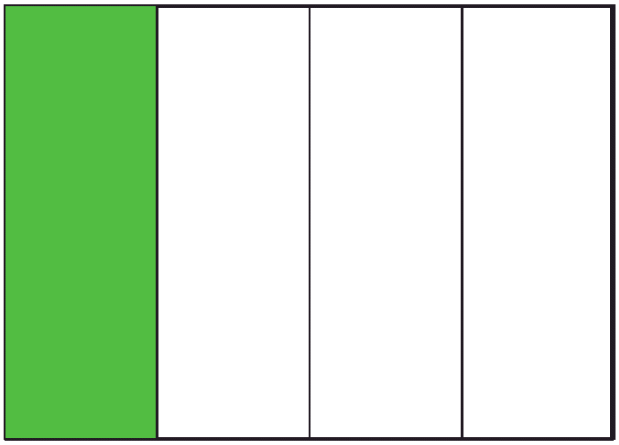
MATH WHISPERER

Where math makes sense

Practice 1: Explaining one fourth

Explain why each of these can be one fourth $\{\frac{1}{4}\}$ when they look so different.





With fractions, everything depends on what "one" is. The "one" is different. When the circle equals one, then the shaded piece shows $\frac{1}{4}$ of the circle. If the rectangle equals one, the shaded piece shows $\frac{1}{4}$ of the rectangle. It all depends on what "one" is.

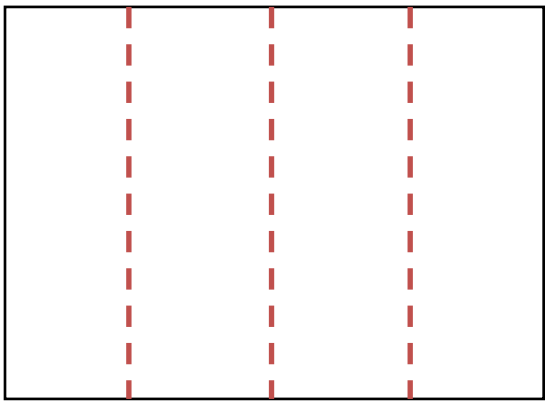
Activity 2: Explaining one fourth again

Divide the rectangles into fourths in two different ways.

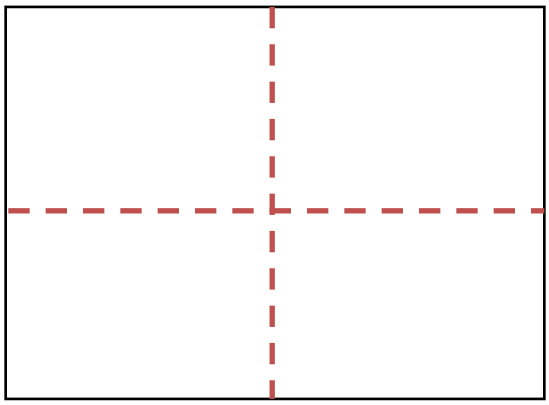
Then compare $\frac{1}{4}$ of rectangle A with $\frac{1}{4}$ of rectangle B. Are they the same? yes

Why or why not?

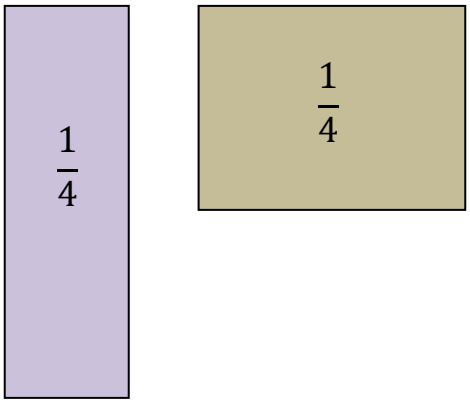
The "one" is the same for both rectangles A and B. They look different as they are different shapes, but in both cases, four of them will exactly cover the "one," so each is $\frac{1}{4}$ of the rectangle. I can cut up the $\frac{1}{4}$ from rectangle A and it is exactly the same as the $\frac{1}{4}$ from rectangle B.

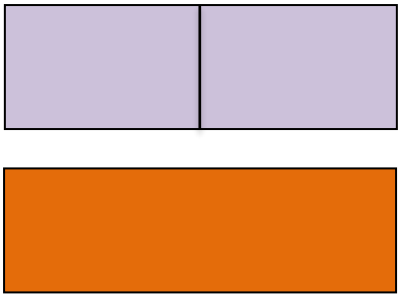


Rectangle A



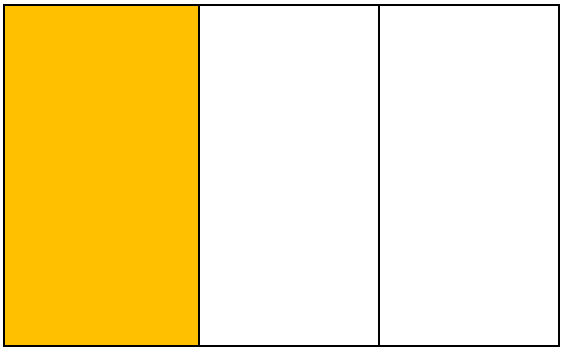
Rectangle B

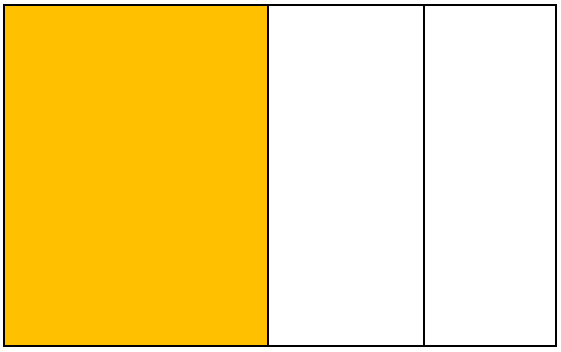




Practice 2: Thirds

Does each picture show $\frac{1}{3}$? no





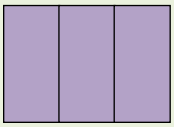
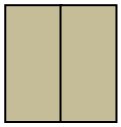
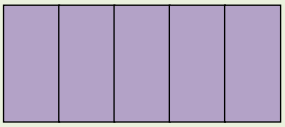
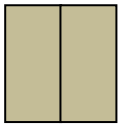
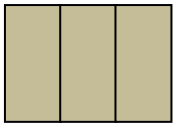
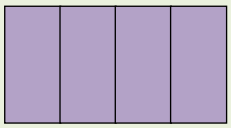
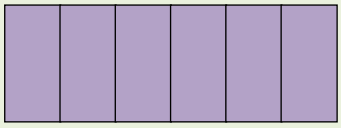
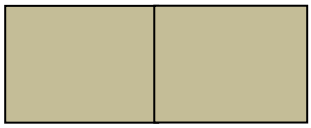
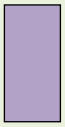
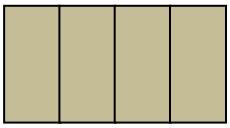
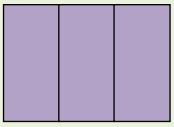
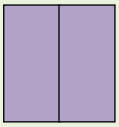
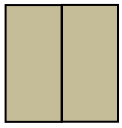
Explain why or why not.

The rectangle on the left does show $\frac{1}{3}$, because the rectangle is divided into 3 equal parts. The rectangle on the right does not show $\frac{1}{3}$. On the right the parts are different sizes. $\frac{1}{3}$ means "one divided into 3 equal parts."

Practice 3: Can you name me now?

If the whole is:	Tell what fraction of the whole each of these is:		
White $= 1$	Pink $=$? $\frac{1}{8}$	Green $=$? $\frac{1}{12}$	Purple $=$? $\frac{1}{3}$
Pink $= 1$	White $=$? 8	Gray $=$? 2	Yellow $=$? 4
Green $= 1$	White $=$? 12	Gray $=$? 3	Purple $=$? 4
Orange $= 1$	Blue $=$? $\frac{1}{3}$	Gray $=$? $\frac{1}{2}$	White $=$ 2
Purple $= 1$	White $=$? 3	Blue $=$? 2	Green $=$? 6

Practice 4: What is "1"?

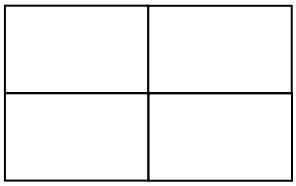
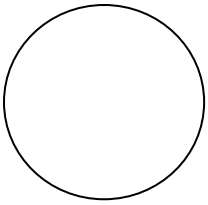
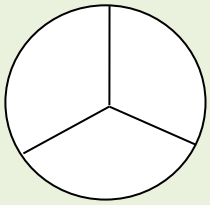
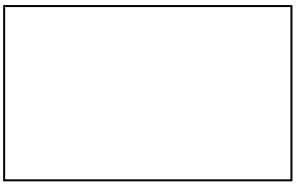
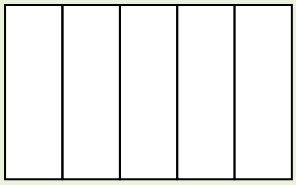
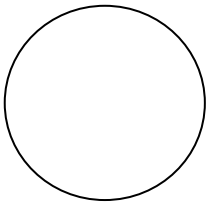
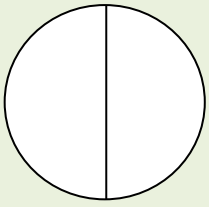
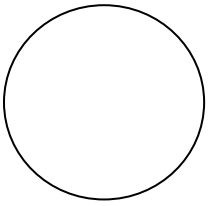
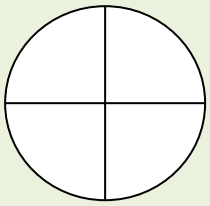
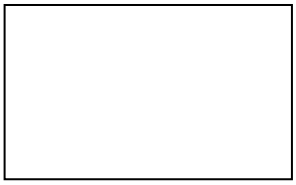
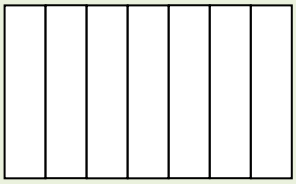
If this is $\frac{1}{3}$ 	draw 1 
If this is $\frac{2}{5}$ 	draw 1 
If this is $\frac{2}{3}$ 	draw 1 
If this is $\frac{3}{4}$ 	draw 1 
If this is $\frac{1}{6}$ 	draw 1 
If this is 2 	draw 1 
If this is 3 	draw 1 
If this is $\frac{4}{3}$ 	draw 1 
If this is $\frac{3}{2}$ 	draw 1 
If this is 2 	draw 1 

Practice 5: Quick summary

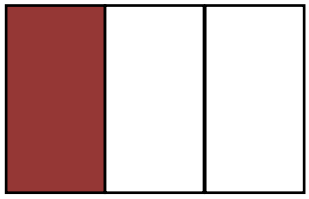
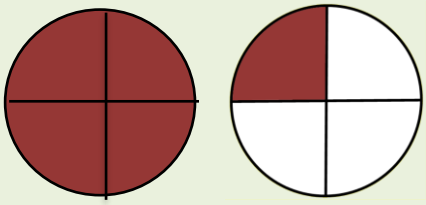
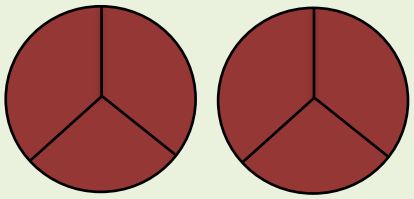
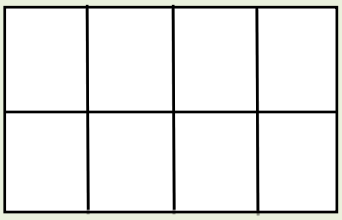
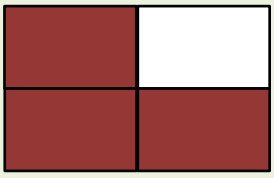
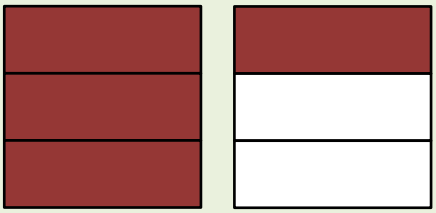
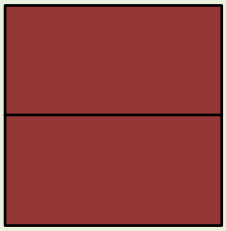
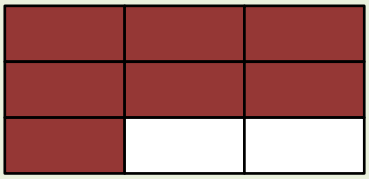
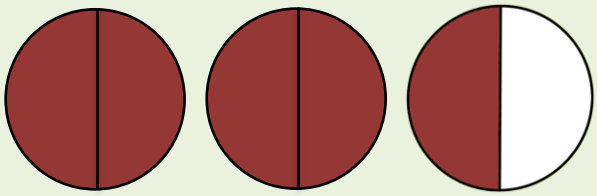
Look back over Practices 1 through 4, and Activities 1 and 2. Why is it so important to know what "one" is?

Everything depends on "one" when you name a fraction. The fraction is part of "one".

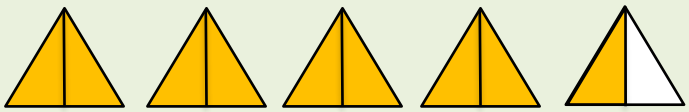
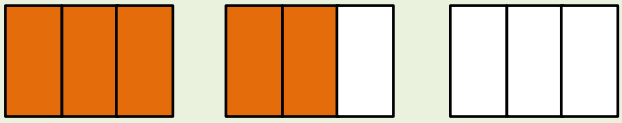
Practice 6: Denominator practice

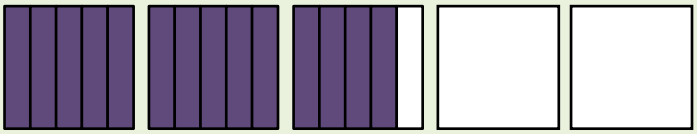
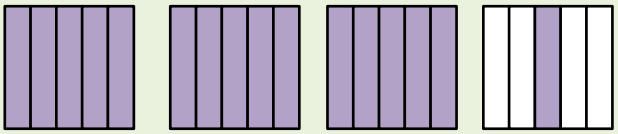
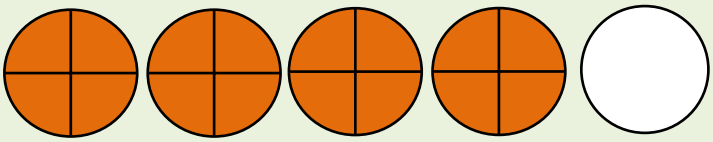
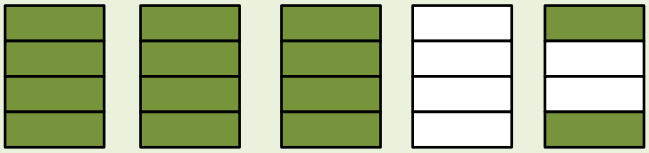
Fraction Symbols	Fraction Name in Words	This is One	Fraction Picture
$\frac{}{4}$	fourths		
$\frac{}{3}$	thirds		
$\frac{}{5}$	fifths		
$\frac{}{2}$	halves		
$\frac{}{4}$	fourths		
$\frac{}{7}$	sevenths		

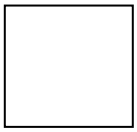
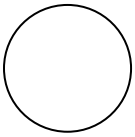
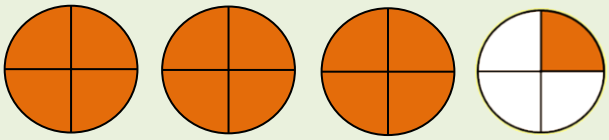
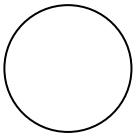
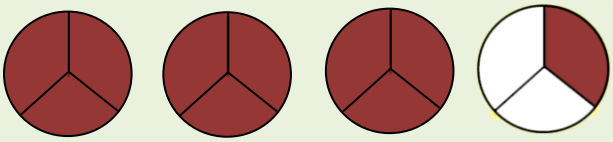
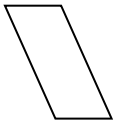
Practice 7: Getting more familiar with numerators and denominators

Box 1: denominator 3, numerator 1	Box 2: denominator 4, numerator 5
 $\frac{1}{3}$	 $\frac{5}{9}$
Box 3: denominator 3, numerator 6	**Box 4: denominator 8, numerator 0**
 $\frac{6}{3}$	 $\frac{0}{8} = 0$
Box 5: denominator 4, numerator 3	**Box 6: denominator 3, numerator 4**
 $\frac{3}{4}$	 $\frac{4}{3}$
Box 7: denominator 2, numerator 2	**Box 8: denominator 9, numerator 7**
 $\frac{2}{2} = 1$	 $\frac{7}{9}$
Box 9: denominator 1, numerator 1	**Box 10: denominator 2, numerator 5**
 $\frac{1}{1} = 1$	 $\frac{5}{2}$

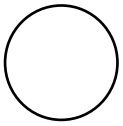
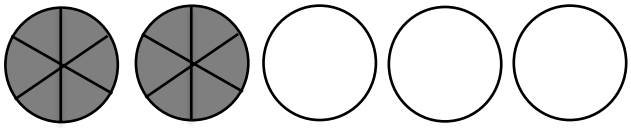
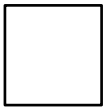
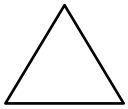
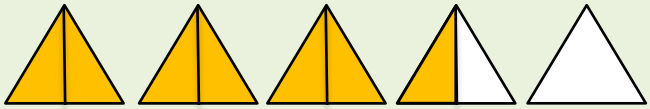
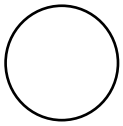
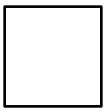
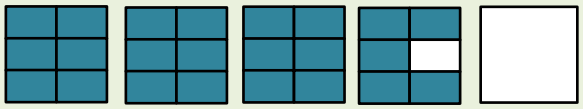
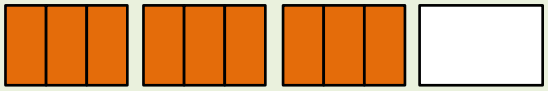
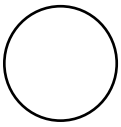
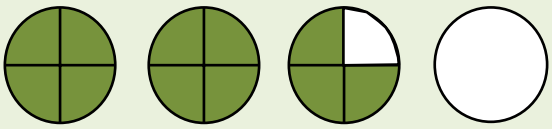
Practice 8: Mixed numbers and "Improper" fractions

One	Picture	Mixed Number	"Improper" Fraction Form
		$3\frac{3}{4}$	$\frac{15}{4}$
		$4\frac{2}{3}$	$\frac{14}{3}$
		$4\frac{1}{2}$	$\frac{9}{2}$
		$1\frac{2}{3}$	$\frac{5}{3}$

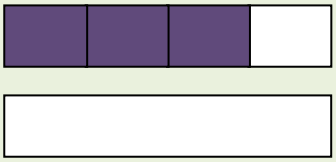
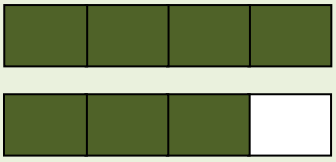
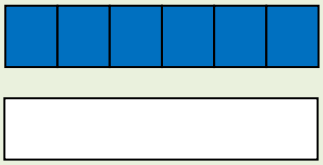
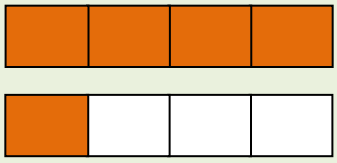
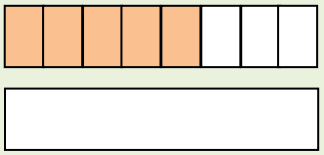
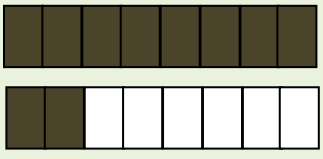
One	Picture	Mixed Number	"Improper" Fraction Form
		$2\frac{4}{5}$	$\frac{14}{5}$
		$3\frac{1}{5}$	$\frac{16}{5}$
		4	$\frac{16}{4}$
		$3\frac{2}{4}$ *or* $3\frac{1}{2}$	$\frac{14}{4}$ *or* $\frac{7}{2}$
For the remaining problems, draw if you need to.			

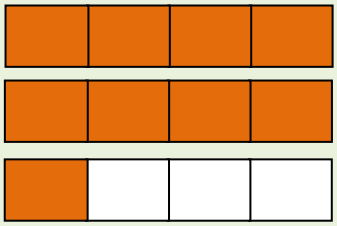
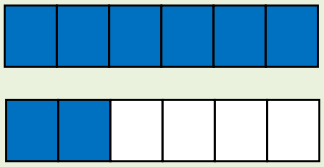
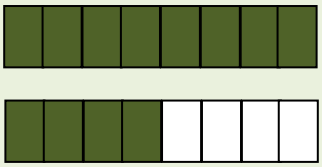
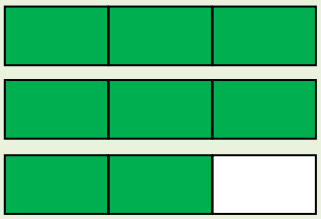
		$2\frac{1}{3}$	$\frac{7}{3}$
		$2\frac{1}{2}$	$\frac{5}{2}$
		$3\frac{1}{4}$	$\frac{13}{4}$
		$3\frac{1}{3}$	$\frac{10}{3}$
		5	$\frac{10}{2}$

Practice 9: More mixed numbers and "Improper" fractions

"Improper" Fraction Form	One	Picture	Mixed Number
$\frac{12}{6}$			2
$\frac{18}{4}$			$4\frac{2}{4}$ *or* $4\frac{1}{2}$
$\frac{7}{2}$			$3\frac{1}{2}$
$\frac{6}{3}$			2
$\frac{23}{6}$			$3\frac{5}{6}$
$\frac{9}{3}$			3
$\frac{11}{4}$			$2\frac{3}{4}$

Practice 10: Practice with fractions greater and less than one

Fraction	One	Picture	Compare to 1 Use >, <, or =	Mixed Number If > 1
$\frac{3}{4}$			$\frac{3}{4} < 1$	
$\frac{7}{4}$			$\frac{7}{4} > 1$	$1\frac{3}{4}$
$\frac{6}{6}$			$\frac{6}{6} = 1$	
$\frac{5}{4}$			$\frac{5}{4} > 1$	$1\frac{1}{4}$
$\frac{5}{8}$			$\frac{5}{8} < 1$	
$\frac{10}{8}$			$\frac{10}{8} > 1$	$1\frac{2}{8}$ or $1\frac{1}{4}$

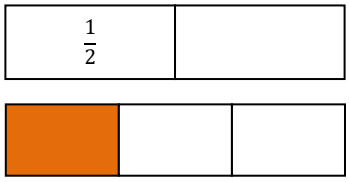
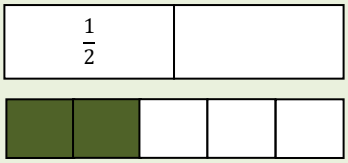
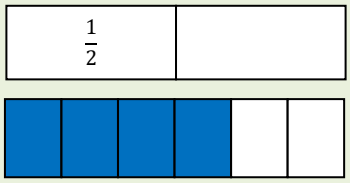
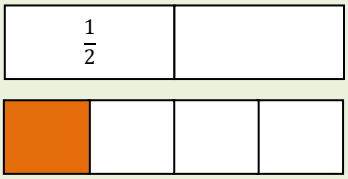
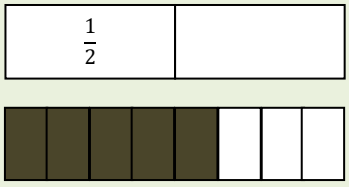
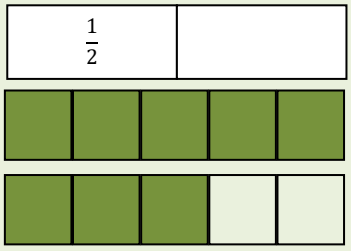
Fraction	One	Picture	Compare to 1 Use >, <, or =)	Mixed Number if > 1
$\frac{9}{4}$			$\frac{9}{4} > 1$	$2\frac{1}{4}$
$\frac{8}{6}$			$\frac{8}{6} > 1$	$1\frac{2}{6}$ or $1\frac{1}{3}$
$\frac{12}{8}$			$\frac{12}{8} > 1$	$1\frac{4}{8}$ or $1\frac{1}{2}$
$\frac{8}{3}$			$\frac{8}{3} > 1$	$2\frac{2}{3}$
$\frac{5}{5}$			$\frac{5}{5} = 1$	

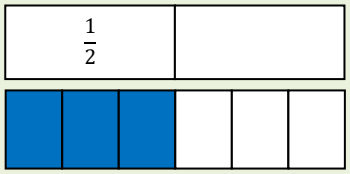
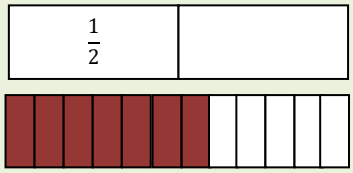
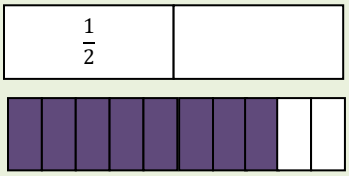
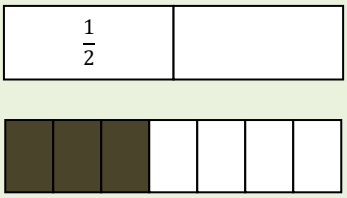
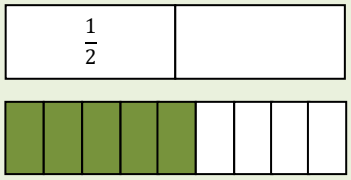
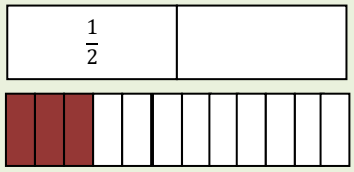
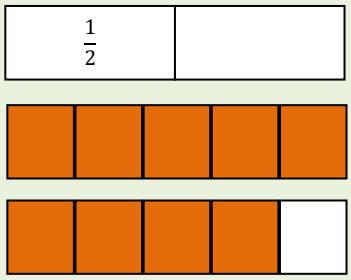
Question: We have seen that $\frac{8}{3} > 1$, and $\frac{12}{8} > 1$. Can you tell just by looking at the fraction if it greater than one?

Yes. If the numerator is greater than the denominator, the fraction is greater than one.

Practice 11: Comparing to $\frac{1}{2}$

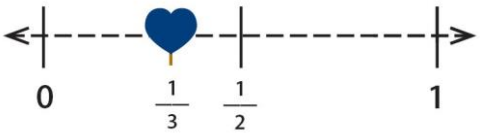
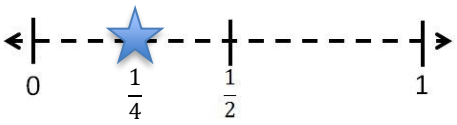
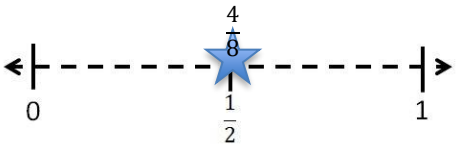
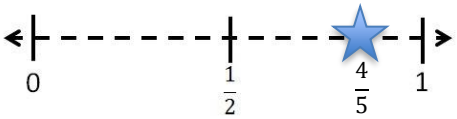
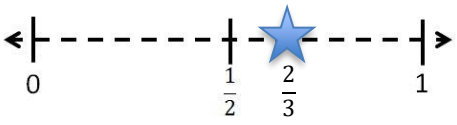
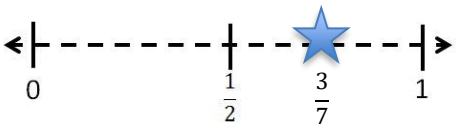
One half can be used as a benchmark to determine the relative size of fractions. For each fraction, shade in the appropriate portion of the rectangle divided into halves. Then write whether the given fraction is greater than, less than, or equal to $\frac{1}{2}$.

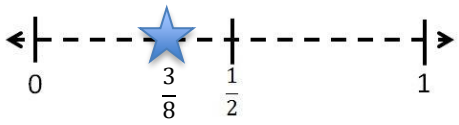
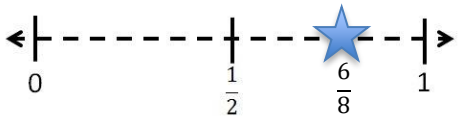
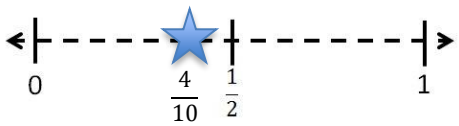
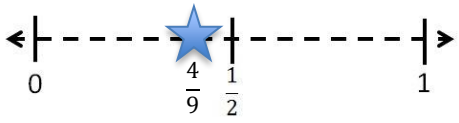
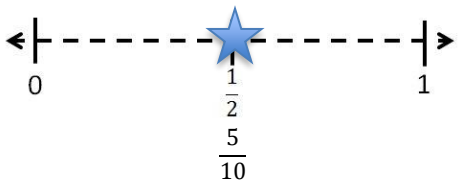
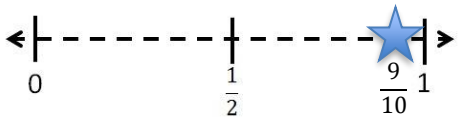
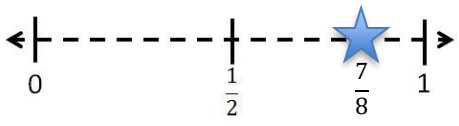
Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{1}{3}$		$\frac{1}{3} < \frac{1}{2}$
$\frac{2}{5}$		$\frac{2}{5} < \frac{1}{2}$
$\frac{4}{6}$		$\frac{4}{6} > \frac{1}{2}$
$\frac{1}{4}$		$\frac{1}{4} < \frac{1}{2}$
$\frac{5}{8}$		$\frac{5}{8} > \frac{1}{2}$
$\frac{8}{5}$		$\frac{8}{5} > \frac{1}{2}$

Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{3}{6}$		$\frac{3}{6} = \frac{1}{2}$
$\frac{7}{12}$		$\frac{7}{12} > \frac{1}{2}$
$\frac{8}{10}$		$\frac{8}{10} > \frac{1}{2}$
$\frac{3}{7}$		$\frac{3}{7} < \frac{1}{2}$
$\frac{5}{9}$		$\frac{5}{9} > \frac{1}{2}$
$\frac{3}{12}$		$\frac{3}{12} < \frac{1}{2}$
$\frac{9}{5}$		$\frac{9}{5} > \frac{1}{2}$

Practice 12: Using $\frac{1}{2}$ as a benchmark

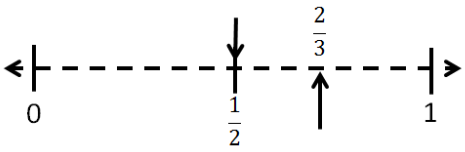
One half can be used as a benchmark to determine the relative size of fractions. Put the fraction on the number line, then write $>$, $<$, or $=$ in the circle to make a true statement.

Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{1}{3}$		$\frac{1}{3}$ $<$ $\frac{1}{2}$
$\frac{1}{4}$		$\frac{1}{4}$ $<$ $\frac{1}{2}$
$\frac{4}{8}$		$\frac{4}{8}$ $=$ $\frac{1}{2}$
$\frac{4}{5}$		$\frac{4}{5}$ $>$ $\frac{1}{2}$
$\frac{2}{3}$		$\frac{2}{3}$ $>$ $\frac{1}{2}$
$\frac{3}{4}$		$\frac{3}{4}$ $>$ $\frac{1}{2}$

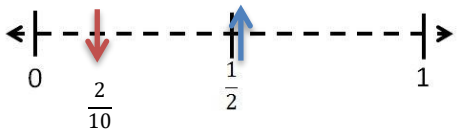
Fraction	Picture	Compare to $\frac{1}{2}$ Use $>$, $<$, or $=$
$\frac{3}{8}$		$\frac{3}{8}$ $<$ $\frac{1}{2}$
$\frac{6}{8}$		$\frac{6}{8}$ $>$ $\frac{1}{2}$
$\frac{4}{10}$		$\frac{4}{10}$ $<$ $\frac{1}{2}$
$\frac{4}{9}$		$\frac{4}{9}$ $<$ $\frac{1}{2}$
$\frac{5}{10}$		$\frac{5}{10}$ $=$ $\frac{1}{2}$
$\frac{9}{10}$		$\frac{9}{10}$ $>$ $\frac{1}{2}$
$\frac{7}{8}$		$\frac{7}{8}$ $>$ $\frac{1}{2}$

Practice 13: Who is larger?

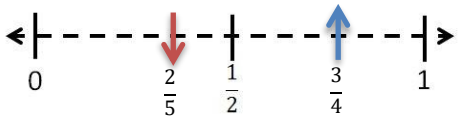
Show where the fractions would be on number line. Write >, <, or = in the circle between the fractions.



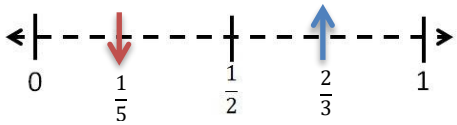
$\frac{2}{3}$ > $\frac{1}{2}$



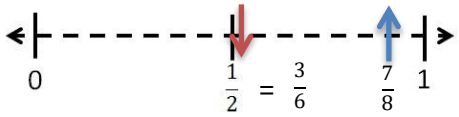
$\frac{2}{10}$ < $\frac{1}{2}$



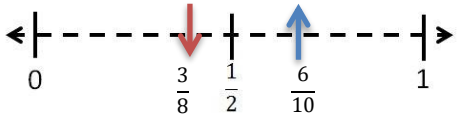
$\frac{3}{4}$ > $\frac{2}{5}$



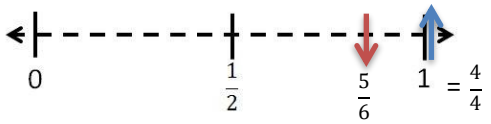
$\frac{1}{5}$ < $\frac{2}{3}$



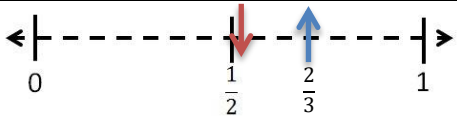
$\frac{3}{6}$ < $\frac{7}{8}$



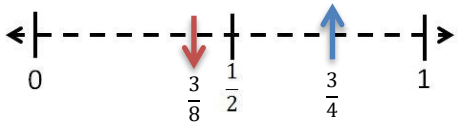
$\frac{6}{10}$ > $\frac{3}{8}$



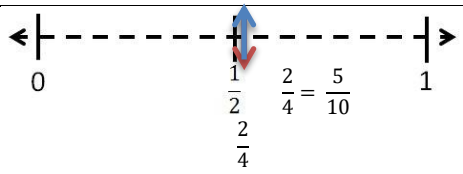
$\frac{5}{6}$ < $\frac{4}{4}$



$\frac{2}{4}$ < $\frac{2}{3}$



$\frac{3}{8}$ < $\frac{3}{4}$



$\frac{1}{2} = \frac{2}{4}$ = $\frac{5}{10}$

Practice 14: Who is right?

Mrs. Robben has a daughter in elementary school who is just learning about fractions. She thinks that because 3 is bigger than 2 then $\frac{1}{3}$ is bigger than $\frac{1}{2}$. Is Mrs. Robben's daughter correct? Explain why or why not, using sentences and pictures.

Mrs. Robben's daughter is incorrect.
Of course, Mrs. Robben's daughter is correct that 3 is bigger than 2.

But $\frac{1}{2}$ means one object divided into two equal parts.

And $\frac{1}{3}$ means one object divided into three equal parts.

Say this is one 

Then this is $\frac{1}{2}$ 

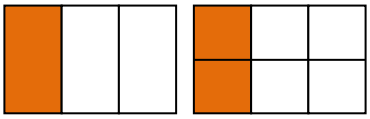
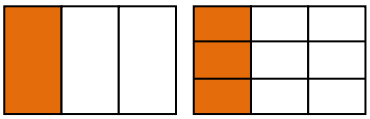
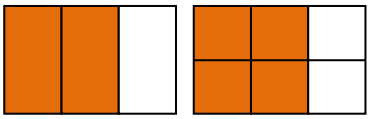
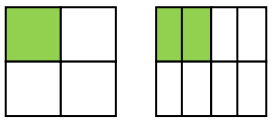
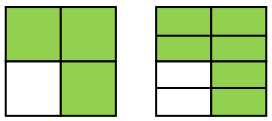
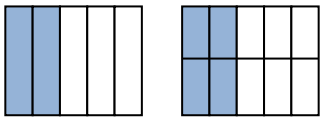
And this is $\frac{1}{3}$ 

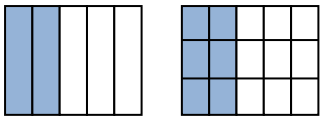
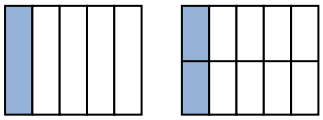
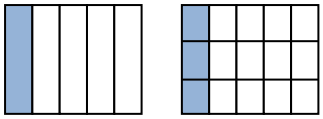
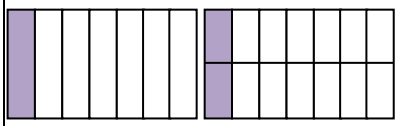
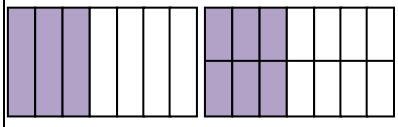
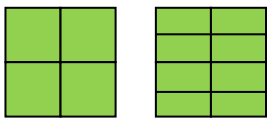
$\frac{1}{3}$ is smaller than $\frac{1}{2}$ because the same one object is divided into more equal pieces.

Activity 3: Order, please

one tenth	$\frac{1}{10}$	seven tenths	$\frac{7}{10}$
one eighth	$\frac{1}{8}$	three fourths	$\frac{3}{4}$
one sixth	$\frac{1}{6}$	four fifths	$\frac{4}{5}$
one fifth	$\frac{1}{5}$	nine tenths	$\frac{9}{10}$
one fourth	$\frac{1}{4}$	five fourths	$\frac{5}{4}$
one third	$\frac{1}{3}$	four thirds	$\frac{4}{3}$
three eighths	$\frac{3}{8}$	three halves	$\frac{3}{2}$
two fifths = Four tenths	$\frac{2}{5} = \frac{4}{10}$	five thirds	$\frac{5}{3}$
one half	$\frac{1}{2}$	four halves	$\frac{4}{2}$
three fifths	$\frac{3}{5}$	five halves	$\frac{5}{2}$
two thirds	$\frac{2}{3}$	six halves	$\frac{6}{2}$

Practice 16: Two looks at equivalent fractions

Equivalent Fractions	Picture	How many equal parts did you make from each original part?	Algebraic form:
$\frac{1}{3} = \frac{2}{6}$		Each of the one thirds is divided into **2** equal pieces	$\frac{1 * 2 = 4}{3 * 2 = 8}$
$\frac{1}{3} = \frac{3}{9}$		Each of the one thirds is divided into **3** equal pieces	$\frac{1 * 3 = 3}{3 * 3 = 9}$
$\frac{2}{3} = \frac{4}{6}$		Each of the one thirds is divided into **2** equal pieces	$\frac{2 * 2 = 4}{3 * 2 = 6}$
$\frac{1}{4} = \frac{2}{8}$		Each of the one fourths is divided into **2** equal pieces	$\frac{1 * 2 = 2}{4 * 2 = 8}$
$\frac{3}{4} = \frac{6}{8}$		Each of the one fourths is divided into **2** equal pieces	$\frac{3 * 2 = 6}{4 * 2 = 8}$
$\frac{2}{5} = \frac{4}{10}$		Each of the one fifths is divided into **2** equal pieces	$\frac{2 * 2 = 4}{5 * 2 = 10}$

Equivalent Fractions	Picture	How many equal parts did you make from each original part?	Algebraic form:
$\frac{2}{5} = \frac{6}{15}$		Each of the one fifths is divided into **3** equal pieces	$\frac{2 * 3 = 6}{5 * 3 = 15}$
$\frac{1}{5} = \frac{2}{10}$		Each of the one fifths is divided into **2** equal pieces	$\frac{1 * 2 = 2}{5 * 2 = 10}$
$\frac{1}{5} = \frac{3}{15}$		Each of the one thirds is divided into **3** equal pieces	$\frac{1 * 3 = 3}{5 * 3 = 15}$
$\frac{1}{7} = \frac{2}{14}$		Each of the one sevenths is divided into **2** equal pieces	$\frac{1 * 2 = 2}{7 * 2 = 14}$
$\frac{3}{7} = \frac{6}{14}$		Each of the one sevenths is divided into **2** equal pieces	$\frac{3 * 2 = 6}{7 * 2 = 14}$
$\frac{4}{4} = \frac{8}{8}$		Each of the one fourths is divided into **2** equal pieces	$\frac{4 * 2 = 8}{4 * 2 = 8}$

Is this a coincidence? Look at some of your examples in the activity you just did.

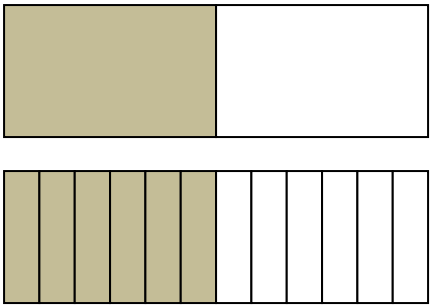
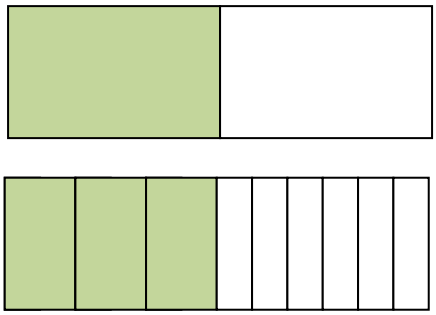
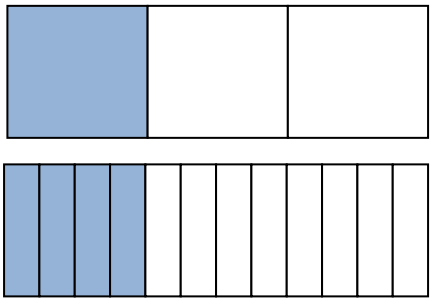
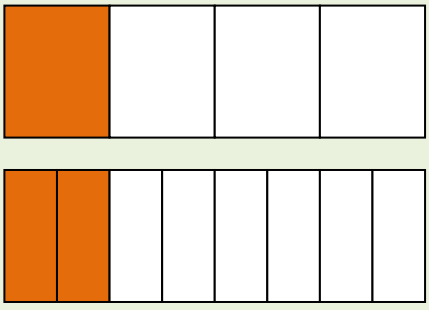
$$\frac{1 * 3 = 3}{3 * 3 = 6}$$ Here each of the two halves was divided into 3 pieces. And so on.

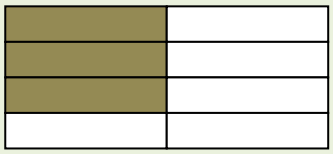
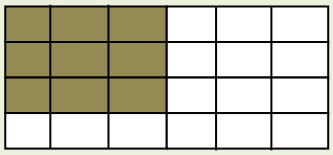
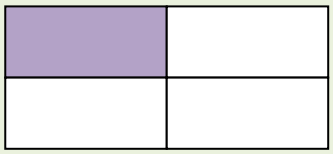
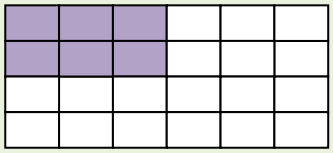
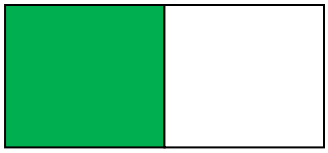
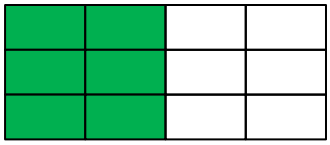
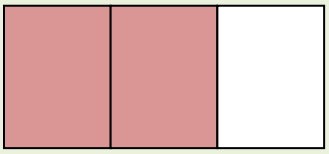
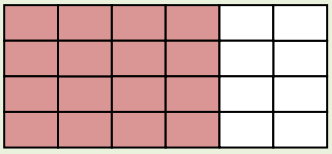
Try to make a general case here. Do your best!

$$\frac{1 * n =}{2 * n =}$$ means each of the halves is divided into **n** (how many) pieces?

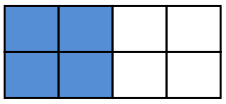
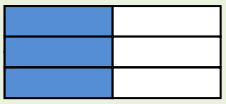
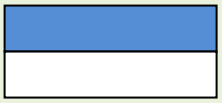
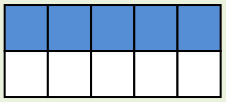
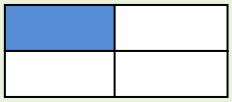
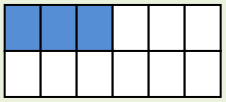
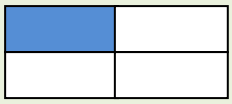
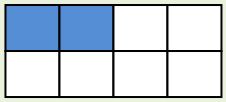
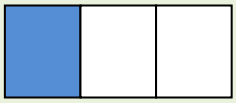
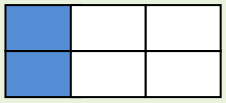
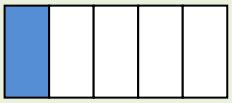
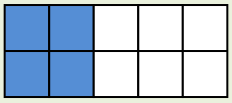
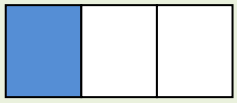
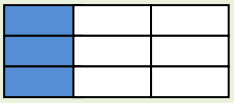
$$\frac{1 * n =}{3 * n =}$$ means each of the thirds is divided into **n** (how many) pieces?

Practice 17: More work with equivalent fractions

Picture	Symbol	Relationship	Explanation
	$\frac{1}{2}$ $\frac{6}{12}$	$\frac{1 * 6}{2 * 6} = \frac{6}{12}$ $\frac{6 \div 6}{12 \div 6} = \frac{1}{2}$	$\frac{1}{2}$ is equivalent to $\frac{6}{12}$ because they cover the same area
	$\frac{1}{2}$ $\frac{3}{6}$	$\frac{1 * 3}{2 * 3} = \frac{3}{6}$ $\frac{3 \div 3}{6 \div 3} = \frac{1}{2}$	$\frac{1}{2}$ is equivalent to $\frac{3}{6}$ because they cover the same area
	$\frac{1}{3}$ $\frac{4}{12}$	$\frac{1 * 4}{3 * 4} = \frac{4}{12}$ $\frac{4 \div 4}{12 \div 4} = \frac{1}{3}$	$\frac{1}{3}$ is equivalent to $\frac{4}{12}$ because they cover the same area
	$\frac{1}{4}$ $\frac{2}{8}$	$\frac{1 * 2}{4 * 2} = \frac{2}{8}$ $\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$	$\frac{1}{4}$ is equivalent to $\frac{2}{8}$ because they cover the same area

Picture	Symbol	Relationship	Explanation
 	$\frac{3}{8}$ $\frac{9}{24}$	$\frac{3 * 3}{8 * 3} = \frac{9}{24}$ $\frac{9 \div 3}{24 \div 3} = \frac{3}{8}$	$\frac{3}{8}$ is equivalent to $\frac{9}{24}$ because they cover the same area
 	$\frac{1}{4}$ $\frac{6}{24}$	$\frac{1 * 6}{4 * 6} = \frac{6}{24}$ $\frac{6 \div 6}{24 \div 6} = \frac{1}{4}$	$\frac{1}{4}$ is equivalent to $\frac{6}{24}$ because they cover the same area
 	$\frac{1}{2}$ $\frac{6}{12}$	$\frac{1 * 6}{2 * 6} = \frac{6}{12}$ $\frac{6 \div 6}{12 \div 2} = \frac{1}{2}$	$\frac{1}{2}$ is equivalent to $\frac{6}{12}$ because they cover the same area
 	$\frac{2}{3}$ $\frac{16}{24}$	$\frac{2 * 8}{3 * 8} = \frac{16}{24}$ $\frac{16 \div 8}{2 \div 8} = \frac{2}{3}$	$\frac{2}{3}$ is equivalent to $\frac{16}{24}$ because they cover the same area

Practice 18: Using pictures to find equivalent fractions – The Area Model

Original Fraction	Picture	Final Fraction	Picture	Factor for Numerator and Denominator
$\frac{1}{2}$		$\frac{4}{8}$		For every 1 piece of $\frac{1}{2}$, there are 4 pieces for $\frac{4}{8}$
$\frac{1}{2}$		$\frac{3}{6}$		For every 1 piece of $\frac{1}{2}$, there are 3 pieces for $\frac{3}{6}$
$\frac{1}{2}$		$\frac{5}{10}$		For every 1 piece of $\frac{1}{2}$, there are 5 pieces for $\frac{5}{10}$
$\frac{1}{4}$		$\frac{3}{12}$		For every 1 piece of $\frac{1}{4}$, there are 3 pieces for $\frac{3}{12}$
$\frac{1}{4}$		$\frac{2}{8}$		For every 1 piece of $\frac{1}{4}$, there are 2 pieces for $\frac{2}{8}$
$\frac{1}{3}$		$\frac{2}{6}$		For every 1 piece of $\frac{1}{3}$, there are 2 pieces for $\frac{2}{6}$
$\frac{2}{5}$		$\frac{4}{10}$		For every 1 piece of $\frac{2}{5}$, there are 2 pieces for $\frac{4}{10}$
$\frac{1}{3}$		$\frac{3}{9}$		For every 1 piece of $\frac{1}{3}$, there are 3 pieces for $\frac{3}{9}$

Activity 4: Finding equivalent fractions with your fraction pieces

Use your fraction pieces for this activity.

$\frac{1}{2}$	$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{5}{10}$
$\frac{1}{3}$	$\frac{2}{6}$ $\frac{4}{12}$
$\frac{1}{5}$	$\frac{2}{10}$
$\frac{2}{3}$	$\frac{4}{6}$ $\frac{8}{12}$
$\frac{2}{5}$	$\frac{4}{10}$

$\frac{5}{10}$	$\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{1}{2}$
$\frac{3}{12}$	$\frac{1}{4}$ $\frac{2}{8}$
$\frac{1}{4}$	$\frac{2}{8}$ $\frac{3}{12}$

Thinking about it:

a. What is the same about the list of equivalent fractions for $\frac{1}{2}$ and $\frac{5}{10}$?

They have the same list of equivalent fractions.

b. What is the same about the list of equivalent fractions for $\frac{3}{12}$ and $\frac{1}{4}$?

They have the same list of equivalent fractions.

c. How can you tell without fraction pieces that two fractions are equivalent?
(more about this later)

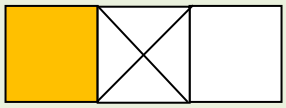
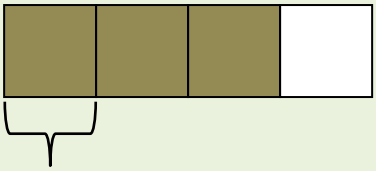
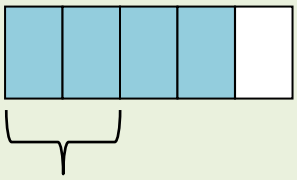
They cover the same area exactly.

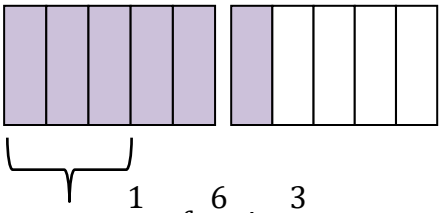
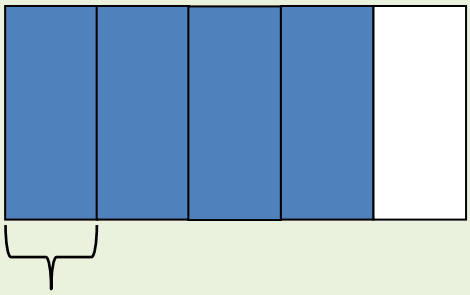
Practice 19: More equivalent fraction practice.

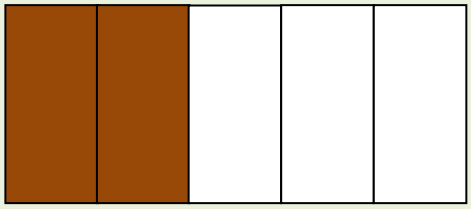
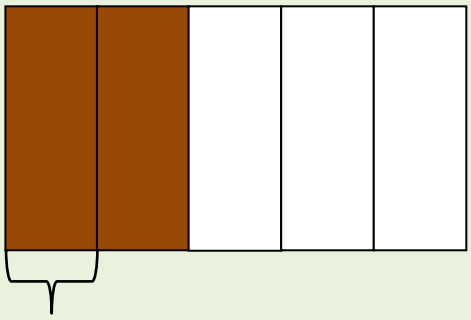
Circle the fractions that are equivalent to the given fraction.

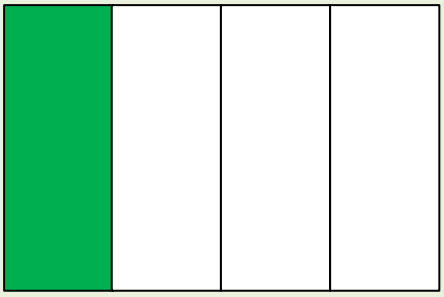
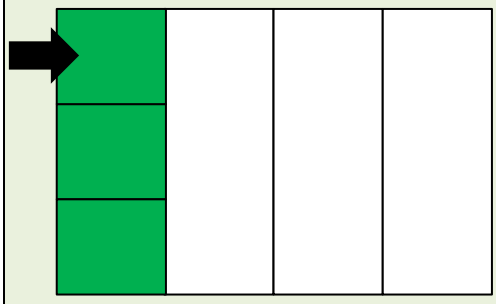
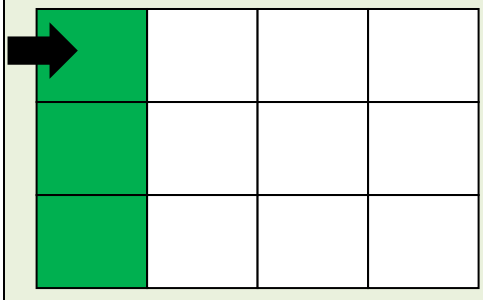
$\frac{1}{2}$	$\frac{4}{8}$ $\frac{120}{240}$ $\frac{2}{1}$ $\frac{50}{25}$ $\frac{3}{6}$ $\frac{75}{150}$ $\frac{16}{36}$ $\frac{5}{12}$ $\frac{11}{20}$ $\frac{82}{164}$ $\frac{4}{2}$ $\frac{14}{18}$ $\frac{350}{700}$
$\frac{1}{3}$	$\frac{9}{3}$ $\frac{120}{300}$ $\frac{3}{1}$ $\frac{50}{150}$ $\frac{3}{6}$ $\frac{2}{6}$ $\frac{16}{46}$ $\frac{4}{12}$ $\frac{11}{33}$ $\frac{30}{10}$ $\frac{6}{2}$ $\frac{14}{42}$ $\frac{300}{900}$
$\frac{1}{4}$	$\frac{4}{12}$ $\frac{120}{480}$ $\frac{4}{1}$ $\frac{50}{25}$ $\frac{4}{8}$ $\frac{25}{100}$ $\frac{75}{150}$ $\frac{3}{12}$ $\frac{11}{42}$ $\frac{80}{320}$ $\frac{8}{2}$ $\frac{4}{14}$ $\frac{100}{400}$
$\frac{2}{5}$	$\frac{4}{20}$ $\frac{200}{500}$ $\frac{5}{1}$ $\frac{50}{125}$ $\frac{10}{5}$ $\frac{20}{100}$ $\frac{12}{50}$ $\frac{5}{10}$ $\frac{22}{55}$ $\frac{8}{20}$ $\frac{5}{2}$ $\frac{12}{30}$ $\frac{20}{5}$
$\frac{2}{3}$	$\frac{4}{12}$ $\frac{100}{300}$ $\frac{12}{13}$ $\frac{50}{75}$ $\frac{9}{6}$ $\frac{30}{20}$ $\frac{16}{24}$ $\frac{4}{6}$ $\frac{22}{33}$ $\frac{80}{120}$ $\frac{3}{2}$ $\frac{9}{12}$ $\frac{250}{350}$

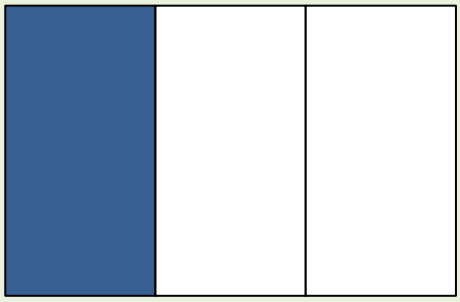
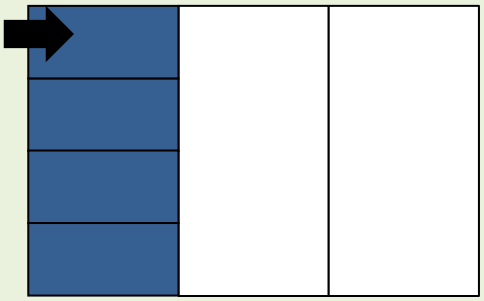
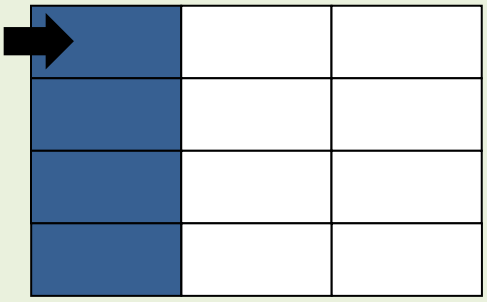
Practice 20: Using pictures and algebra to multiply fractions

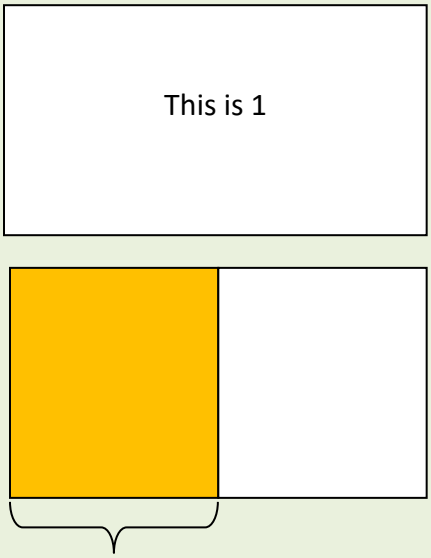
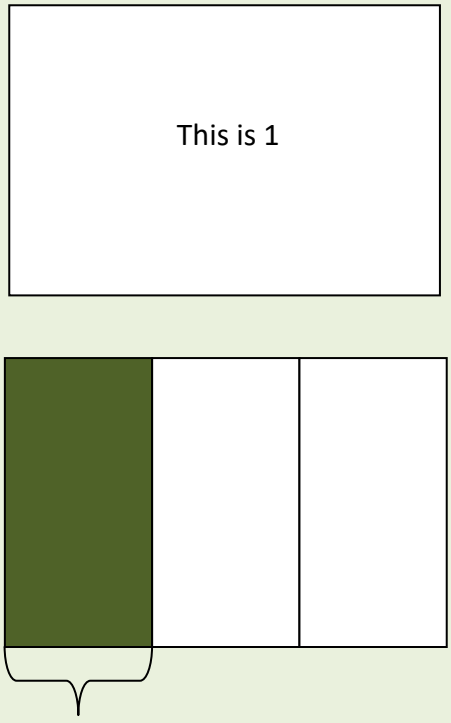
$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a*c}{b*d}$
$\frac{1}{2} * \frac{2}{3} =$	Start with $\frac{2}{3}$  Do the "of"  Result $\frac{1}{3}$  $\frac{1}{2}$ of $\frac{2}{3}$ is $\frac{2}{6} = \frac{1}{6}$	$\frac{1}{2} * \frac{2}{3} = \frac{1*2}{2*3} = \frac{2}{6} = \frac{1}{3}$
$\frac{1}{3} * \frac{3}{4} =$	 $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$	$\frac{1}{3} * \frac{3}{4} = \frac{1*3}{3*4} = \frac{3}{12} = \frac{1}{4}$
$\frac{1}{2} * \frac{4}{5} =$	 $\frac{1}{2}$ of $\frac{4}{5}$ is $\frac{2}{5}$	$\frac{1}{2} * \frac{4}{5} = \frac{1*4}{2*5} = \frac{4}{10} = \frac{2}{5}$

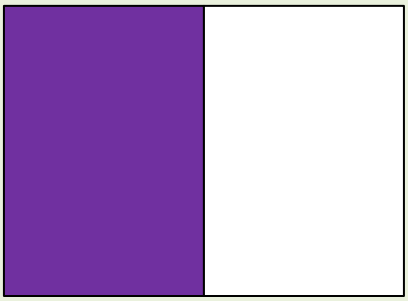
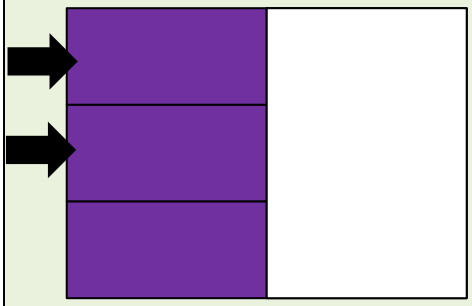
$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{2} * \frac{6}{5} =$	 $\frac{1}{2}$ of $\frac{6}{5}$ is $\frac{3}{5}$	$\frac{1}{2} * \frac{6}{5} = \frac{1 * 6}{2 * 5} = \frac{6}{10} = \frac{3}{5}$
$\frac{1}{4} * \frac{4}{5} =$ means $\frac{1}{4}$ *of* $\frac{4}{5}$	Step 1: Show $\frac{4}{5}$  Step 2: Show $\frac{1}{4}$ of $\frac{4}{5}$ $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{1}{5}$	$\frac{1}{4} * \frac{4}{5} = \frac{1 * 4}{4 * 5} = \frac{4}{20} = \frac{1}{5}$

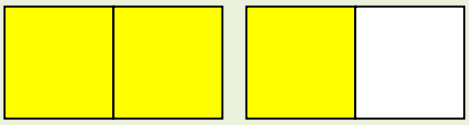
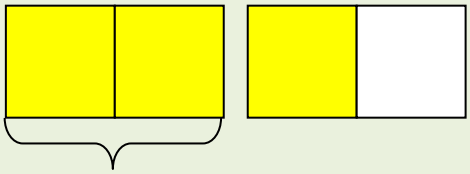
$\frac{a}{b} * \frac{c}{d}$	Picture:	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{2} * \frac{2}{5} =$ means $\frac{1}{2}$ *of* $\frac{2}{5}$	Step 1: Show $\frac{2}{5}$  Step 2: Show $\frac{1}{2}$ of $\frac{2}{5}$  $\frac{1}{2}$ of $\frac{2}{5}$ is $\frac{1}{5}$	$\frac{1}{2} * \frac{2}{5} = \frac{1 * 2}{2 * 5} = \frac{2}{10} = \frac{1}{5}$

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{3} * \frac{1}{4} =$ means $\frac{1}{3}$ *of* $\frac{1}{4}$	Step 1: Show $\frac{1}{4}$  Step 2: Show $\frac{1}{3}$ of $\frac{1}{4}$ This is $\frac{1}{3}$ of $\frac{1}{4}$  Step 3: To figure out the name of $\frac{1}{3}$ of $\frac{1}{4}$ I can draw lines across and see that it is $\frac{1}{12}$ of 1 This is $\frac{1}{12}$  $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$	$\frac{1}{3} * \frac{1}{4} = \frac{1 * 1}{3 * 4} = \frac{1}{12}$

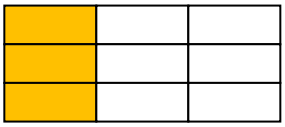
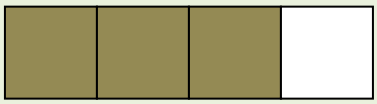
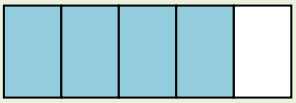
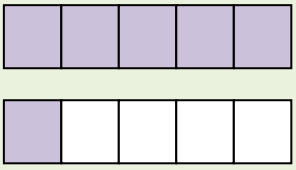
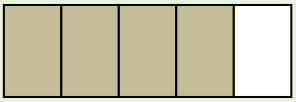
$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{4} * \frac{1}{3} =$ means $\frac{1}{4}$ *of* $\frac{1}{3}$	Step 1: Show $\frac{1}{3}$  Step 2: Show $\frac{1}{4}$ of $\frac{1}{3}$ This is $\frac{1}{4}$ of $\frac{1}{3}$  Step 3: To figure out the name of $\frac{1}{4}$ of $\frac{1}{3}$ I can draw lines across and see that it is $\frac{1}{12}$ of 1 This is $\frac{1}{12}$  $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$	$\frac{1}{4} * \frac{1}{3} = \frac{1 * 1}{4 * 3} = \frac{1}{12}$

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{1}{2} * 1 =$	 $\frac{1}{2}$ of 1 is $\frac{1}{2}$	$\frac{1}{2} * 1 = \frac{1}{2} * \frac{1}{1} = \frac{1 * 1}{2 * 1} = \frac{1}{2}$
$\frac{1}{3} * 1 =$	 $\frac{1}{3}$ of 1 is $\frac{1}{3}$	$\frac{1}{3} * 1 = \frac{1}{3} * \frac{1}{1} = \frac{1 * 1}{3 * 1} = \frac{1}{3}$

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{2}{3} * \frac{1}{2} =$ means $\frac{2}{3}$ of $\frac{1}{2}$	Step 1: Show $\frac{1}{2}$  Step 2: Show $\frac{2}{3}$ of $\frac{1}{2}$ This is $\frac{2}{3}$ of $\frac{1}{2}$  Step 3: To figure out the name of $\frac{2}{3}$ of $\frac{1}{3}$ I can draw lines across and see that it is $\frac{2}{6}$ of 1 This is $\frac{2}{6}$  $\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$ or $\frac{1}{3}$	$\frac{2}{3} * \frac{1}{2} = \frac{2 * 1}{3 * 2} = \frac{2}{6} = \frac{1}{3}$

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$
$\frac{2}{3} * \frac{3}{2} =$	This is 1  This is $\frac{3}{2}$  $\frac{2}{3}$ of $\frac{3}{2}$ is 1	$\frac{2}{3} * \frac{3}{2} = \frac{2 * 3}{3 * 2} = \frac{6}{6} = 1$

Practice 21: Using pictures and algebra to multiply fractions

$\frac{a}{b} * \frac{c}{d}$	Picture	Algorithm: $\frac{a}{b} * \frac{c}{d} = \frac{a*c}{b*d}$
$\frac{1}{2} * \frac{2}{3} =$	 $\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$ or $\frac{1}{3}$	$\frac{1}{2} * \frac{2}{3} = \frac{1*2}{2*3} = \frac{2}{6} = \frac{1}{3}$
$\frac{1}{3} * \frac{3}{4} =$	 $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$	$\frac{1}{3} * \frac{3}{4} = \frac{1*3}{3*4} = \frac{3}{12} = \frac{1}{4}$
$\frac{1}{2} * \frac{4}{5} =$	 $\frac{1}{2}$ of $\frac{4}{5}$ is $\frac{2}{5}$	$\frac{1}{2} * \frac{4}{5} = \frac{1*4}{2*5} = \frac{4}{10} = \frac{2}{5}$
$\frac{1}{2} * \frac{6}{5} =$	 $\frac{1}{2}$ of $\frac{6}{5}$ is $\frac{3}{5}$	$\frac{1}{2} * \frac{6}{5} = \frac{1*6}{2*5} = \frac{6}{10} = \frac{3}{5}$
$\frac{1}{4} * \frac{4}{5} =$	 $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{1}{5}$	$\frac{1}{4} * \frac{4}{5} = \frac{1*4}{4*5} = \frac{4}{20} = \frac{1}{5}$
$\frac{1}{2} * \frac{2}{5} =$	 $\frac{1}{2}$ of $\frac{2}{5}$ is $\frac{1}{5}$	$\frac{1}{2} * \frac{2}{5} = \frac{1*2}{2*5} = \frac{2}{10} = \frac{1}{5}$

Practice 22: More multiplication of fractions practice

a. $\frac{1}{3} * \frac{2}{5} = \frac{1 * 2}{3 * 5} = \frac{2}{15}$	b. $\frac{1}{5} * \frac{2}{7} = \frac{1 * 2}{5 * 7} = \frac{2}{35}$
c. $\frac{1}{3} * \frac{11}{7} = \frac{1 * 11}{3 * 7} = \frac{11}{21}$	d. $\frac{1}{3} * \frac{4}{5} = \frac{1 * 4}{3 * 5} = \frac{4}{15}$
e. $\frac{1}{4} * \frac{1}{8} = \frac{1 * 1}{4 * 8} = \frac{1}{32}$	f. $\frac{1}{3} * \frac{1}{3} = \frac{1 * 1}{3 * 3} = \frac{1}{9}$
g. $\frac{2}{3} * \frac{2}{5} = \frac{2 * 2}{3 * 5} = \frac{4}{15}$	h. $\frac{1}{4} * \frac{3}{4} = \frac{1 * 3}{4 * 4} = \frac{3}{16}$
i. $\frac{1}{5} * \frac{2}{11} = \frac{1 * 2}{5 * 11} = \frac{2}{55}$	j. $\frac{1}{10} * \frac{3}{7} = \frac{1 * 3}{10 * 7} = \frac{3}{70}$
k. $\frac{10}{3} * \frac{2}{7} = \frac{10 * 2}{3 * 7} = \frac{20}{21}$	l. $\frac{1}{7} * \frac{4}{5} = \frac{1 * 4}{7 * 5} = \frac{4}{35}$
m. $\frac{1}{3} * \frac{2}{3} = \frac{1 * 2}{3 * 3} = \frac{2}{9}$	n. $\frac{5}{3} * \frac{2}{3} = \frac{5 * 2}{3 * 3} = \frac{10}{9}$
o. $\frac{1}{17} * \frac{2}{3} = \frac{1 * 2}{17 * 3} = \frac{2}{51}$	p. $\frac{10}{7} * \frac{3}{1} = \frac{10 * 3}{7 * 1} = \frac{30}{7}$

Practice 23: And even more practice multiplying fractions

a. $\frac{1}{3} * \frac{w}{5} = \frac{1 * w}{3 * 5} = \frac{1w}{15} = \frac{w}{15}$	b. $\frac{1}{y} * \frac{2}{7} = \frac{1 * 2}{y * 7} = \frac{2}{7y}$
c. $\frac{1}{3} * \frac{11}{x} = \frac{1 * 11}{3 * x} = \frac{11}{3x}$	d. $\frac{r}{3} * \frac{4}{5} = \frac{r * 4}{3 * 5} = \frac{4r}{15}$
e. $\frac{x}{4} * \frac{1}{8} = \frac{x * 1}{4 * 8} = \frac{x}{32}$	f. $\frac{1}{s} * \frac{2}{3} = \frac{1 * 2}{s * 3} = \frac{2}{3s}$
g. $\frac{2}{3} * \frac{2}{5} = \frac{2 * 2}{3 * 5} = \frac{4}{15}$	h. $\frac{x}{y} * \frac{3}{4} = \frac{x * 3}{y * 4} = \frac{3x}{4y}$
i. $\frac{1}{5} * \frac{r}{11} = \frac{1 * r}{5 * 11} = \frac{r}{55}$	j. $\frac{t}{10} * \frac{3}{7} = \frac{t * 3}{10 * 7} = \frac{3t}{70}$
k. $\frac{10}{3} * \frac{k}{7} = \frac{10 * k}{3 * 7} = \frac{10k}{21}$	l. $\frac{1}{7} * \frac{x}{5} = \frac{1 * x}{7 * 5} = \frac{x}{35}$
m. $\frac{1}{3} * \frac{m}{3} = \frac{1 * m}{3 * 3} = \frac{m}{9}$	n. $\frac{5}{q} * \frac{2}{3} = \frac{5 * 2}{q * 3} = \frac{10}{3q}$
o. $\frac{1}{7} * \frac{2}{3} = \frac{1 * 2}{7 * 3} = \frac{2}{21}$	p. $\frac{1}{7} * \frac{3}{1} = \frac{1 * 3}{7 * 1} = \frac{3}{7}$

9. Help for Helpers

I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as "I know you can do this. I believe in you." And then leave the student alone to do the work.

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn't going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don't generally want to ask our boss or friend: "Am I right? Am I right?" As an adult, we have to know the answer is right ourselves. The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student's life. You will gain confidence in your students as you watch them be successful on their own.

Fractions are a huge part of math. Also, they are useful in everyday life. But their major importance is that they are a critical part of algebra. All Algebra 1 consists of is: fraction operations and the distributive law, with positive and negative numbers and variables, like x. So, understanding fractions and fraction operations is the key to Algebra success.