MATH WHISPERER

Where math makes sense

Research Edition

The Operations Series

Addition

Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information, please go to www.mathwhisperer.com

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DEDICATION

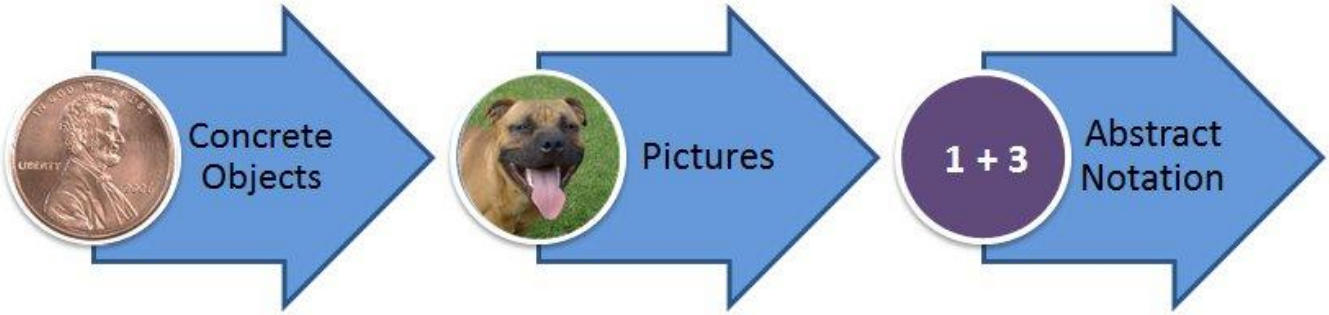
Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

Contents

Introduction 5
Addition Day 1 6
Practice 1: What is Addition Really? 7
Activity 1: Addition Cards 9
Addition Card Deck 11
Activity 2: Addition Card Game with Recording 15
Optional Reading: Know as Much as Your Math Teacher 16
Activity 3: The "Preferred Way" to Show Numbers 18
Ten Frames Sheet and Practice 20
Activity 4: Card Deck to Help Students Visualize Numbers in a Preferred Way 31
Activity 5: How Do You Know? 32
Addition with Ten Frames 35
The Japanese Way to Add 36
Practice 2: Adding with Color 37
Activity 6: Modeling with Pennies 39
Activity 7: Adding the Japanese Way 40
Practice 3: Ten Frame Practice 45
Practice 4: More Ten Frame Practice 46
Practice 5: Circle Pairs 47
Practice 6: Equals 10 48
Practice 7: Grouping 49
Practice 8: Additional Practice 50
Activity 8: Modeling the Addition Algorithm 51
Activity 9: Modeling the Addition Algorithm with Popsicle Sticks 53
Practice 9: More Addition 58
Practice 10: Enrichment 60
Solutions/Answers 62
Help for Helpers 76

1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is actually supposed to make sense. When you start with hands-on objects, math can make sense.



You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three step progression works for everybody.

It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. Won't it feel great to never have to learn this again? The math will stick with you with the three step progression.

Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you.

Addition Day 1

Learning objectives

There are two parts to being able to add reliably. One is mastering addition facts (such as 4 + 3 = 7) and the second is combining addition facts with place value (such as 18 + 14 = 32). It is also helpful to be able to estimate an answer to make sure their computation is correct. We will work on all three of these.

There are four operations in mathematics: addition, subtraction, multiplication and division. Pretty much all of math until calculus is adding, subtracting, multiplying and dividing different kinds of numbers and variables.

It is very helpful to be able to add quickly. The Japanese have a marvelous way to teach addition quickly, and that is what I will show you here.

I have seen so many middle and high school students adding on their fingers underneath their desks. It makes me sad for two reasons. One, it is slow. Two, students adding on their fingers generally feels ashamed. If you are one of those students, it is not your fault. It takes a while to learn how to add, and you didn't get enough time.

I urge you to take the time, probably 5 hours total with learning and practice, to learn to add in a much easier and faster way. I looked around the world for the best way to learn to add, and that is what is here.

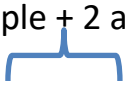
For this lesson collection you will need:

- Addition card deck, pages 11 and 13
- Pennies (about 15)
- Popsicle sticks (about 200)
- Rubber bands (about 20)

Practice 1: What is addition really?

Write the results for the following additions.

1 apple + 2 apples totals _____



(added to)

1 apple + 2 oranges totals _____

1 grapefruit + 2 oranges totals _____

1 apple + 2 iPods totals _____

1 apple + 2 seconds totals _____

Which of the results above can give 3 of something that makes sense?

_____

_____

Why?

_____

Explain how you would show that 1 apple + 2 apples totals 3 apples.

_____

What is the same about these two problems:

1 apple + 2 apples = ?

1 apple + 2 bananas = ?

What is different?

_____

Answers page 63

What is addition really?

This is a good place to introduce a new vocabulary word **REFERENT.** The referent is the label attached to the number.

Grammar may help. The number is an adjective, and the referent is the noun that the adjective describes. (We do not use the word "unit" because this also connotes "one.")

For example: 3 apples – 3 is an adjective, and apples is the noun. Apples is the referent.

To state a rule about when it makes sense to add two things**. You can add two things when they are the same, or they can be made the same**. In other words, you can add when the referents are the same.

To explain how you would show that 1 apple + 2 apples totals 3 apples.

I would draw a picture:

1 apple + 2 apples is the same as 3 apples

Activity 1: Addition Cards

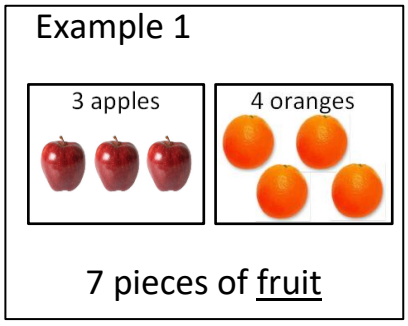
Materials:

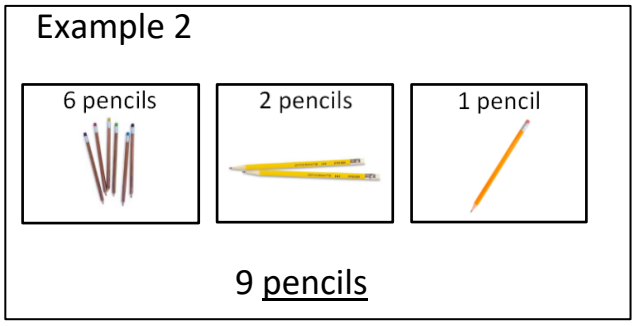
- Addition card set (page 11 and 13)
- Scissors and cut the cards out.

Students will play two card games. The first game is more informal. The second game is similar to the first game, but students are required to record their plays.

All of the cards in the card deck are laid out face up.

Game 1: The first student puts together 2 or 3 of the cards that can be added. He or she states the referent. The referent may have to be changed, and if so, he or she must state the new referent.

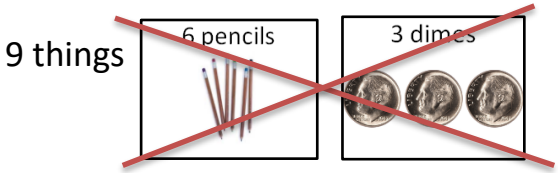




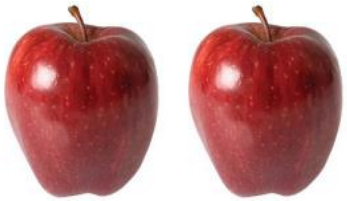
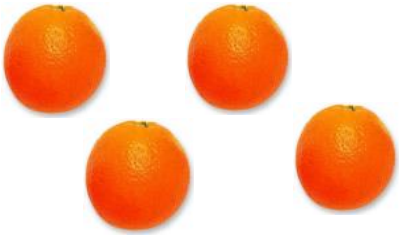
Then the second student takes a turn and also puts together 2 or 3 cards… Then students get another turn. When they are done they record how many cards are left over, i.e. cards with numbers and referents that cannot be combined by addition.

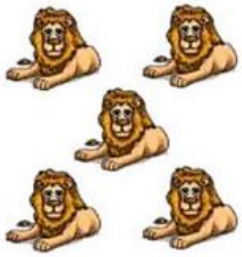
Then the student who went second goes first.

Rule: It is not acceptable to have a referent of "things." For example, it is not allowed to take the "6 pencils" card and the "3 dimes" card and say "9 things."



Addition Card Deck

1 penny 	3 dimes 	1 nickel 
4 pennies 	1 quarter 	2 apples 
3 apples 	4 oranges 	2 bananas 
6 pencils 	1 pencil 	3 pens 
4 markers 	2 maple trees 	3 maple trees 

4 oak trees 	2 puppies 	5 puppies 
6 puppies 	1 kitten 	3 kittens 
5 kittens 	3 tulips 	4 tulips 
1 daisy 	3 daisies 	5 lions 
2 turtles 	4 turtles 	2 pencils 

Activity 2: Addition Card Game with Recording

This is the second of the two card games. The goal is to find pairs that can be added. Changing referents is allowed, with the exception of the referent "things" or a synonymous word. Then students fill out a form like this:

Write expression with referents	No changes needed	Changes needed	Resulting sum

What values cannot be paired to add?

Optional Reading: Know as Much as Your Math Teacher

I rarely had a teacher explain to me why I was being taught a certain way. So I want to give you that option. I think it is really interesting, and I hope you will read on. It's your decision.

There is such a thing as "learning theory." For over 100 years, some people have studied how we learn. The learning theory followed in my lessons uses this model:

Concrete Objects (hands-on objects)	Iconic Representation (pictures)	Abstract Notation (symbols)

What this means is that learning math should begin with concrete objects, things you can hold in your hand. This is true for everyone. When a five or six year old is learning to add, that is how he or she should be taught.

What happens all too often is that these little children are taught with Abstract Notation first and only. That means they start with symbols, like "3."

No one is born understanding such a thing as "3." Five year olds can understand 3 cookies, 3 friends, 3 concrete objects. When kids are not given enough instruction using learning theory, they can end up counting on their fingers in high school. It's not their fault, and it generally doesn't feel good.

Using Fingers Is OK!

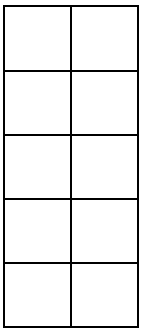
After all, fingers are concrete objects. It's just slow.

There is such a better way and here it is:

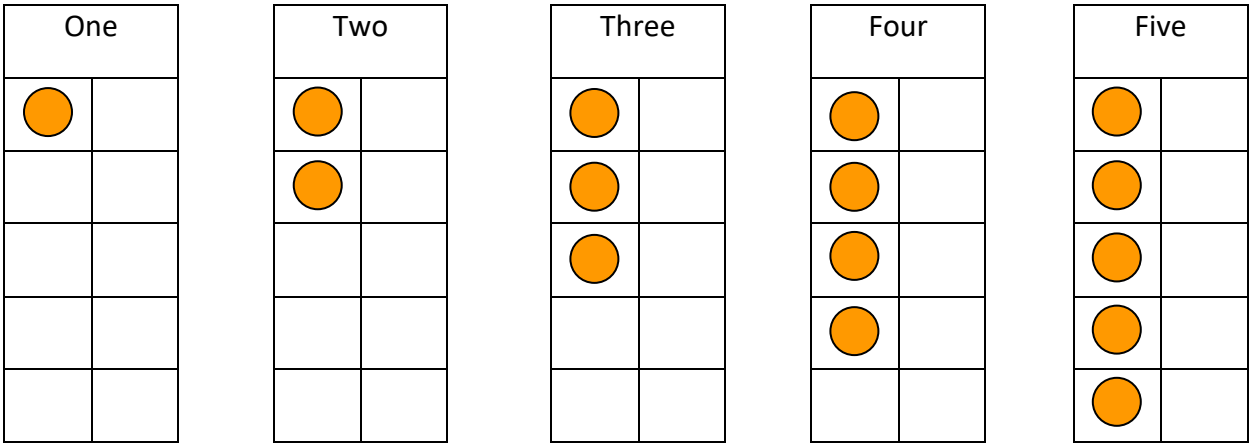
The Japanese Way to Add

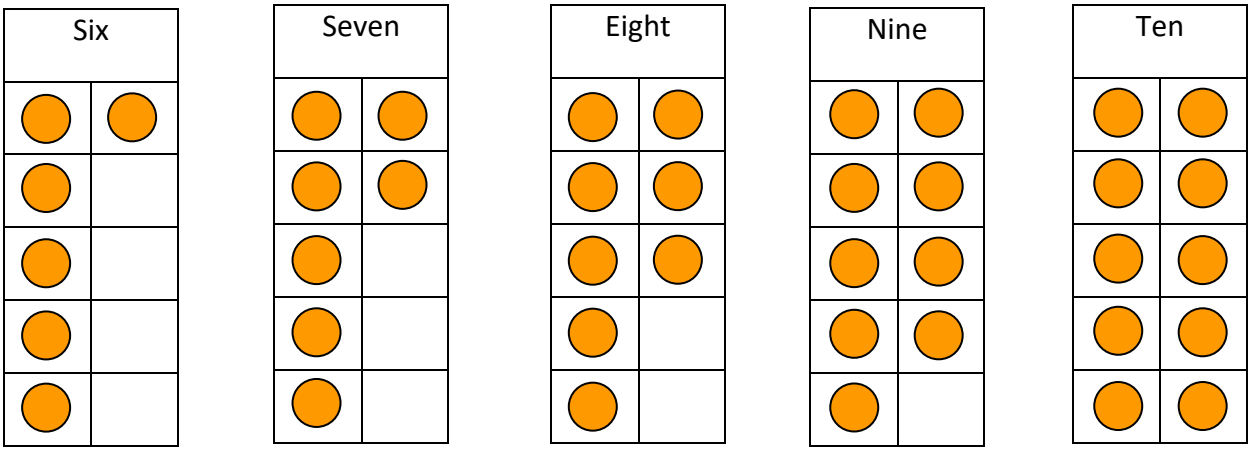
A more efficient way to add is to use the fill-ups method for teaching addition facts. This is the way the Japanese teach their students. The basic idea is that people can visualize five items in a row.

This is the template for visualizing numbers in fives.



There are many ways to show any number, but the following way leads to the easy way to add.

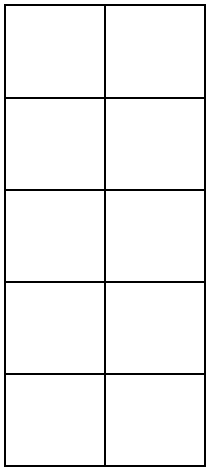




The key to the technique is to visualize the numbers in the manner shown, the "preferred way."

Activity 3: The "Preferred Way" to Show Numbers

Ten is the basis of our number system around the world. The Japanese math teachers figured out that people can see five. So they split ten into two fives, like this:

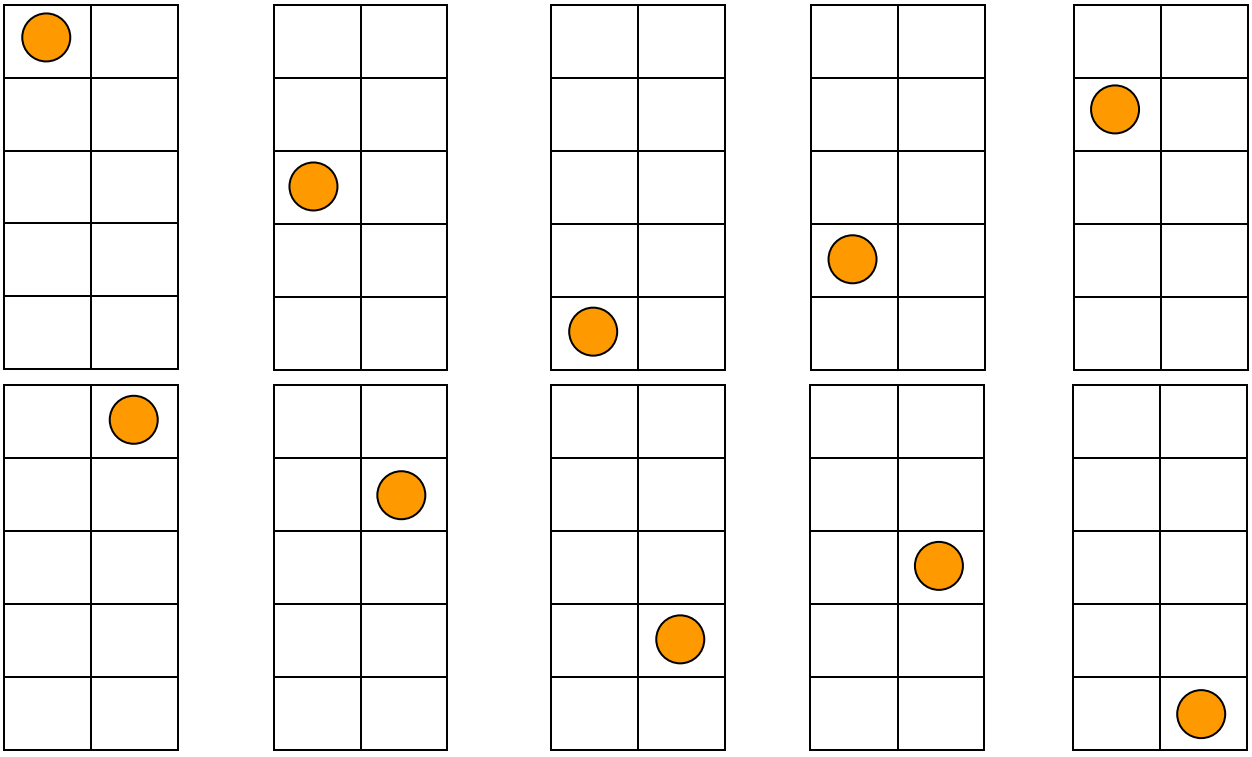


There are many ways to show each number, but only one way is the "preferred way." The preferred way leads to an easy way to add.

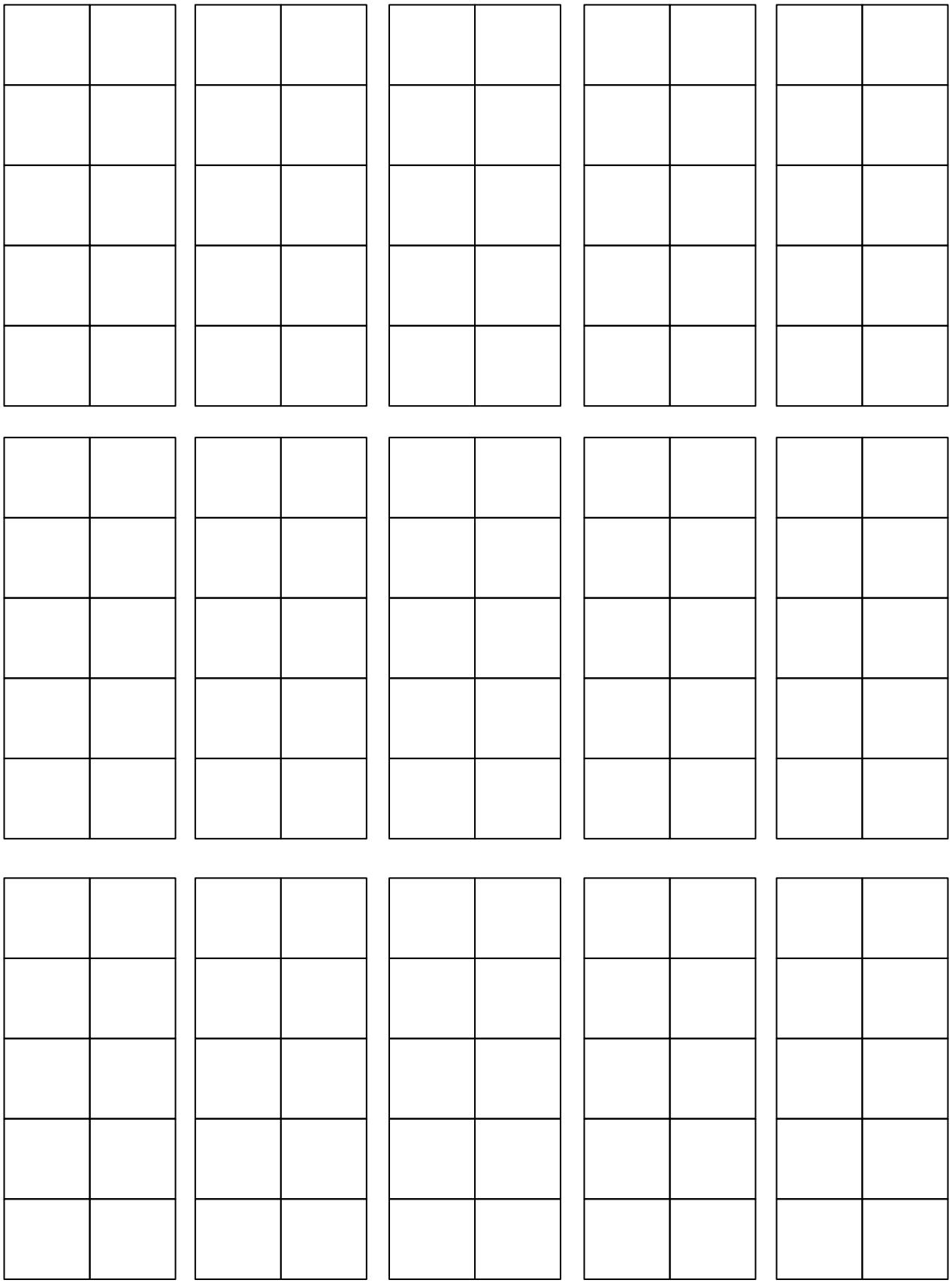
Materials:

- Ten from page 20
- 16 pennies

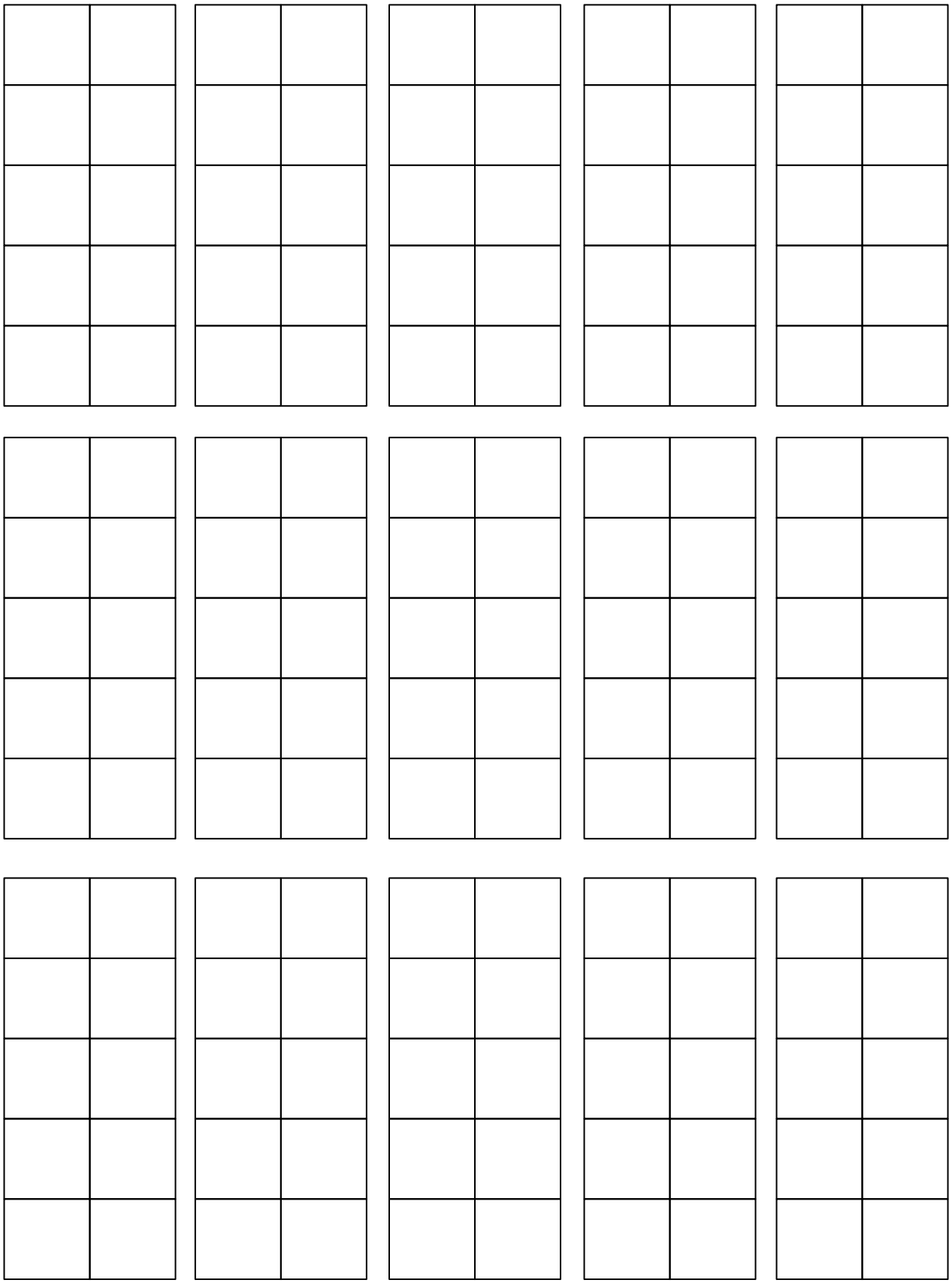
Show all the ways to show "one" on your small ten-frames. Here are the possibilities:



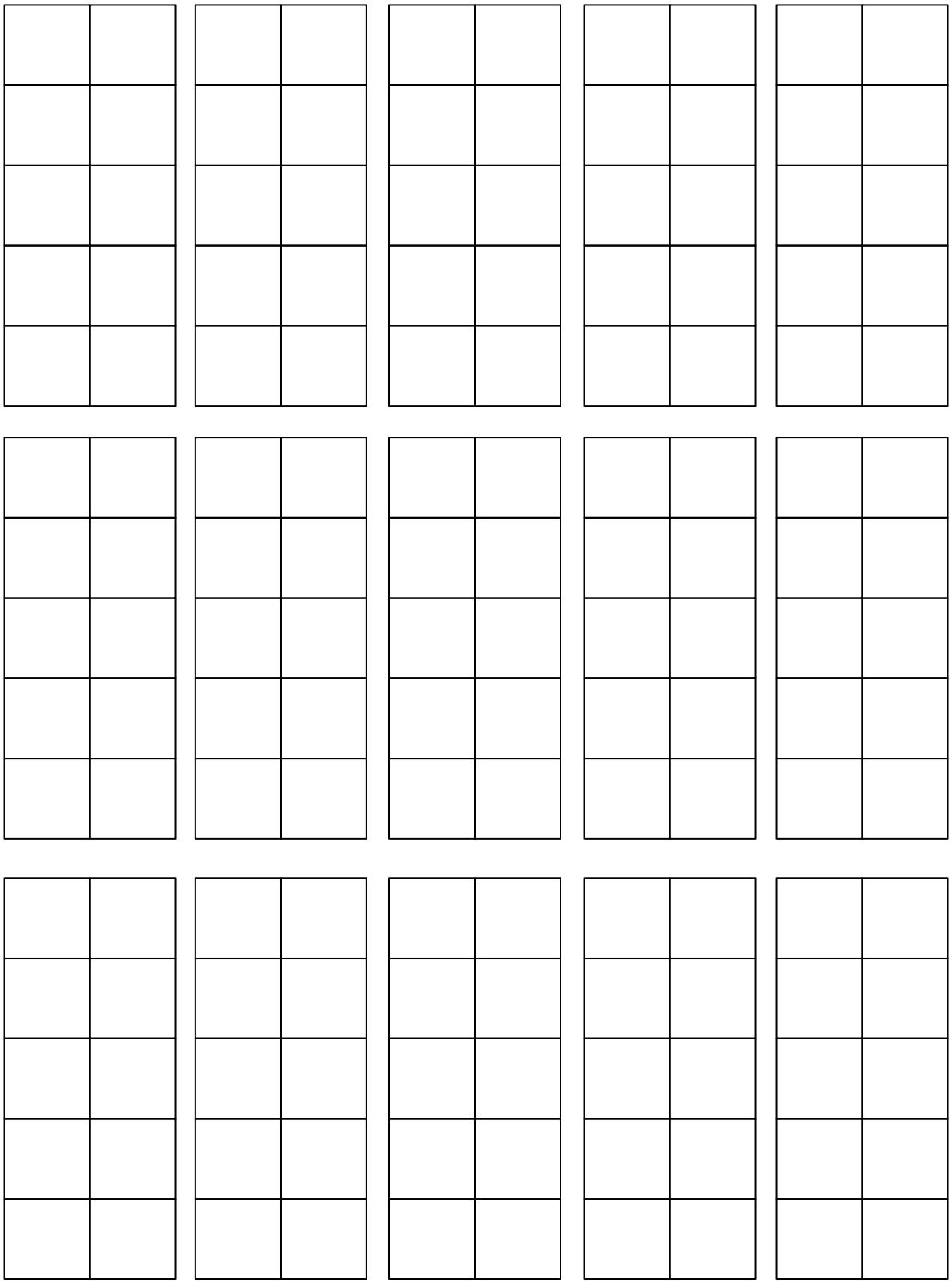
What are different ways to show 2?



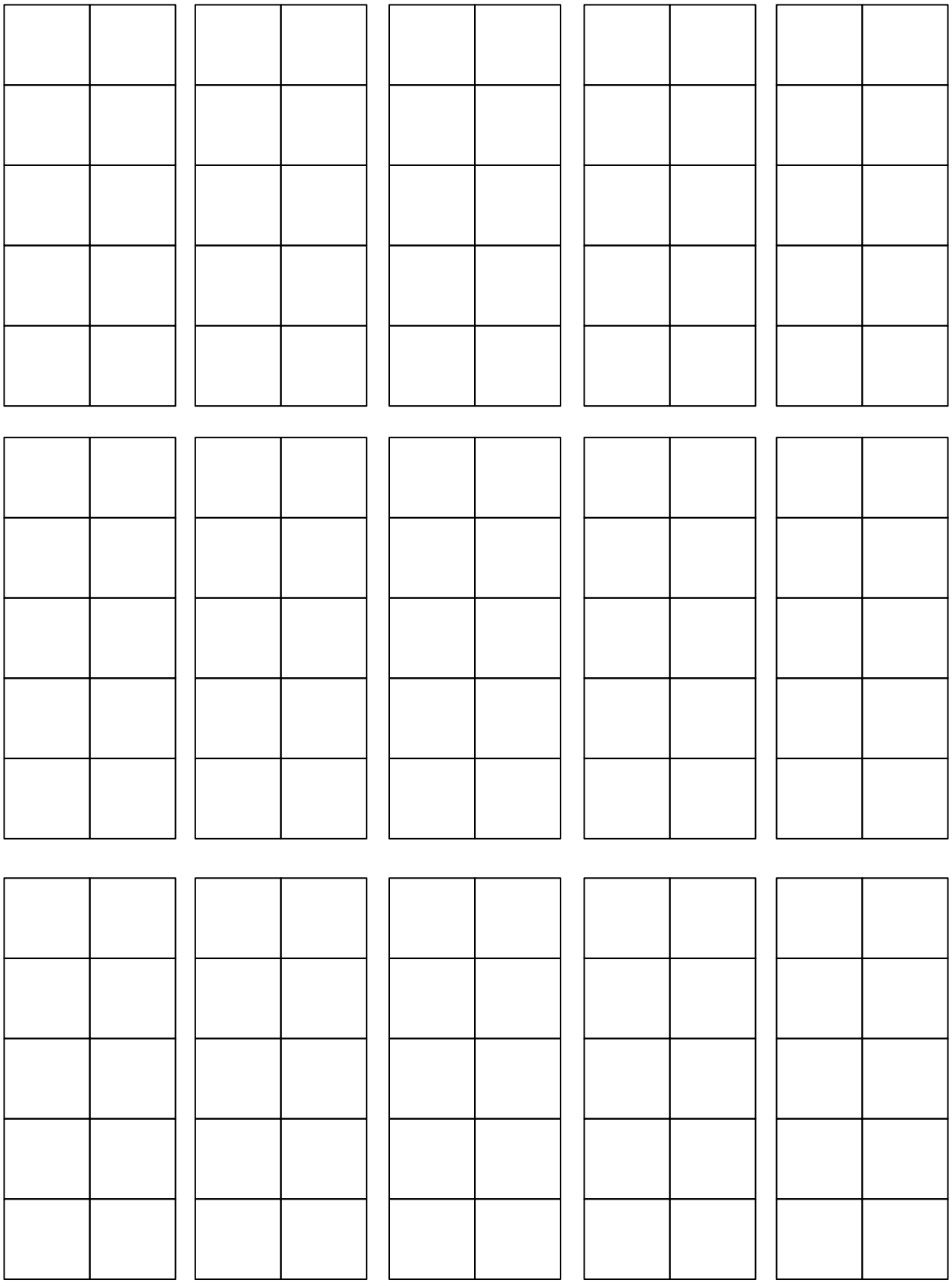
What are different ways to show 3?



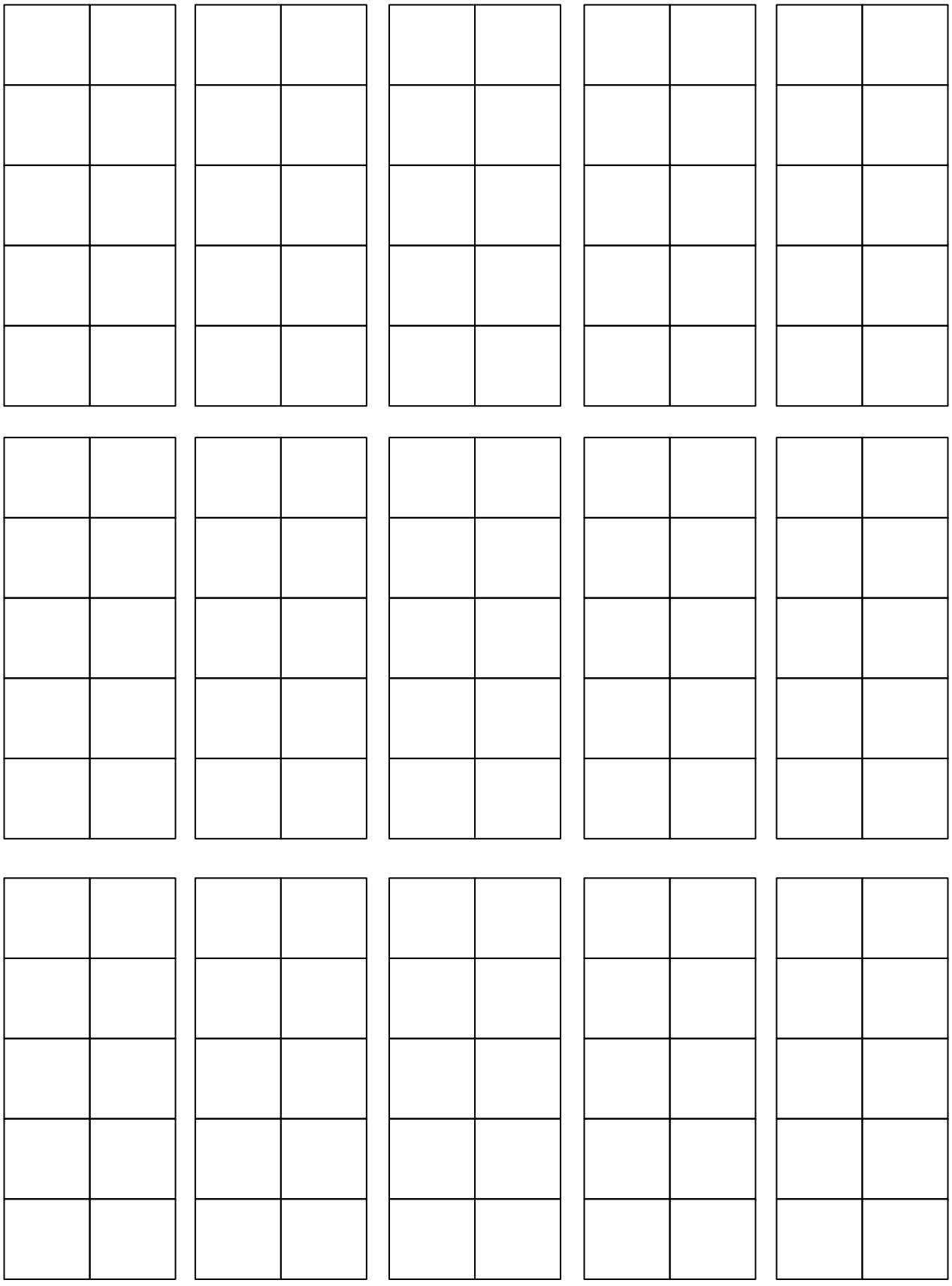
What are different ways to show 4?



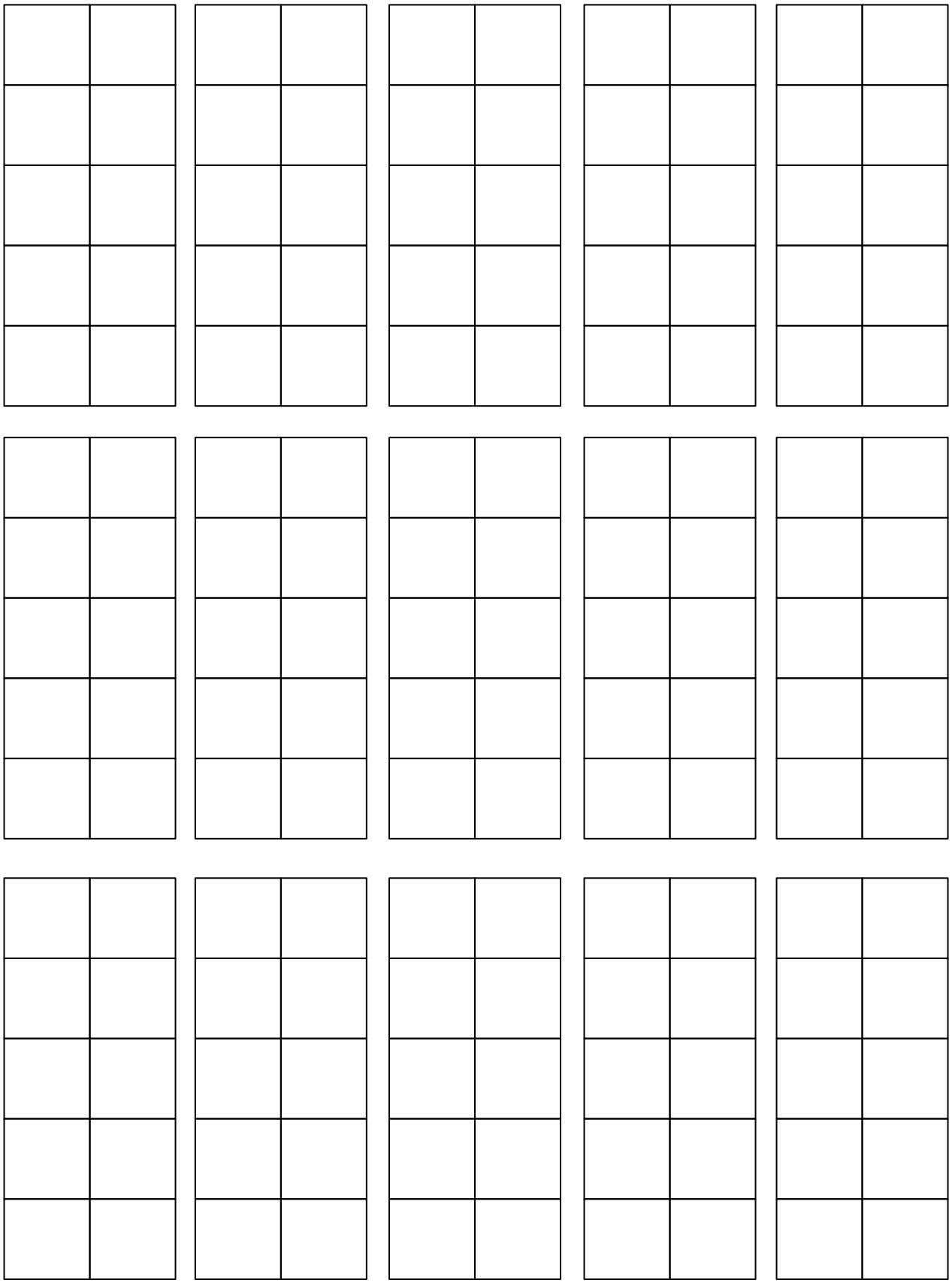
What are different ways to show 5?



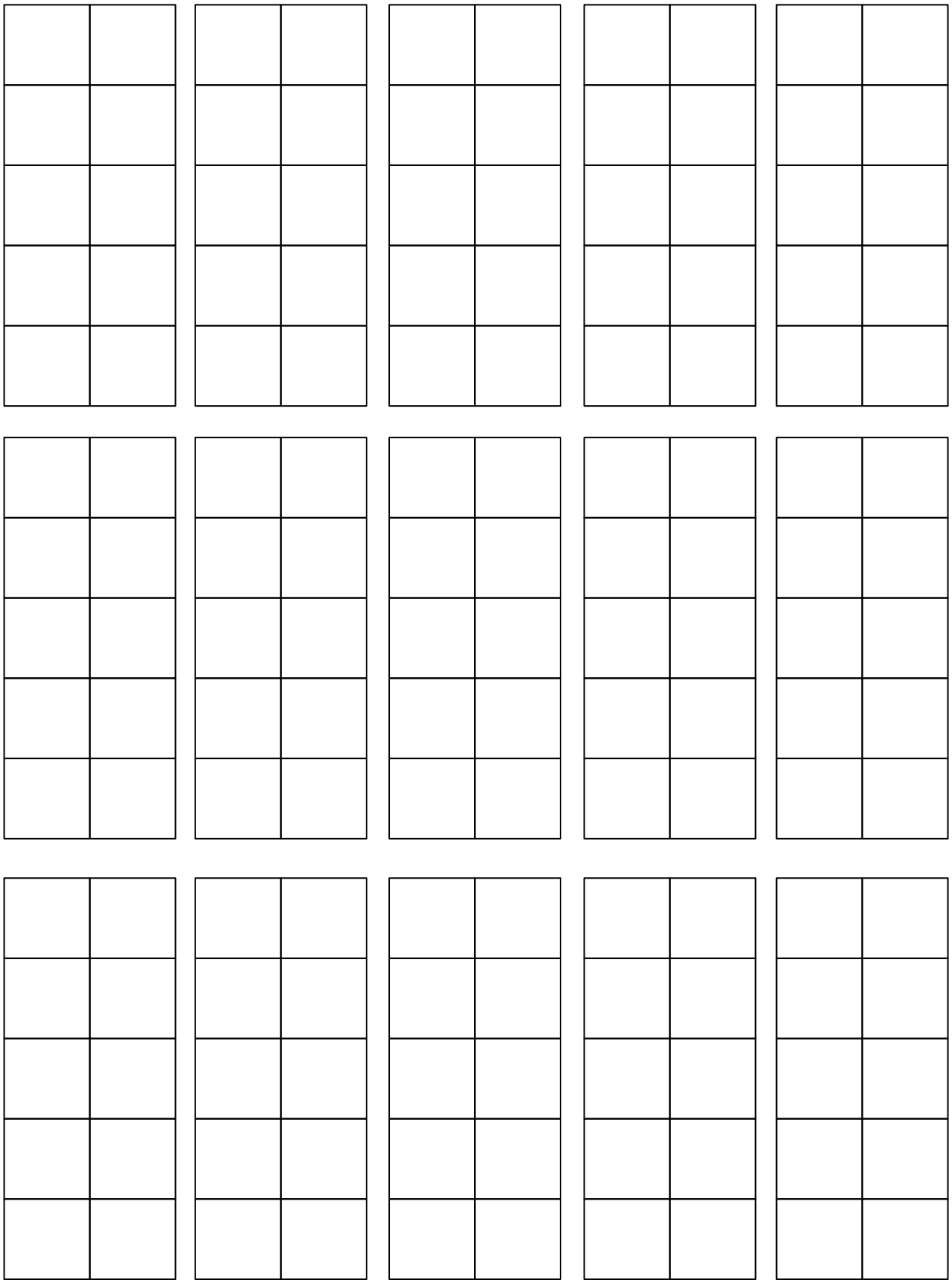
What are different ways to show 6?



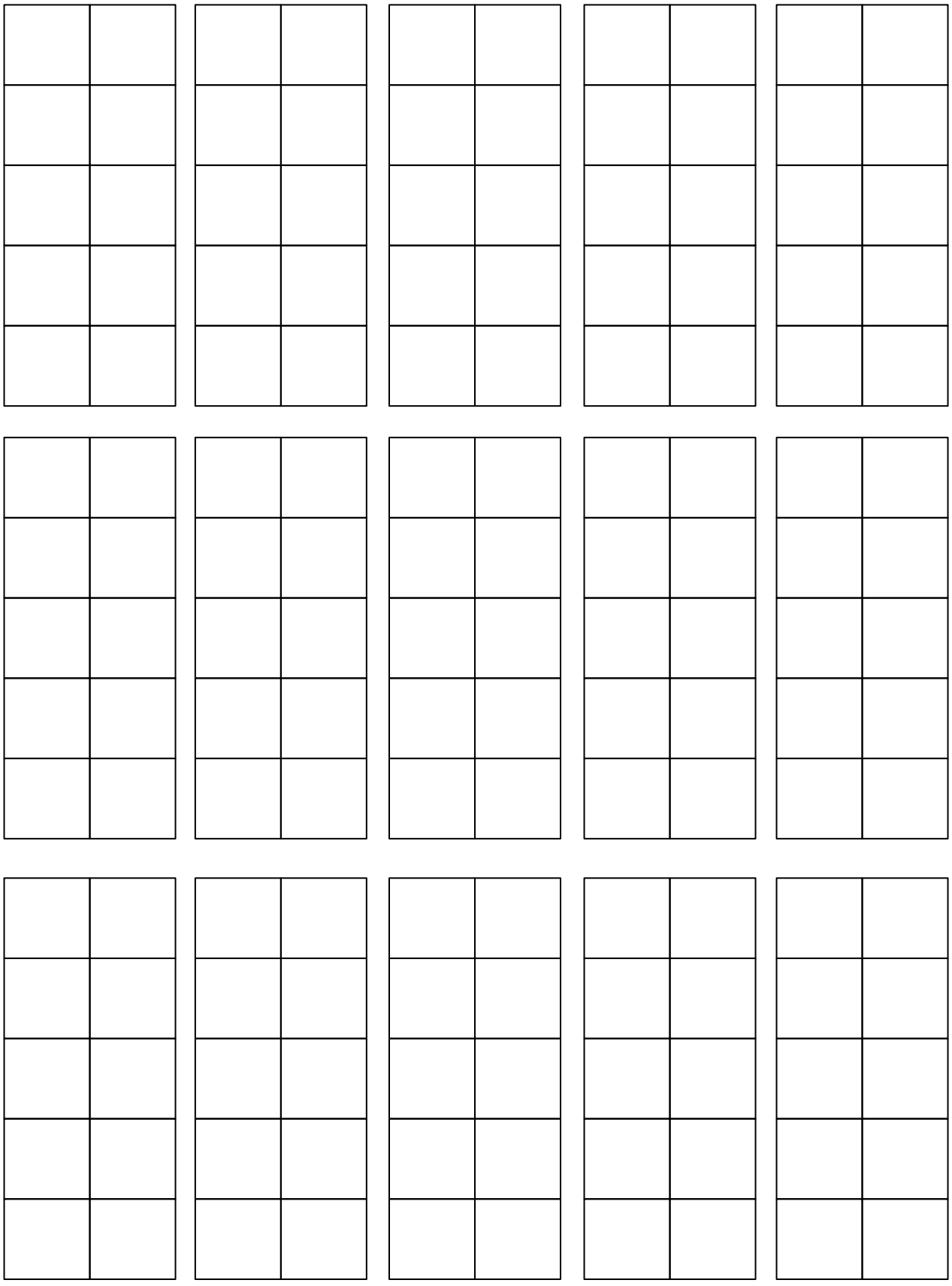
What are different ways to show 7?



What are different ways to show 8?



What are different ways to show 9?



Model each of these numbers with the pennies and ten frames. Then draw the number indicated in the preferred way.

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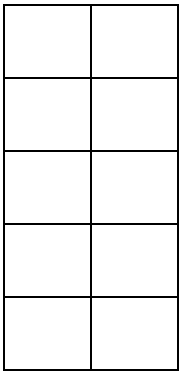

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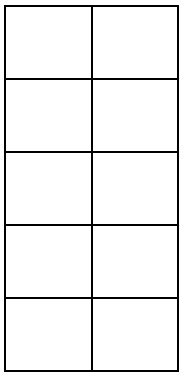

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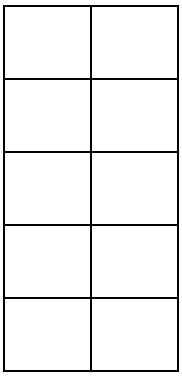

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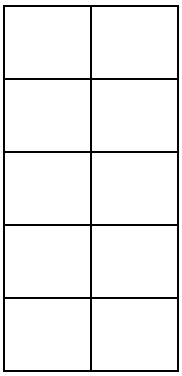

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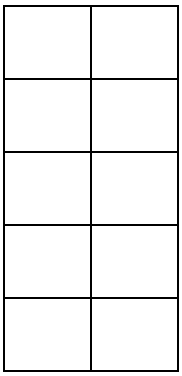

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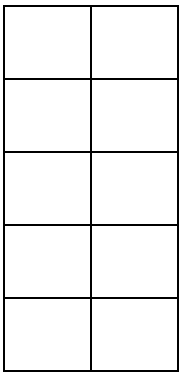

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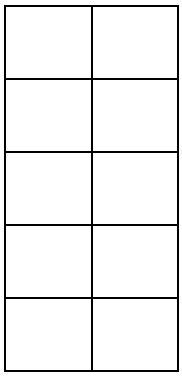

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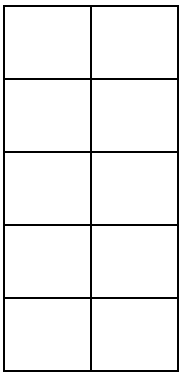

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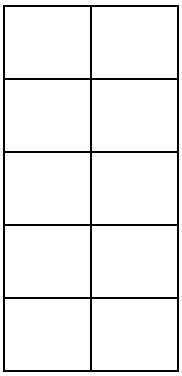

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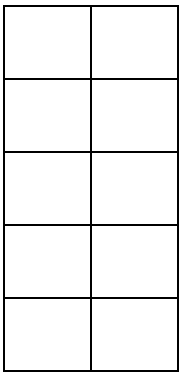

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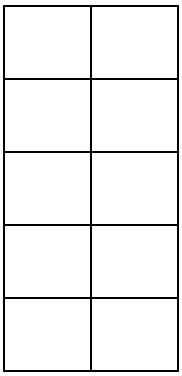

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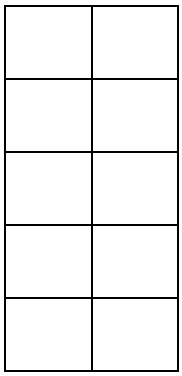

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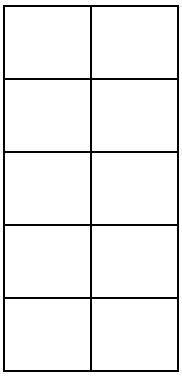

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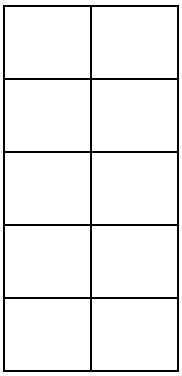

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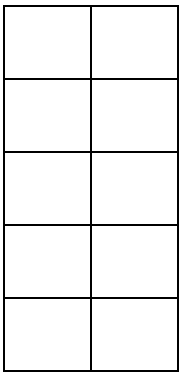

Model each of these numbers with the pennies and ten frames. Then draw the number indicated in the preferred way.

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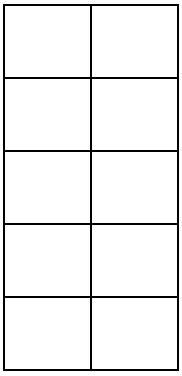

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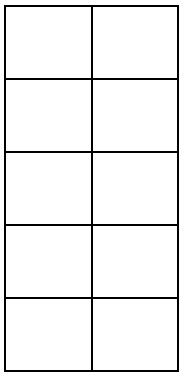

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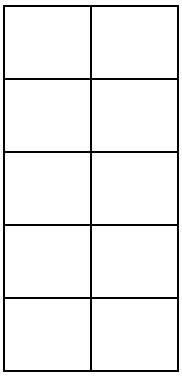

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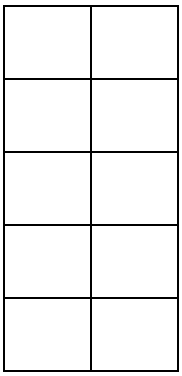

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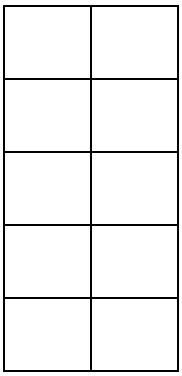

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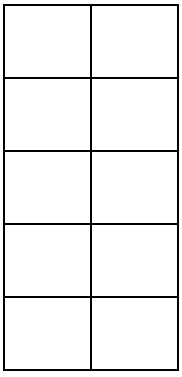

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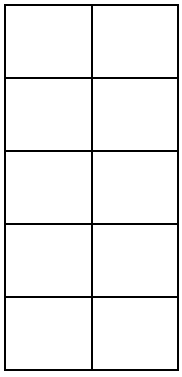

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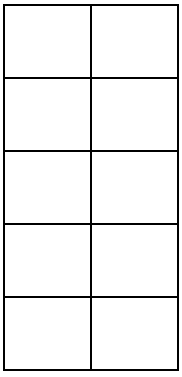

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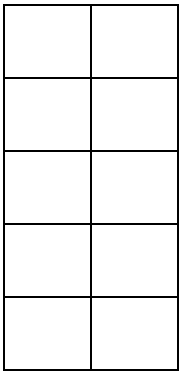

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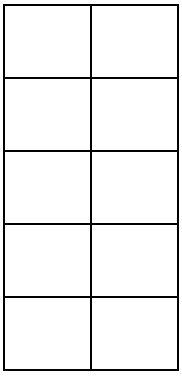

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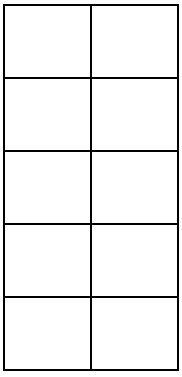

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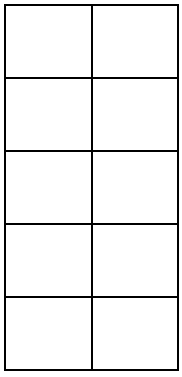

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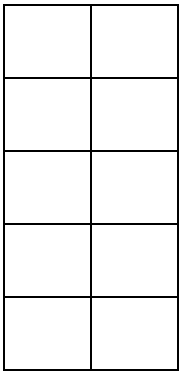

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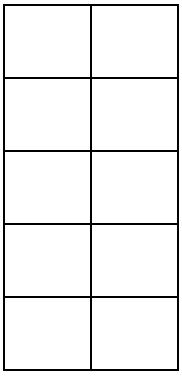

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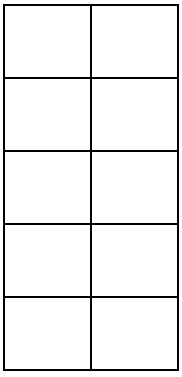


Activity 4: Card Deck to Help You Visualize Numbers in a Preferred Way

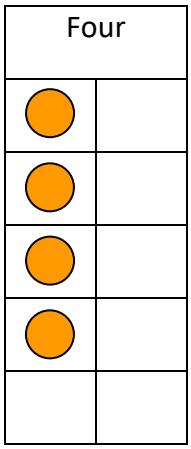
Materials:

- Card deck representing all the numbers from 1 to 10.
  Note: Students will be using this card deck later for addition.

If you have a partner that's great! If not, you can do this by yourself.

Put the card deck face down in front of you. Turn a card over and say what number is represented.

For example this is 4



Your partner can check your work.

Note: This deck will be used again for addition practice

Activity 5: How Do You Know?

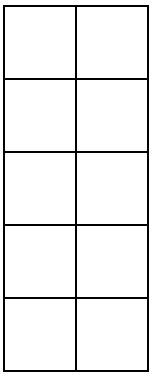
Materials

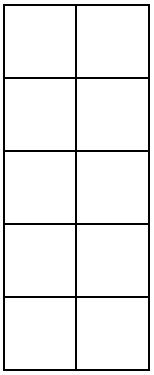
- The card deck

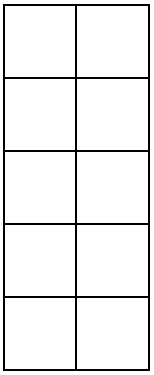
As you turn each card over, say its value then say how do you know. When you are done fill in this in:

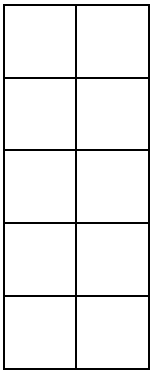
		How I know

How I know

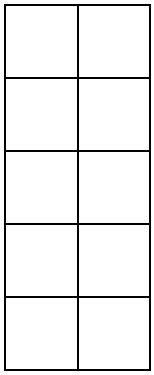


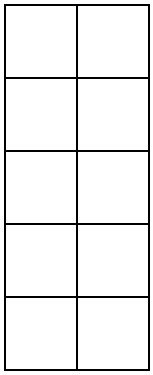


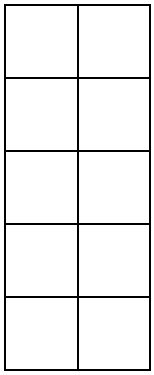


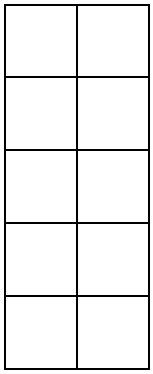


How I know









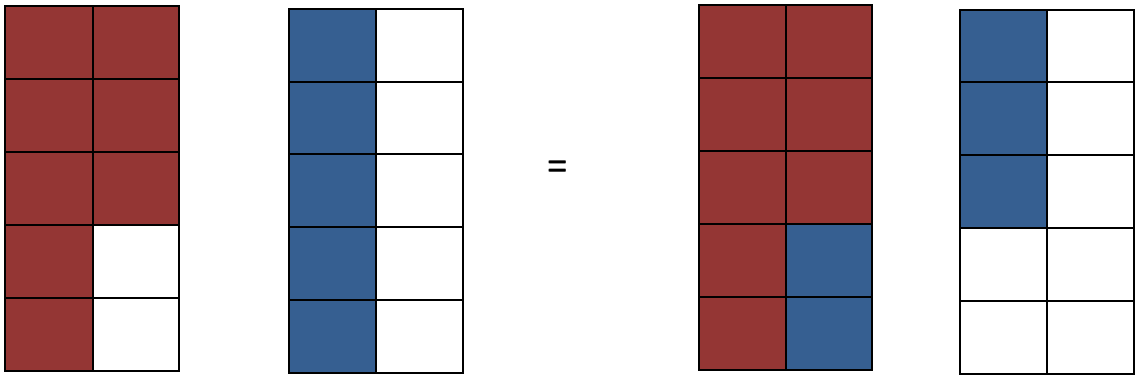
Addition with Ten Frames

The ten frames can be used to add more efficiently by using groups.

The way the Japanese method of adding works is:

- visualize the first addend on the ten-frame
- visualize the second addend on a different ten-frame
- slide the second addend when appropriate to find the sum

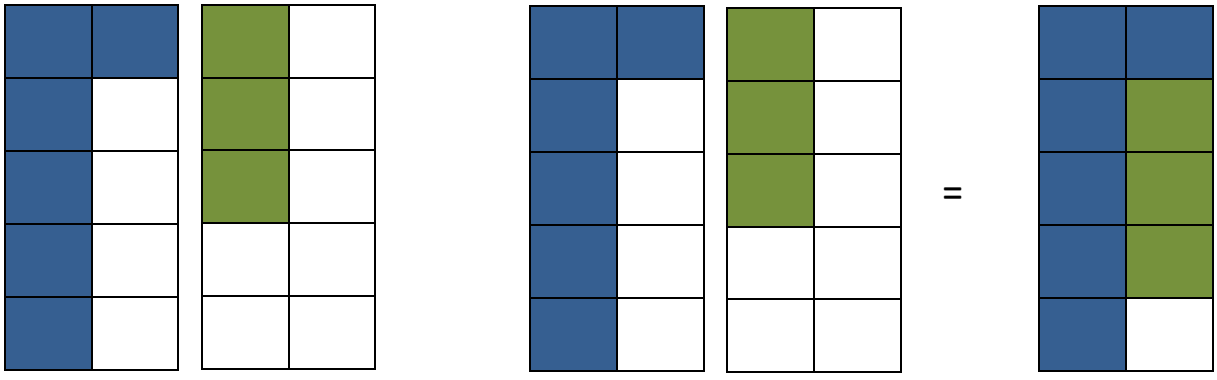
Here is another example, 8 + 5 = 13



8 plus 5 equals 13

Here is an example:

For 6 + 3, we can put the two together, and slide the three onto the six.

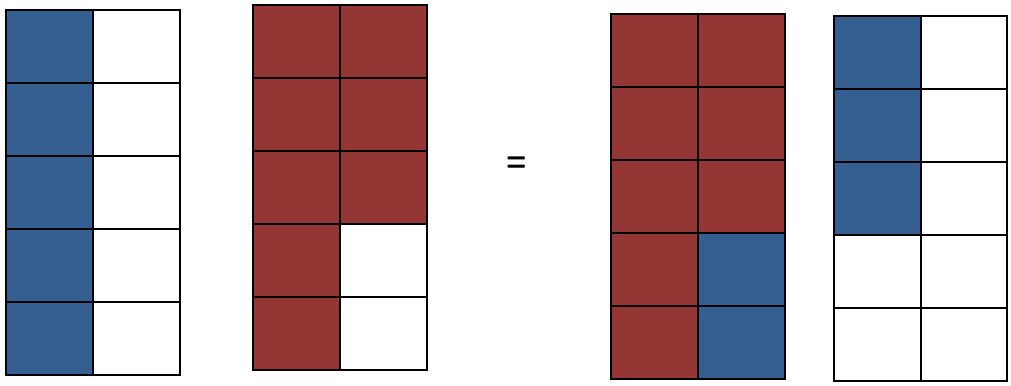


Here is 6 Here is 3 6 plus 3 equals 9

The Japanese Way to Add

- visualize the first addend on the ten-frame
- visualize the second addend on a different ten-frame
- slide the second addend when appropriate to find the sum

Here is another example, 8 + 5 = 13



8 plus 5 equals 13

3 steps: visualize 8, visualize 5, slide

Compare to a different way:

Start with 8. Then count 9, 10, 11, 12, 13

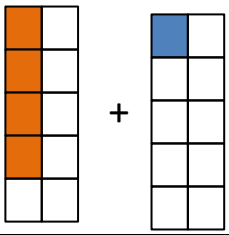
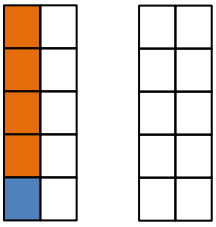
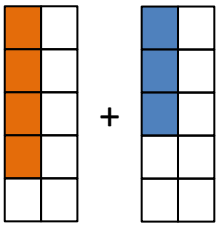
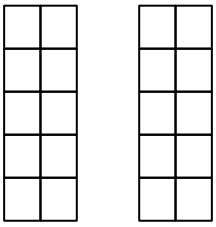
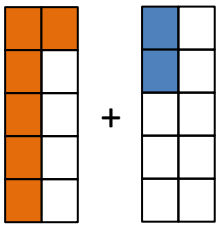
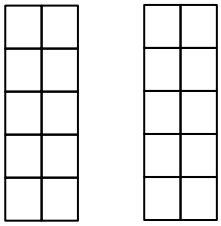
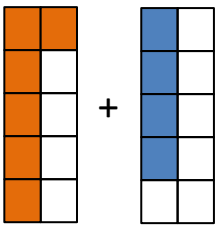
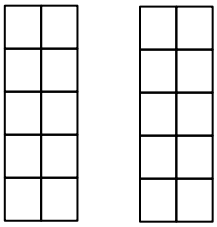
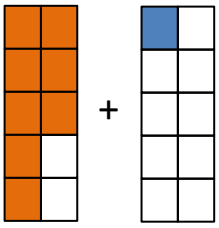
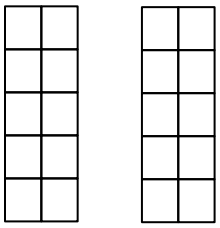
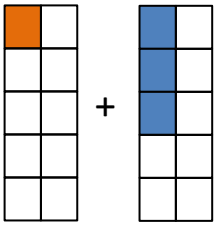
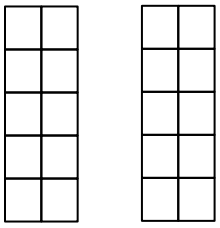
Adding 5

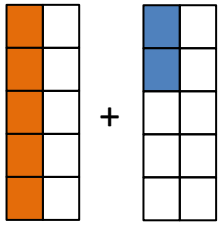
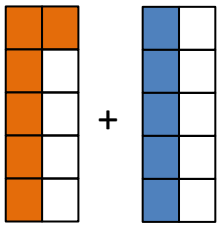
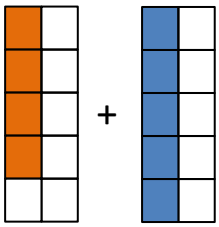
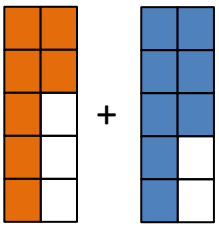
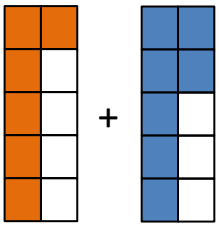
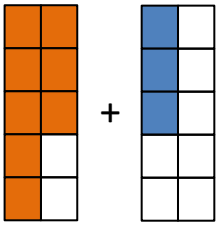
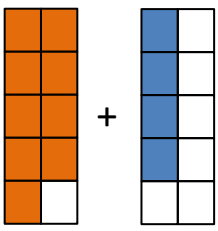
6 steps: start with 8, then 9, 10, 11, 12, 13

So the Japanese way is faster.

Practice 2: Adding with Color

Materials: You will need colored pencils

Drawing	Sum	Symbols
		4 + 1 = 5
		
		
		
		
		

Drawing	Sum	Symbols
		
		
		
		
		
		
		

Answers page 65

Activity 6: Modeling with Pennies

Use the pennies and ten-frames to model each addition problem. Then write the sum.

This activity is not about the answer; you can get the answer other ways. The purpose of this activity is for you to practice adding using ten-frames.

a. 3 + 4 =

b. 5 + 7 =

c. 6 + 3 =

d. 6 + 7 =

e. 8 + 2 =

f. 9 + 4 =

g. 9 + 6 =

h. 1 + 7 =

i. 2 + 4 =

j. 8 + 8 =

k. 7 + 8 =

l. 4 + 7 =

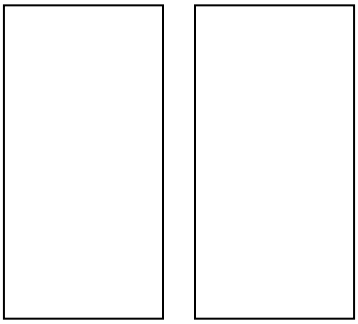
Activity 7: Adding the Japanese Way

Materials:

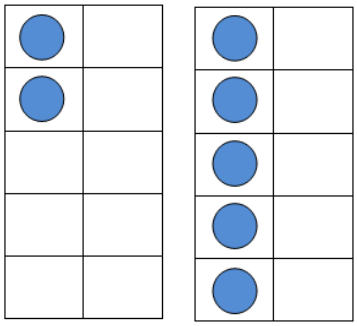
- Card Deck page pages 42 – 44

Cut out the card deck

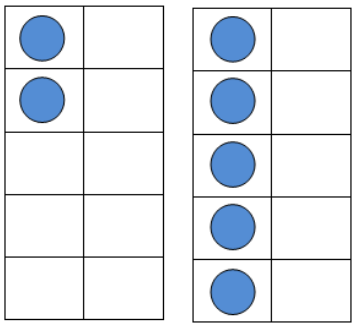
Put the cards into two piles (face down).



Turn the top card over from each pile.

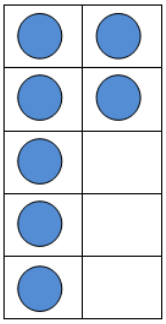


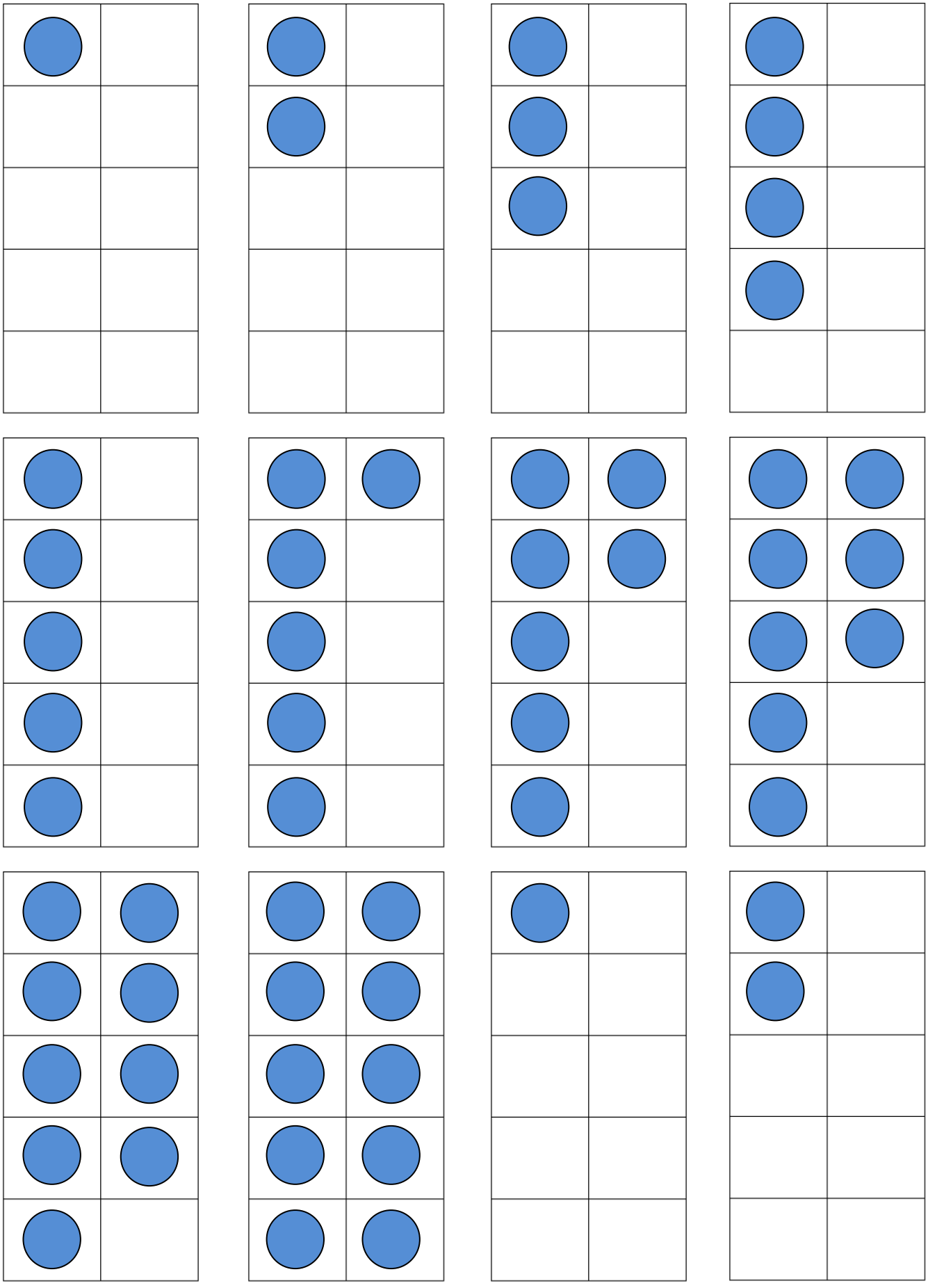
Add in your head by visualization.

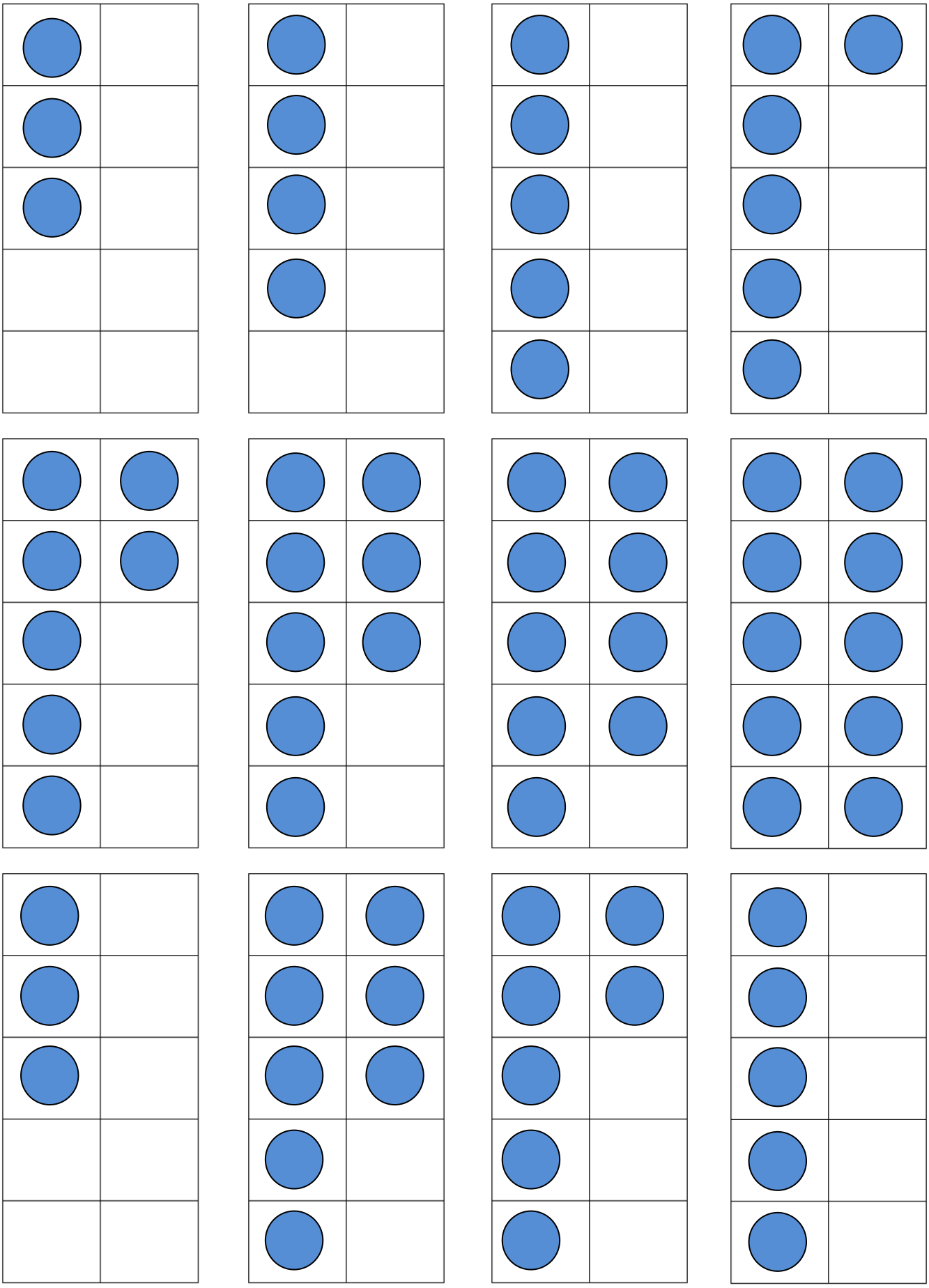


2 5

I can see

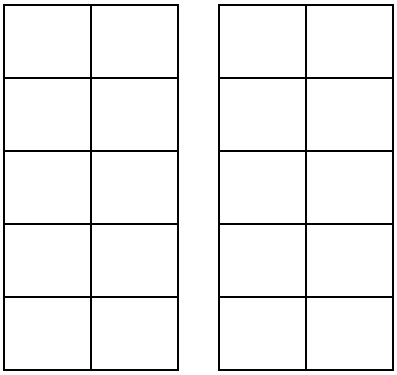
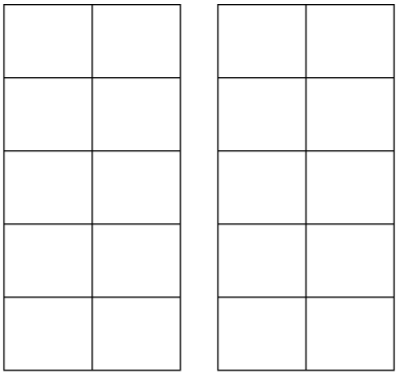
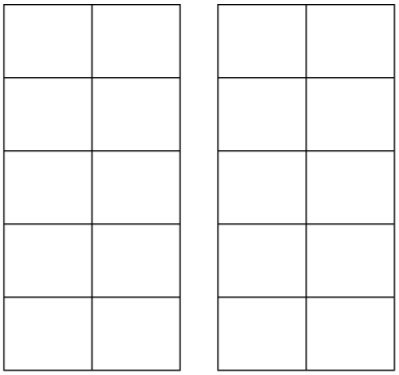
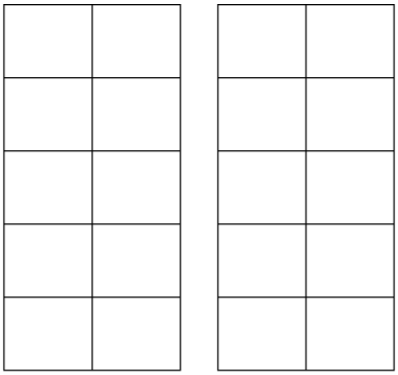
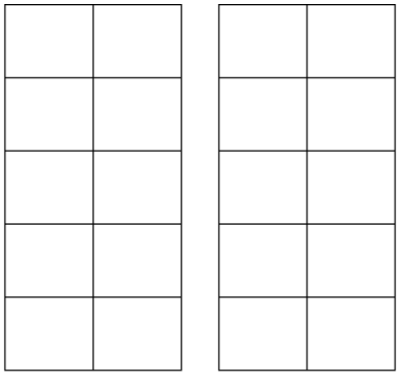
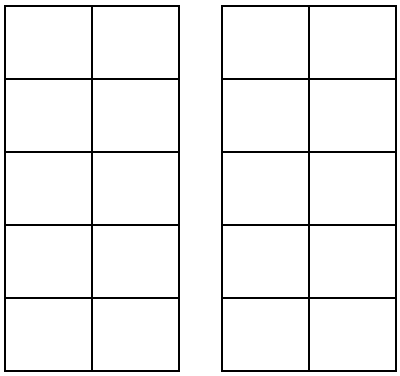






Practice 3: Ten Frame Practice

1. Shade each addend separately.
2. Visualize moving the pennies.
3. Give the sum.

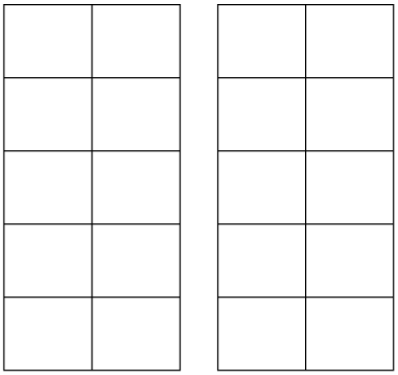
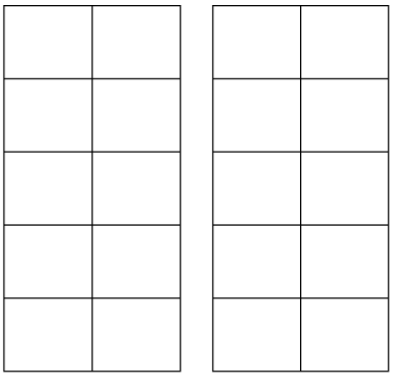
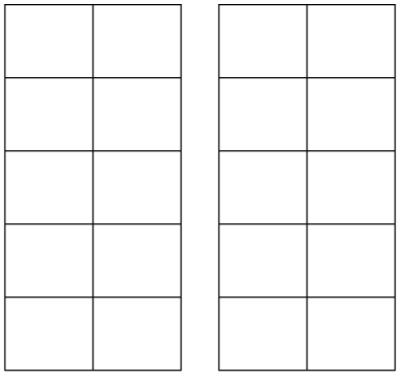
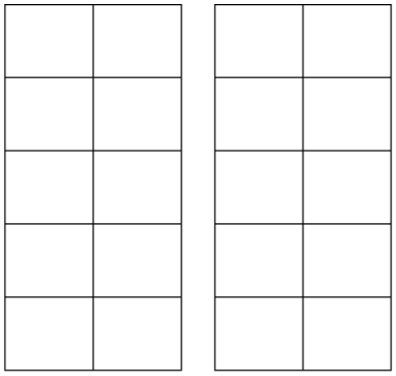
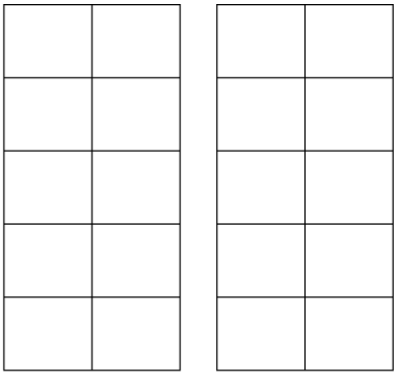
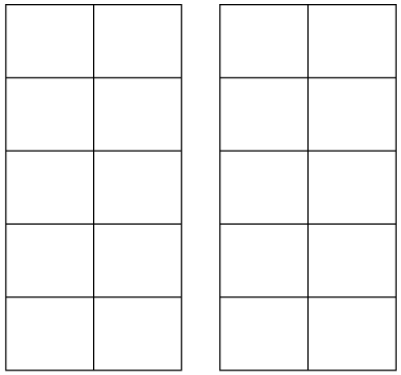
3 + 8 	8 + 5 
7 + 9 = ___ 	8 + 6 = ___ 
8 + 2 	4 + 7 = _____ 

Answers page 66

Practice 4: More Ten Frame

1. Imagine the pennies in the 10–Frames.

   Try not to draw them!

7 + 6 	3 + 9 
4 + 8 = ___ 	8 + 5 = ____ 
1 + 10 	3 + 7 = ___ 

Answers page 67

Practice 5: Circle Pairs

Circle pairs that make tens, then give the sum of all the numbers added together.

2 4 5 5 6 sum	3 2 2 8 1 sum	5 3 5 8 7 sum
7 4 6 6 3 sum	4 2 4 6 8 sum	5 7 3 5 7 sum
1 7 3 4 6 9 sum	5 3 5 6 7 3 sum	8 2 2 3 8 5 sum

Answers page 68

Practice 6: Equals Ten

Provide the missing addend.

7 + _____ = 10	2 + _____ = 10	_____ + 4 = 10
_____ + 6 = 10	_____ + 9 = 10	5 + _____ = 10

Circle the <u>two</u> numbers that will make 10. Write the sum of all numbers in the box.

4 6 3 sum ☐	9 4 1 sum ☐	8 2 3 sum ☐	2 6 4 sum ☐
3 8 7 sum ☐	4 3 6 sum ☐	10 6 0 sum ☐	7 4 3 sum ☐
6 4 2 sum ☐	1 8 9 sum ☐	3 4 6 sum ☐	5 5 6 sum ☐

Answers page 69

Practice 7: Grouping

Add by grouping for tens, then write the sum.

2 + 2 + 7 + 8 + 1 = _____	6 + 4 + 3 + 3 + 4 = _____
1 + 2 + 7 + 6 + 5 = _____	5 + 3 + 1 + 9 + 2 = _____
7 + 4 + 2 + 1 + 5 + 1 = _____	5 + 5 + 10 + 1 + 7 + 2 = _____
9 + 4 + 1 + 5 + 1 + 4 = _____	7 + 7 + 3 + 6 + 3 = _____
6 + 1 + 3 + 1 + 8 + 2 = _____	3 + 9 + 2 + 5 + 1 + 7 = _____
6 + 2 + 3 + 5 + 4 = _____	6 + 3 + 7 + 4 + 9 = _____

Answers page 70

Practice 8: Additional Practice

Addition Practice

a. 8 + 3 =	3 + 8 =
b. 2 + 5 =	5 + 2 =
c. 1 + 9 =	9 + 1 =
d. 7 + 6 =	6 + 7 =
e. 3 + 9 =	9 + 3 =
f. 8 + 7 =	7 + 8 =
g. 6 + 5 =	5 + 6 =
h. 9 + 6 =	6 + 9 =
i. 4 + 8 =	8 + 4 =
j. 8 + 9 =	9 + 8 =
k. 2 + 8 =	8 + 2 =
l. 9 + 2 =	2 + 9 =
m. 7 + 5 =	5 + 7 =
n. 9 + 7 =	7 + 9 =
o. 3 + 7 =	7 + 3 =

Which is the easier problem to solve in each pair? Explain.

Do you think that a + b = b + a ? Explain your reasoning.

Answers page 71

Activity 8: Modeling the Addition Algorithm

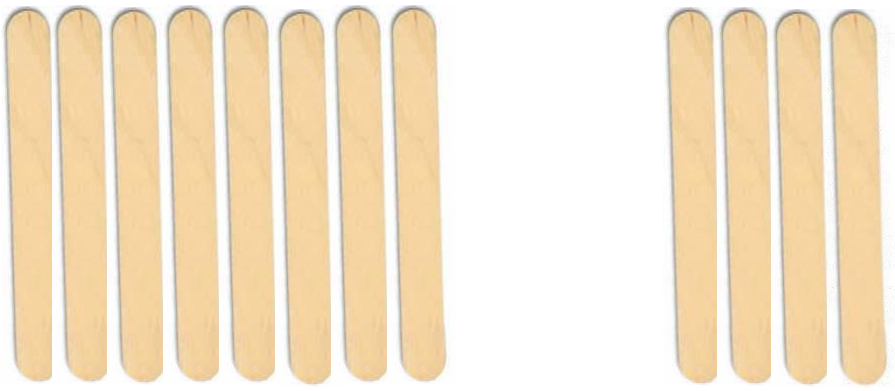
Materials:

- Popsicle sticks, about 50
- Rubber bands, about 7
- 8 ½ by 11 sheet of paper, divided like this:

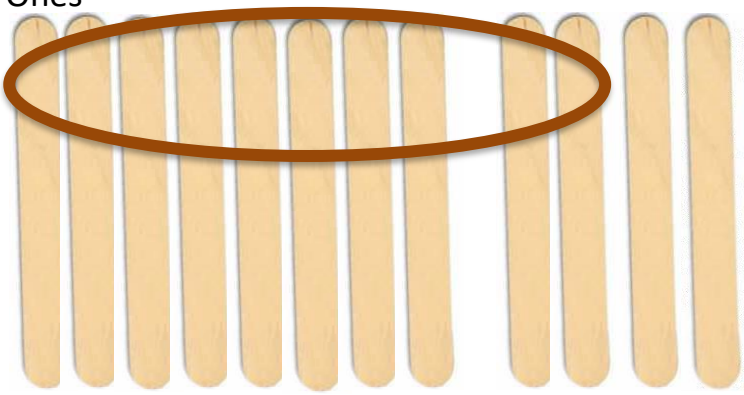
Place value mat

Tens	Ones

This is how the we can add 8 + 4 using popsicle sticks and a place value mat.

Tens	Ones
	

Then take ten of the popsicle sticks and put a rubber band around them:

Tens	Ones
	

The rubber-banded ten popsicle sticks can be moved to the "tens" side of the page, to show the answer of 12.

Tens	Ones
	

This corresponds exactly to the traditional American addition algorithm:

$$\begin{array}{r} 8 \\ +\ 4 \\ \hline \end{array} \qquad \begin{array}{r} {}^{1}8 \\ +\ 4 \\ \hline 12 \end{array}$$

where the 1 superscript corresponds to the "ten."

There are other algorithms.

Activity 9: Modeling the Addition Algorithm with Popsicle Sticks

Materials:

- Place value mat
- About 35 popsicle sticks
- Several rubber bands
- Copy of the addition problems to model and solve that follows

You will need a place value mat like this: (you can make your own with a piece of paper)

Place value mat

Tens	Ones

Model each addition problem with your popsicle sticks. Then find the sum.

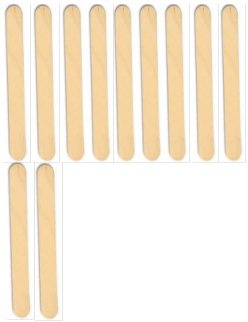
a. 12 + 13 =

b. 8 + 12 =

c. 19 + 4 =

d. 19 + 14 =

e. 27 + 16 =

Tens	Ones
	

=

Tens	Ones
	

$$\begin{array}{r} 9 \\ +2 \\ \hline 11 \end{array}$$

9 + 2 = 11

Tens	Ones

=

Tens	Ones

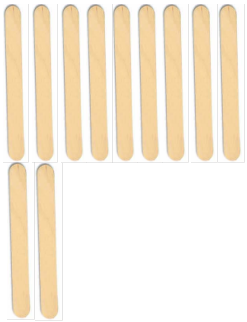
9 + 6 =

Tens	Ones

=

Tens	Ones

12 + 8 =

Tens	Ones
	

=

Tens	Ones

12 + 9 =

Tens	Ones

=

Tens	Ones

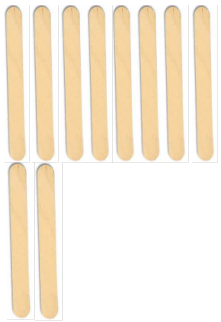
14 + 7 =

Tens	Ones

=

Tens	Ones

38 + 17 =

Tens	Ones
	

=

Tens	Ones

22 + 8 =

Tens	Ones

=

Tens	Ones

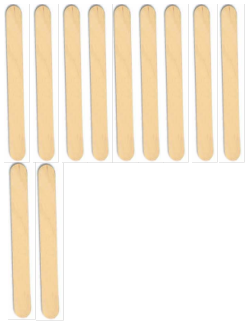
15 + 18 =

Tens	Ones

=

Tens	Ones

12 + 8 =

Tens	Ones
	

=

Tens	Ones

29 + 2 =

Tens	Ones

=

Tens	Ones

41 + 18 =

Tens	Ones

=

Tens	Ones

18 + 16 =

Practice 9: More Addition

25 + 18 =	34 + 17 =
36 + 5 =	29 + 14 =
18 + 20 =	19 + 10 =
11 + 34 =	45 + 9 =
32 + 15 =	23 + 51 =
43 + 8 =	28 + 17 =
28 + 16 =	24 + 13 =

Answers page 72

26 + 7 =	14 + 23 =
17 + 10 =	9 + 15 =
31 + 4 =	17 + 8 =
32 + 8 =	13 + 22 =
53 + 12 =	18 + 17 =

Answers page 73

Practice 10: Enrichment

You may be able to do some of the problems but not others. If you want, you can save this paper in your notebook, and come back to it later.

Words	With common referent	Total
1 tenth + 3 tenths =		
1 tenth + 3 hundredths =		
1 whole + 3 wholes =		
1 whole + 3 tenths =		
1 fifth + 3 fifths =		
1 fifth + 3 fourths =		
1 foot + 3 feet =		
1 foot + 3 miles =		
1 x + 3x =		
3x + 5x + 2 + 6 =		

Optional Practice Enrichment

How many addition signs should be put between digits of the number 987654321 and where should we put them to get a total of 99?

Answers page 75

Solutions / Answers

MATH WHISPERER

Where math makes sense

Practice 1: What is addition really?

Write the results for the following additions.

1 apple + 2 apples totals 3 apples

(added to)

1 apple + 2 oranges totals 3 pieces of fruit

1 grapefruit + 2 oranges totals 3 pieces of fruit

1 apple + 2 iPods totals doesn't make sense

1 apple + 2 seconds totals doesn't make sense

Which of the results above can give 3 of something that makes sense?

Why? because the label for them is either the same or can be changed to something that is the same

Explain how you would show that 1 apple + 2 apples totals 3 apples.

What is the same about these two problems:

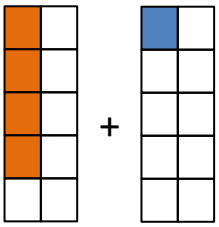
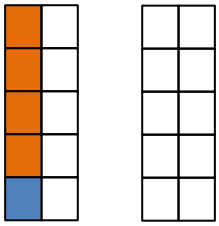
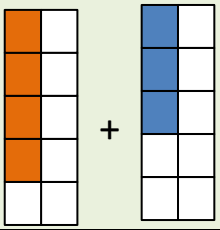
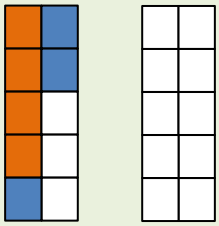
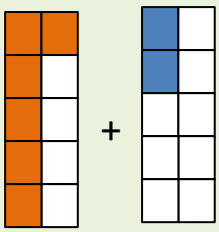
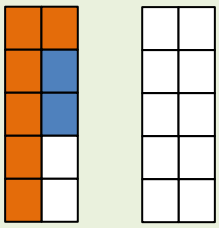
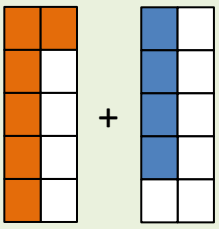
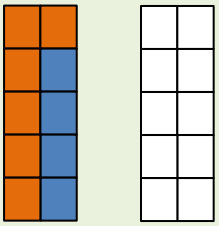
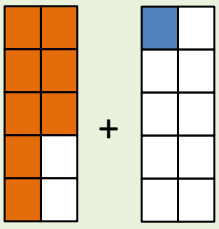
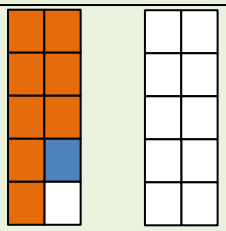
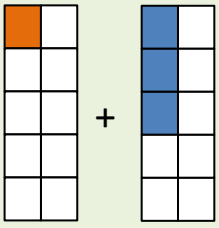
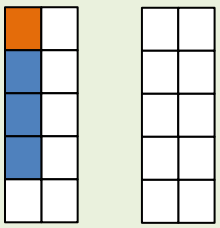
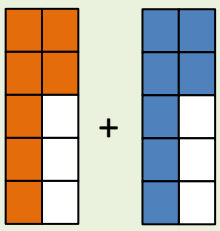
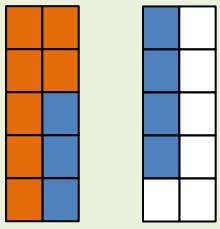
1 apple + 2 apples = ?

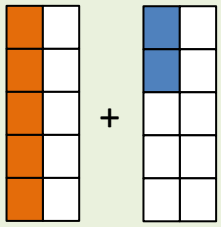
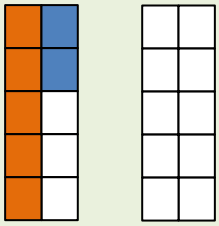
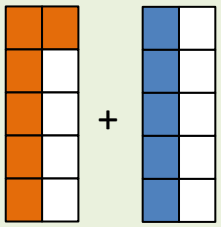
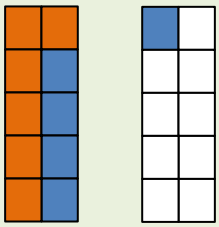
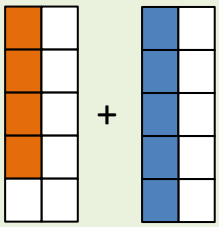
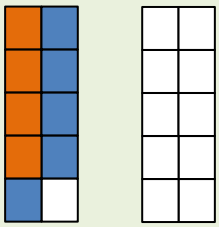
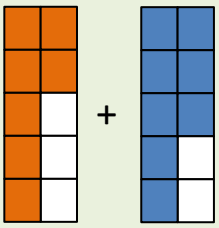
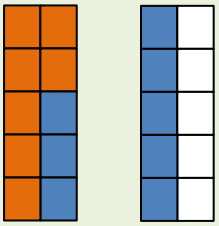
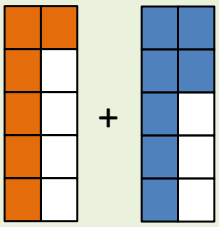
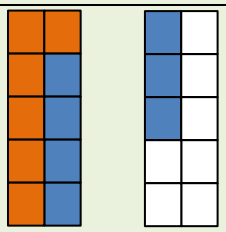
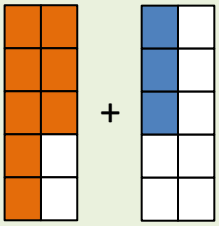
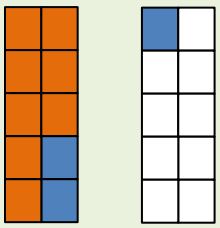
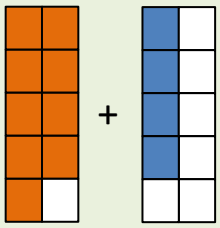
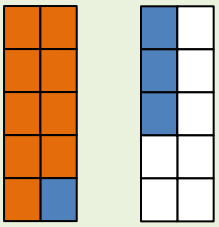
1 apple + 1 banana = ?

What is different?

When adding apples plus apples, the total is apples.
When adding an apple and a banana, the total has to include both of them, so the result is two pieces of fruit.

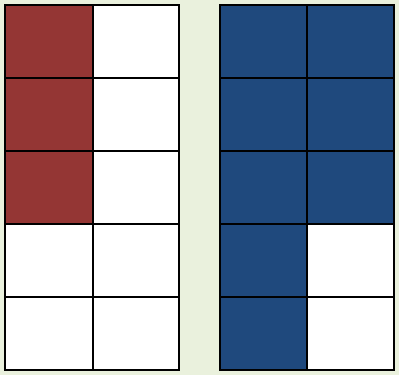
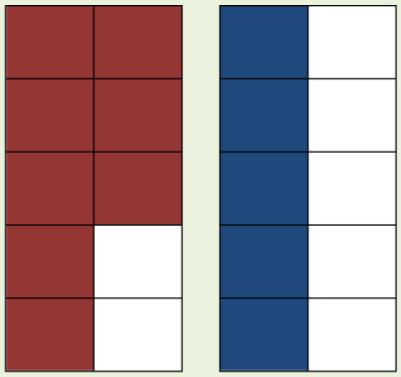
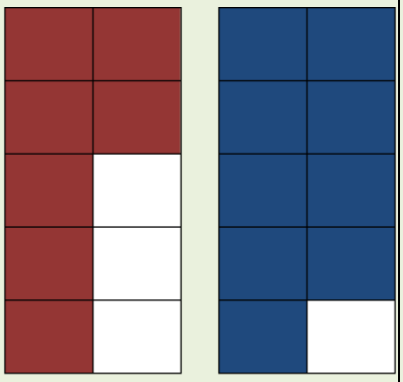
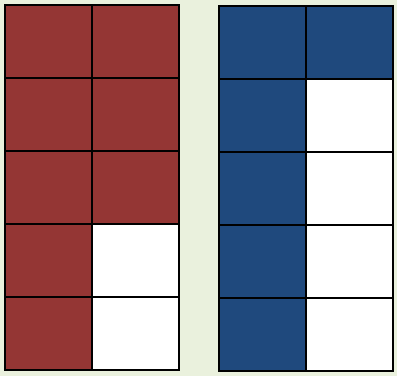
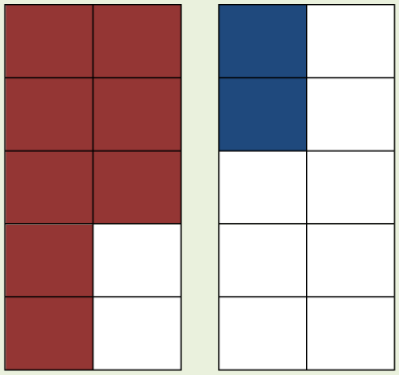
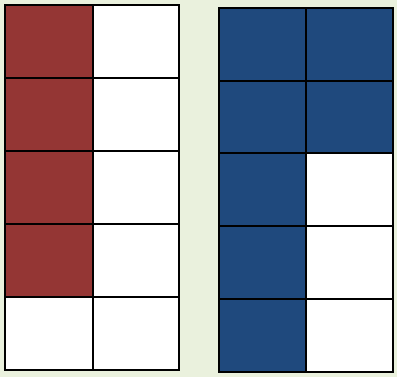
Practice 2: Adding with Color

Drawing	Sum	Symbols
		4 + 1 = 5
		4 + 3 = 7
		6 + 2 = 8
		6 + 4 = 10
		8 + 1 = 9
		1 + 3 = 4
		7 + 7 = 14

Drawing	Sum	Symbols
		5 + 2 = 7
		6 + 5 = 11
		4 + 5 = 9
		7 + 8 = 15
		6 + 7 = 13
		8 + 3 = 11
		9 + 4 = 13

Practice 3: Ten Frame Practice

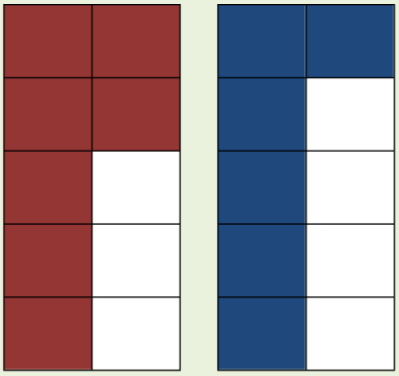
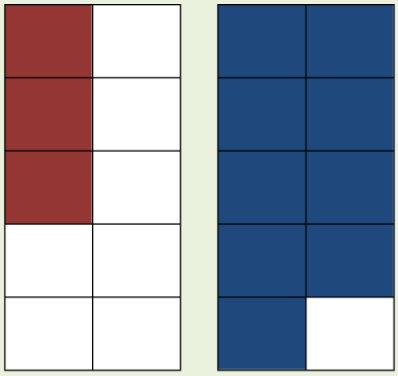
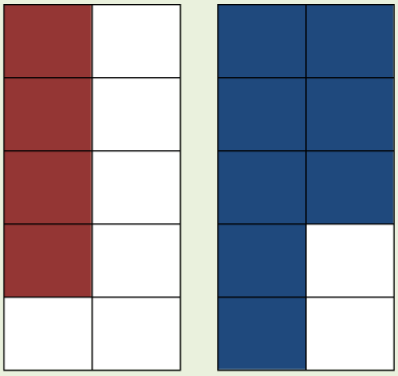
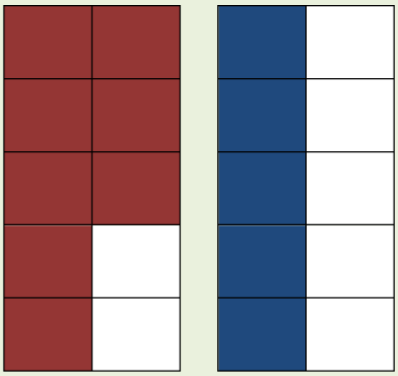
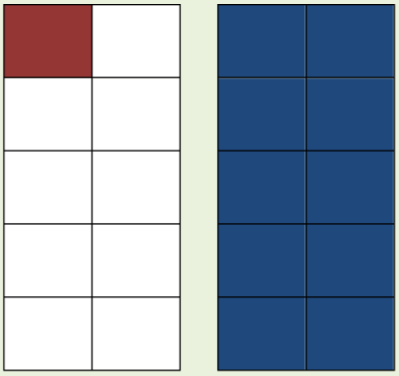
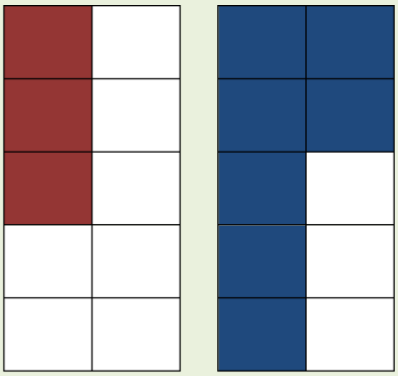
1. Shade each addend separately.
2. Visualize moving the pennies.
3. Give the sum.

$\begin{array}{r} 3 \\ +\ 8 \\ \hline 11 \end{array}$ 	$\begin{array}{r} 8 \\ +\ 5 \\ \hline 13 \end{array}$ 
$7 + 9 = 16$ 	$8 + 6 = 14$ 
$\begin{array}{r} 8 \\ +\ 2 \\ \hline 10 \end{array}$ 	$4 + 7 = 11$ 

Practice 4: More Ten Frame

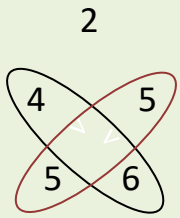
1. Imagine the pennies in the 10–Frames.

   Try not to draw them!

$\begin{array}{r} 7 \\ +\ 6 \\ \hline 13 \end{array}$ 	$\begin{array}{r} 3 \\ +\ 9 \\ \hline 12 \end{array}$ 
$4 + 8 = 12$ 	$8 + 5 = 13$ 
$\begin{array}{r} 1 \\ +\ 10 \\ \hline 11 \end{array}$ 	$3 + 7 = 10$ 

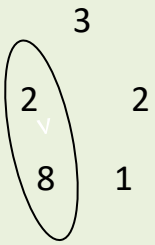
Practice 5: Circle Pairs

Circle pairs that make tens, then give the sum of all the numbers added together.



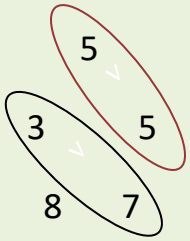
sum

22



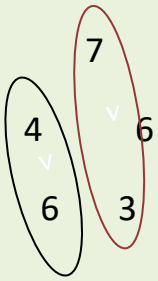
sum

16



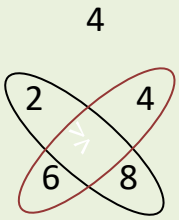
sum

28



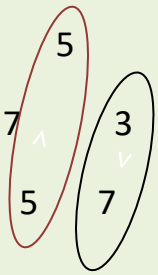
sum

26



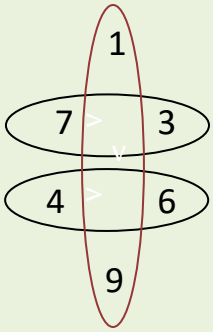
sum

24



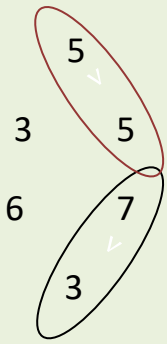
sum

27



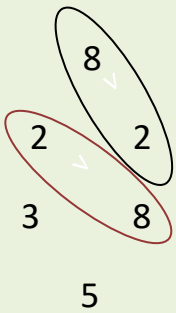
sum

30



sum

29



sum

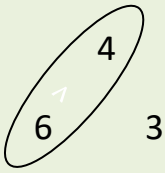
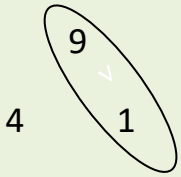
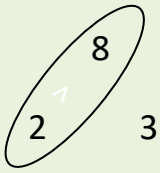
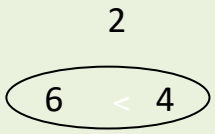
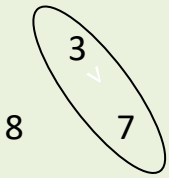
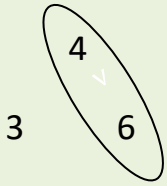
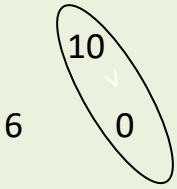
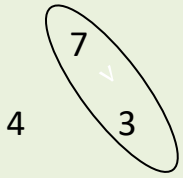
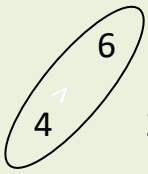
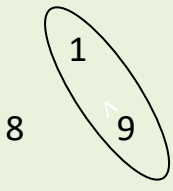
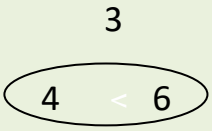
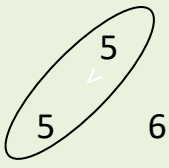
28

Practice 6: Equals Ten

Provide the missing addend.

7 + ___3___ = 10	2 + ___8___ = 10	___6___ + 4 = 10
___4___ + 6 = 10	___1___ + 9 = 10	5 + ___5___ = 10

Circle the <u>two</u> numbers that will make 10. Write the sum of all numbers in the box.

 sum 13	 sum 14	 sum 13	 sum 12
 sum 18	 sum 13	 sum 16	 sum 14
 sum 12	 sum 18	 sum 13	 sum 16

Practice 7: Grouping

Add by grouping for tens, then write the sum.

2 + 2 + 7 + 8 + 1 = __20____	6 + 4 + 3 + 3 + 4 = ___20___
1 + 2 + 7 + 6 + 5 = ___21___	5 + 3 + 1 + 9 + 2 = ___20___
7 + 4 + 2 + 1 + 5 + 1 = ___20___	5 + 5 + 10 + 1 + 7 + 2 = __30____
9 + 4 + 1 + 5 + 1 + 4 = __24____	7 + 7 + 3 + 6 + 3 = ___26___
6 + 1 + 3 + 1 + 8 + 2 = __21____	3 + 9 + 2 + 5 + 1 + 7 = __27____
6 + 2 + 3 + 5 + 4 = __20____	6 + 3 + 7 + 4 + 9 = __29____

Practice 8: Additional Practice

Addition Practice

a. 8 + 3 = 11	3 + 8 = 11
b. 2 + 5 = 7	5 + 2 = 7
c. 1 + 9 = 10	9 + 1 = 10
d. 7 + 6 = 13	6 + 7 = 13
e. 3 + 9 = 12	9 + 3 = 12
f. 8 + 7 = 15	7 + 8 = 15
g. 6 + 5 = 11	5 + 6 = 11
h. 9 + 6 = 15	6 + 9 = 15
i. 4 + 8 = 12	8 + 4 = 12
j. 8 + 9 = 17	9 + 8 = 17
k. 2 + 8 = 10	8 + 2 = 10
l. 9 + 2 = 11	2 + 9 = 11
m. 7 + 5 = 12	5 + 7 = 12
n. 9 + 7 = 16	7 + 9 = 16
o. 3 + 7 = 10	7 + 3 = 10

Which is the easier problem to solve in each pair? Explain.

For which is easier: This is your personal answer. My answer (which may not be yours) is that it's easier for me to start with the bigger number.

Do you think that a + b = b + a ? Explain your reasoning.

Practice 9: More Addition

25 + 18 = 43	34 + 17 = 51
36 + 5 = 41	29 + 14 = 43
18 + 20 = 38	19 + 10 = 29
11 + 34 = 45	45 + 9 = 54
32 + 15 = 47	23 + 51 = 74
43 + 8 = 51	28 + 17 = 45
28 + 16 = 44	24 + 13 = 37

26 + 7 = 33	14 + 23 = 37
17 + 10 = 27	9 + 15 = 24
31 + 4 = 35	17 + 8 = 25
32 + 8 = 40	13 + 22 = 35
53 + 12 = 65	18 + 17 = 35

Practice 10: Enrichment

You may be able to do some of the problems but not others. If you want, you can save this paper in your notebook, and come back to it later.

Words	With common referent	Total
1 tenth + 3 tenths =	1 tenth + 3 tenths	4 tenths
1 tenth + 3 hundredths =	10 hundredths + 3 hundredths	4 hundredths
1 whole + 3 wholes =	1 whole + 3 wholes	4 wholes
1 whole + 3 tenths =	1 tenth + 3 tenths	4 tenths
1 fifth + 3 fifths =	1 fifth + 3 fifths	4 fifths
1 fifth + 3 fourths =	twentieths is the common referent, 4 twentieths + 15 twentieths	19 twentieths
1 foot + 3 feet =	1 foot + 3 feet	4 feet
1 foot + 3 miles =	1 foot + 3 miles = 1 foot + 15840 feet	15841 feet (feet is the common referent)
1 x + 3x =	1x + 3x	4x (x is the common referent)
3x + 5x + 2 + 6 =	common referents are x's and 1's	8x + 8 is the total

Optional Practice Enrichment

How many addition signs should be put between digits of the number 987654321 and where should we put them to get a total of 99?

9 + 8 + 7 + 6 + 5 + 43 + 21

Help for Helpers

I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as "I know you can do this. I believe in you." And then leave the student alone to do the work.

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn't going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don't generally want to ask our boss or friend: "Am I right? Am I right?" As an adult, we have to know the answer is right ourselves. The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student's life. You will gain confidence in your students as you watch them be successful on their own.

The goals for this lesson collection is for students to:

1. Understand the operation of addition.
2. Be able to add single digits quickly and reliably.
3. Be able to use an algorithm for double digit addition.