MATH WHISPERER

Where math makes sense

Research Edition

The Operations Series

Division

Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information, please go to www.mathwhisperer.com

ISBN-13: 978-1543223132
ISBN-10: 1543223133

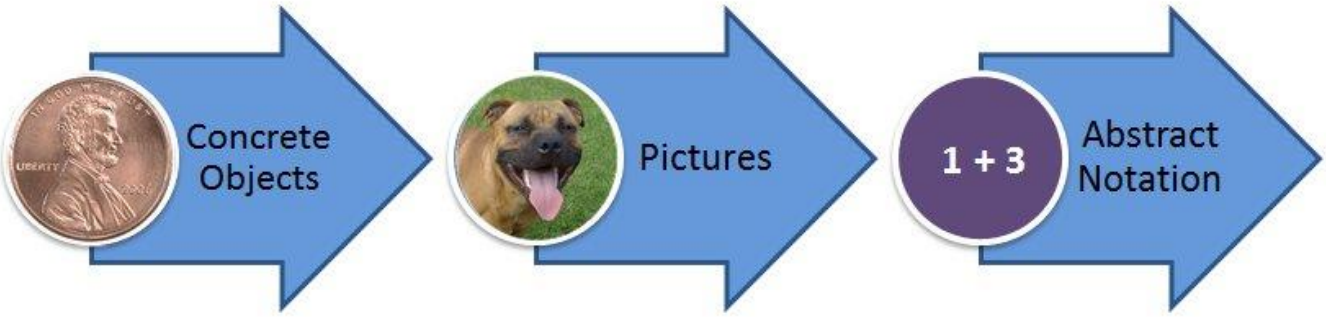
DEDICATION

Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

Contents

1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is actually supposed to make sense. When you start with hands-on objects, math can make sense.



You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three-step progression works for everybody.

It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three-step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. Won't it feel great to never have to learn this again? The math will stick with you with the three-step progression.

Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you!

What is Division?

Division is a way to split up objects into groups. The groups are of equal size.

Most of math until calculus is about the four operations: addition, subtraction, multiplication and division. You use these operations on different types of numbers: Whole numbers like 1, 2, 3, fractions, decimals, and negative numbers. Here is division!

This lesson collection is about the concept of division mainly, and how to divide whole numbers.

Activity 1: Division with popsicle sticks

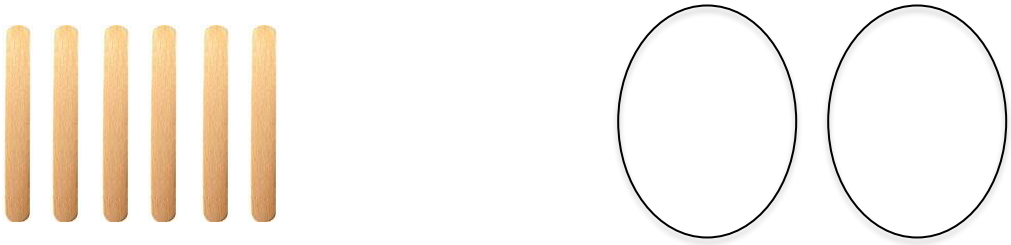
This is one way to solve the problem 6 divided by 2 – this means 6 objects divided into 2 piles.

You will need:

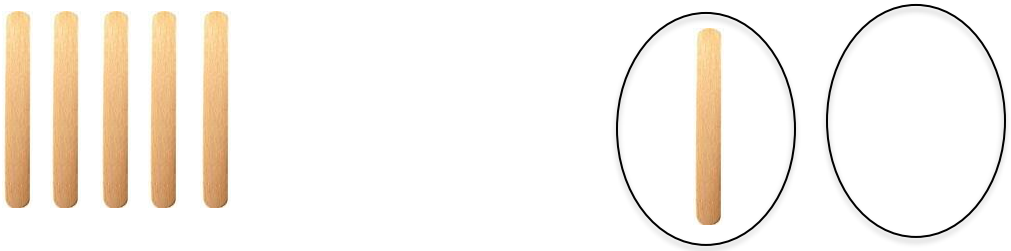
- 20 popsicle sticks

Note: the oval represents a "pile"

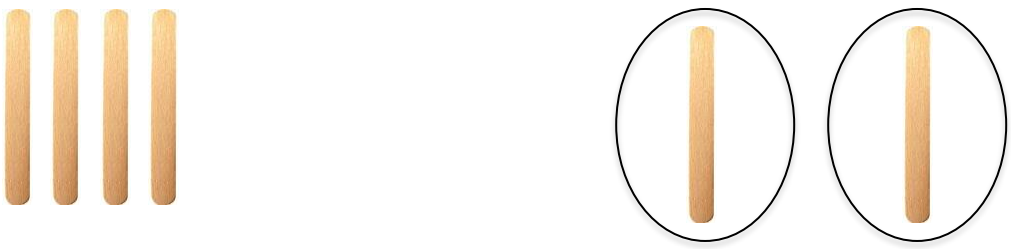
Step 1: 6 popsicle sticks, 2 "piles" shown



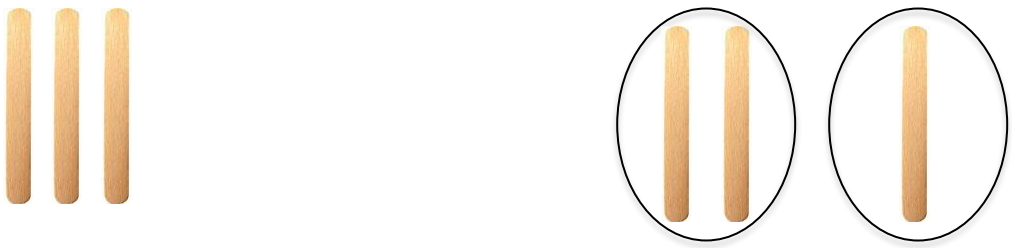
Step 2. Put the first popsicle stick into one pile



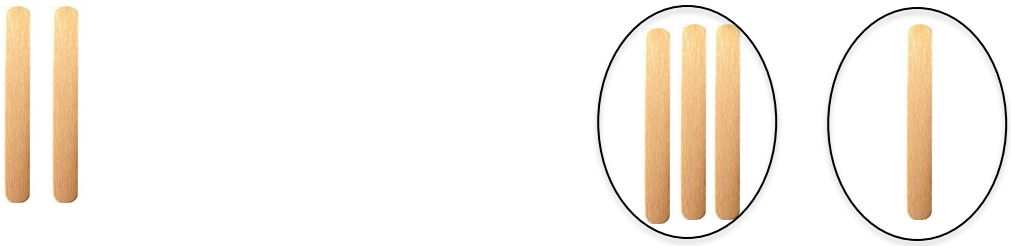
Step 3. Put the second popsicle stick into the second pile



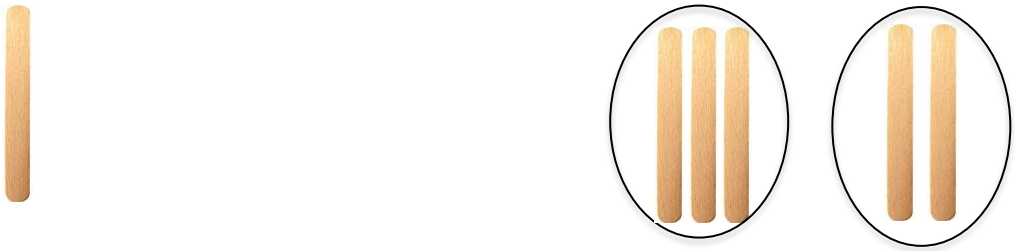
Step 4. Put the third popsicle stick into the first pile



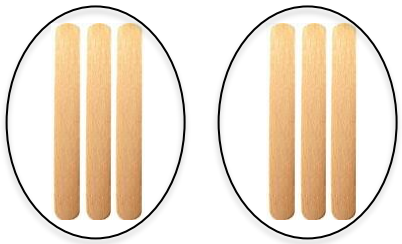
Step 5. Put the fourth popsicle stick into the second pile



Step 6. Put the fifth popsicle stick into the first pile



Step 7. Put the sixth popsicle stick into the second pile



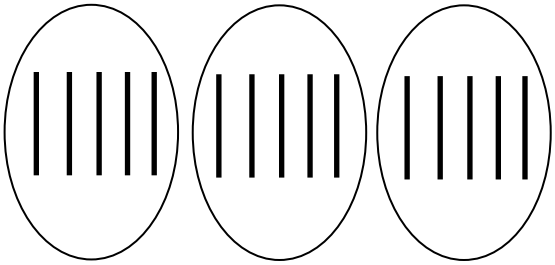
Now all six popsicle sticks are divided equally into two piles. There are three popsicle sticks in each pile. This shows 6 divided by 2, $6 \div 2$.

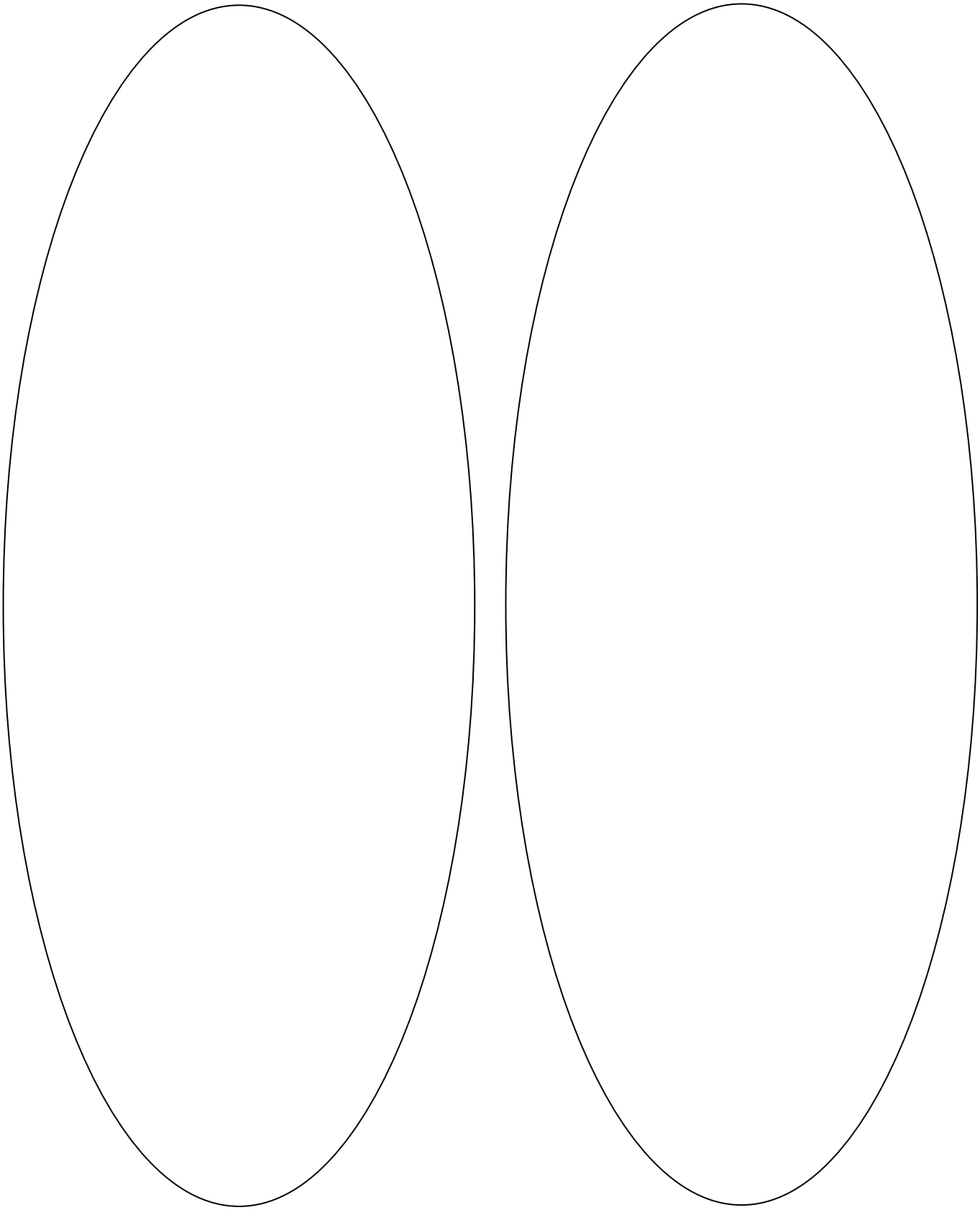
Use popsicle sticks to find the answer to each division computation. Write the answer.

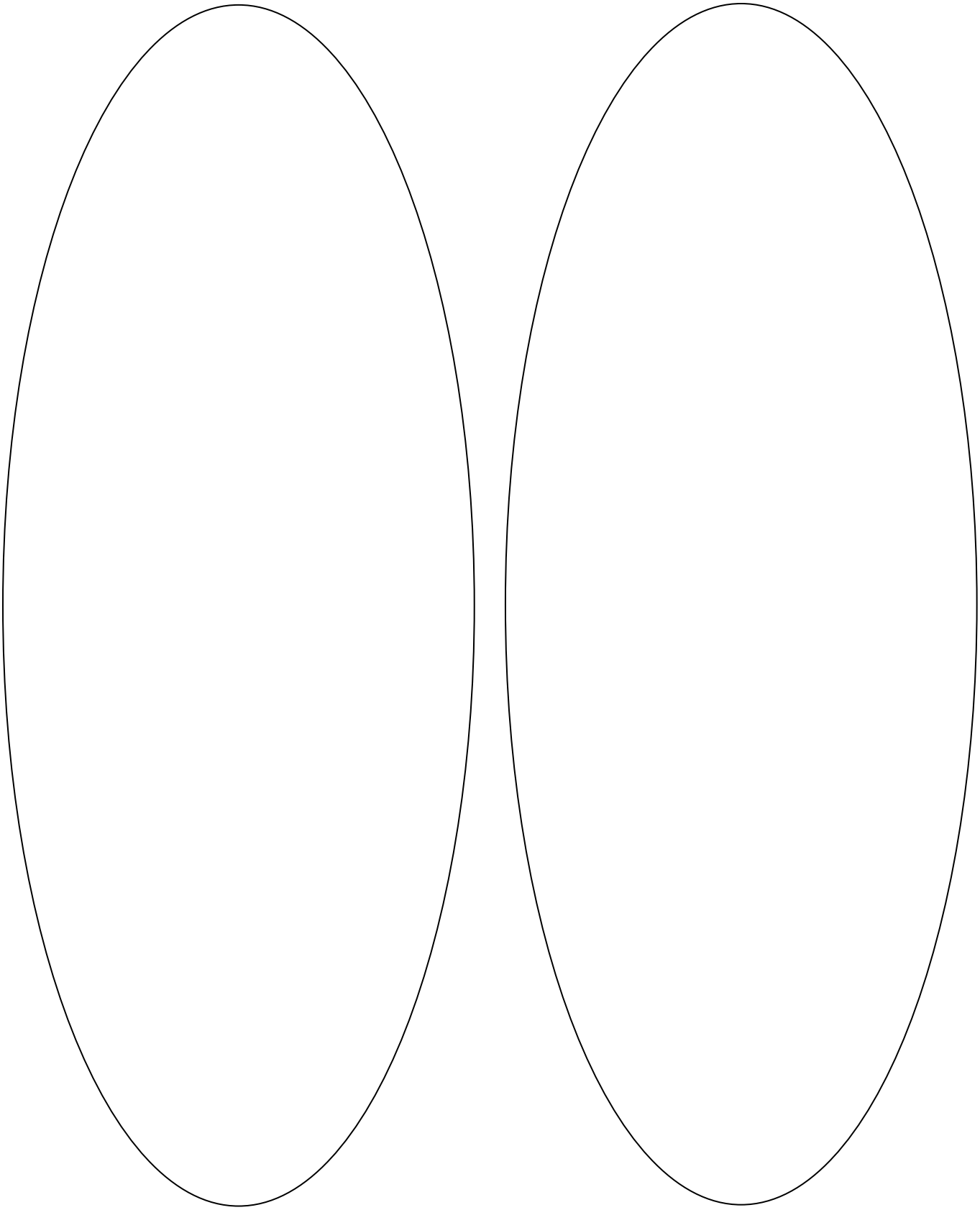
15 ÷ 3 =
12 ÷ 4 =
12 ÷ 3 =
8 ÷ 2 =
15 ÷ 3 =

Activity 2: More Division with popsicle sticks

Use popsicle sticks to find the answer to each division computation and draw the picture. Write the answer. You can use the piles on page

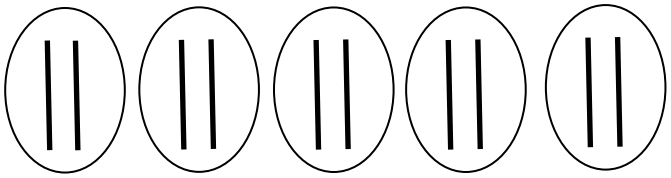
15 ÷ 3 = 	15 ÷ 3 = 5 There are 5 popsicle sticks in each pile.
12 ÷ 4 =	
12 ÷ 3 =	
8 ÷ 2 =	
20 ÷ 4	





Practice 1: Division with popsicle sticks

Draw the popsicle sticks, find the answer to the division expression, and write the words that go with the division equation. Remember to read left to right.

10 ÷ 5 = 2 10 popsicle sticks divided into 5 piles equals 2. 
12 ÷ 6 =
14 ÷ 7 =
15 ÷ 5 =
18 ÷ 3 =
6 ÷ 1 =
10 ÷ 2 =
21 ÷ 7 =
15 ÷ 3 =

Answers page 50

Another way to look at Division – related to Multiplication

Division is related to multiplication the way subtraction is related to addition. There are addition and subtraction number families, such as:

Addition and subtraction number family example:

$4 + 5 = 9$ $\quad$ $5 + 4 = 9$

$9 - 4 = 5$ $\quad$ $9 - 5 = 4$

And there are multiplication and division number families such as:

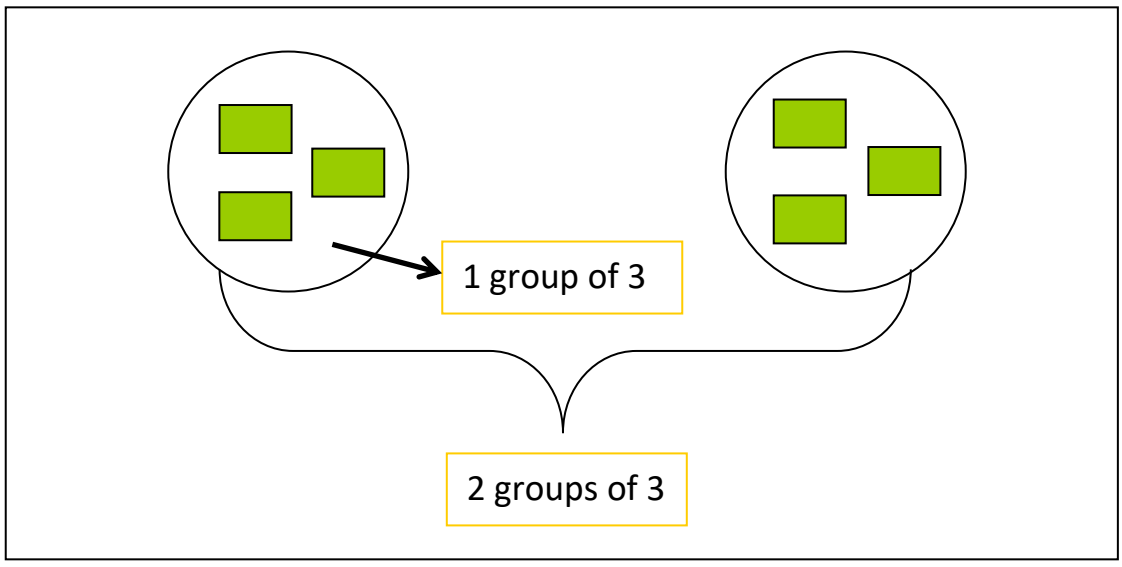
$2 * 3 = 6$ $\quad$ $3 * 2 = 6$

$6 \div 2 = 3$ $\quad$ $6 \div 3 = 2$

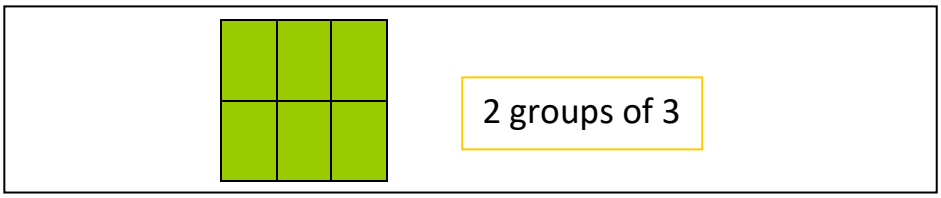
Here is an explanation:

Multiplication means "groups of."

So, $2 * 3$ means 2 groups of 3.



Arranged in an array, or grid, this is:



And, the multiplication and division family that goes with these two representations are:

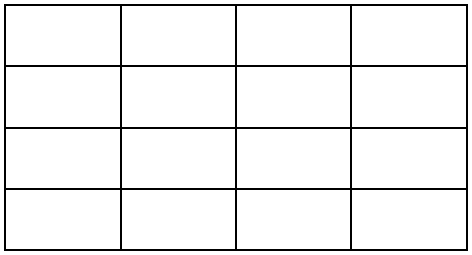
$2 * 3 = 6$

$3 * 2 = 6$

$6 \div 2 = 3$

$6 \div 3 = 2$

Practice 2: Multiplication and Division number families

Multiplication or division fact	Grid	Other members of the fact family
4 * 5 = 20		5 * 4 = 20 20 ÷ 4 = 5 20 ÷ 5 = 4
3 * 6 = 18		
1 * 9 = 9		
16 ÷ 2 = 8		
16 ÷ 4 = 2		

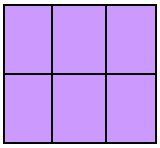
Answers page 51

Multiplication or division fact	Grid	Other members of the fact family
7 * 3 = 21		
18 ÷ 2 = 9		
25 ÷ 5 = 5		
24 ÷ 6 = 4		
27 ÷ 3 = 9		

Answers page 52

Practice 3: Multiplication and Division are a number family

These are the multiplication and division number family statement for this grid.



$2 * 3 = 6$ $3 * 2 = 6$ $6 \div 2 = 3$ $6 \div 3 = 2$

Write the other 3 members of each multiplication/division number family.

$4 * 5 = 20$
$8 \div 4 = 2$
$3 * 7 = 21$
$2 \overline{)12} = 6$
$56 \div 8 = 7$
$a \div b = c$

Answers page 53

Practice 4: Word problems using Division

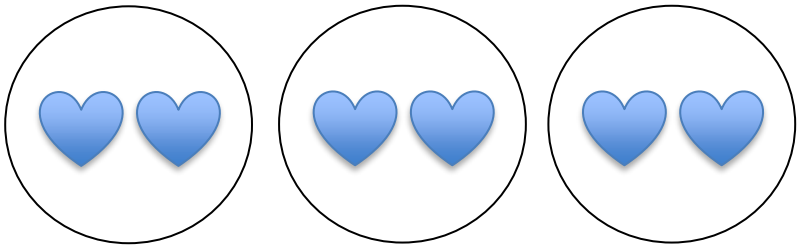
Divide 12 apples into groups of 3 apples each. How many groups are there? What division fact does this represent?
Divide 10 bugs into five groups. How many bugs in each group? What division fact does this represent?
Divide 20 candy kisses into four equal piles. What division fact does this represent?

Answers page 54

The Concept of "Remainder"

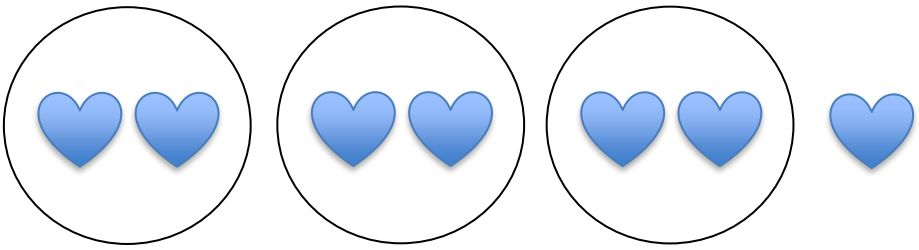
THE CONCEPT OF "REMAINDER"

So far, we have looked at only a small percentage of division problems, those that divide evenly, like 6 ÷ 3 = 2. 6 objects divided into 3 groups puts 2 objects into each group:



But often, there are some left over, a remainder.

A specific example that demonstrates this is: 7 ÷ 3



What to do with the remaining  ?

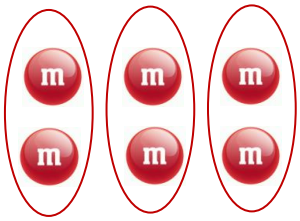
Activity 3: Modeling Remainder

You will need:

- At least 10 m&m's
- A Kleenex to keep the m&m's clean
- About 20 sheets of scratch paper
- A copy of page 15 for each student

Let's look at **7 ÷ 3 = ?**

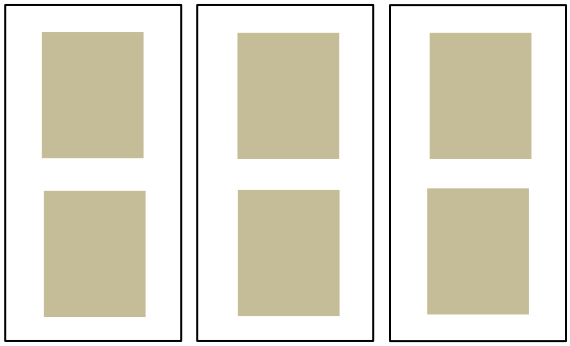
First, take 7 m&m's and divide them into 3 piles



3 piles

1 remaining

Next take 7 pieces of paper and divide them into 3 piles



3 piles

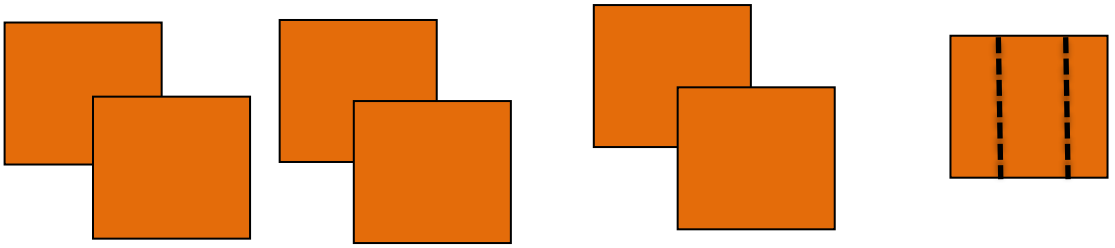
1 remaining

Remainder as R makes sense for m&m's: The m&m cannot be divided into equal parts to share (unless the partner is ok with biting it into pieces, not an exact process!). So, expressing a remainder as R 1 makes sense.

Remainder as a fraction or decimal makes sense for paper: The paper can be torn or cut into equal parts to share. A remainder of 1/3 makes sense. Fractions and decimals can be used to describe the remainder.

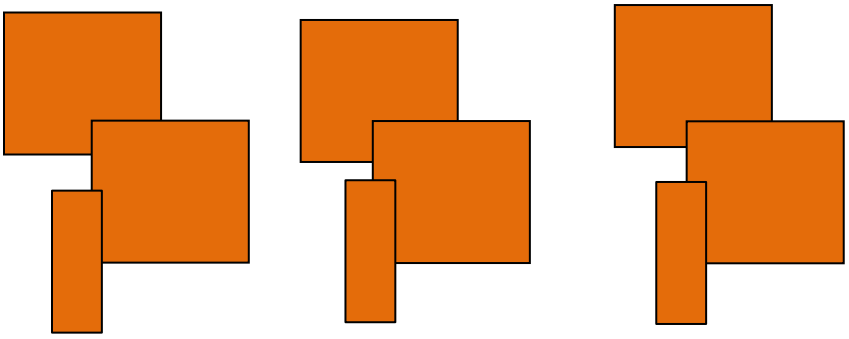
The one remaining piece of paper can be divided equally into three parts.

Each part is 1/3, one out of three.



3 piles 1 remaining

So, each pile has 2 1/3 pieces of paper.



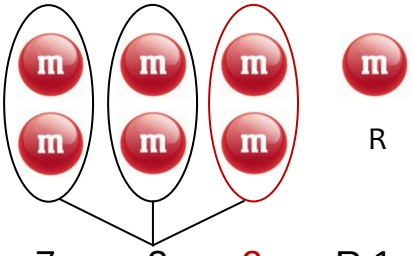
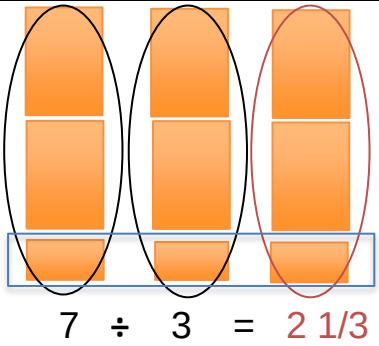
3 piles

Some people prefer using fractions for remainders, and some people prefer using decimals. It's your choice.

Activity 4: Remainders with m&m's and paper

Model and sketch each of the following division problems using m&m's and sheets of paper.

One m&m represents 'one.'  A sheet of paper represents 'one.' 

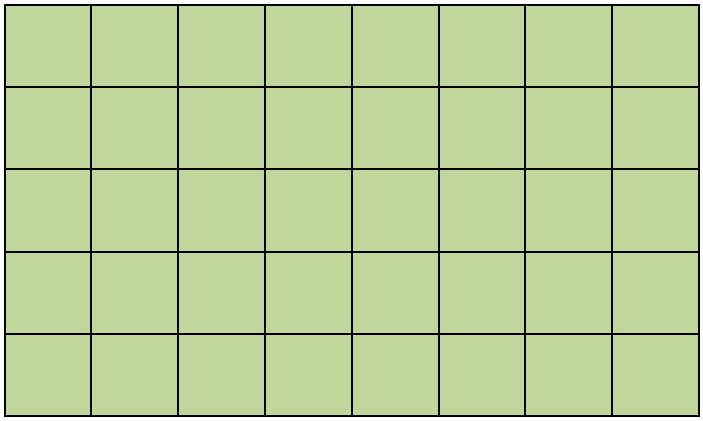
	M & M's	Paper
7 ÷ 3 =		
5 ÷ 2 =		
9 ÷ 4 =		
9 ÷ 2 =		

Using grids to understand remainders

Say we have **42 divided by 8 = x**

An easier problem is 40 ÷ 8, because 40 ÷ 8 = 5, with no remainder.

Here is the grid representation for 5 ∗ 8 = 40

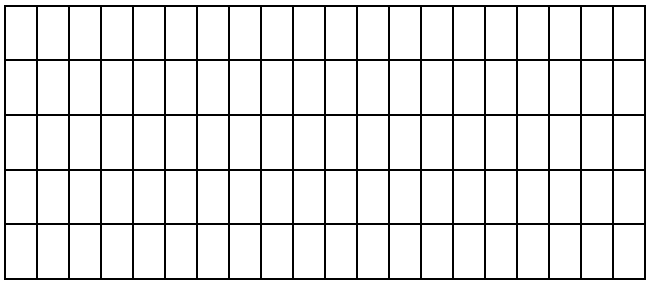
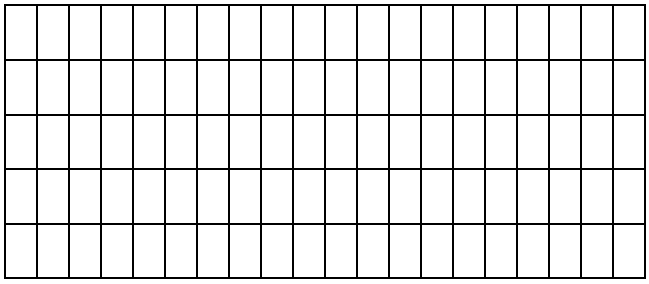
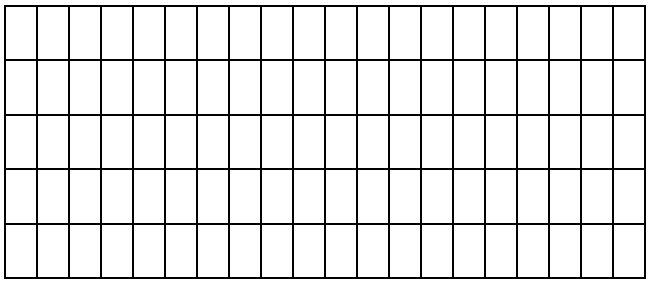
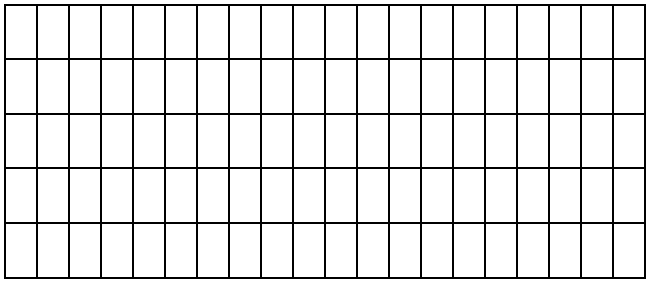
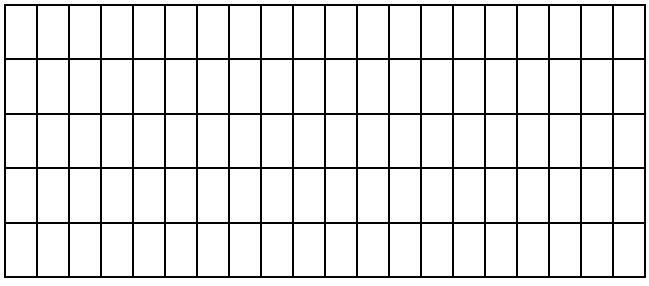


Now let's tackle 42 divided by 8 = ?

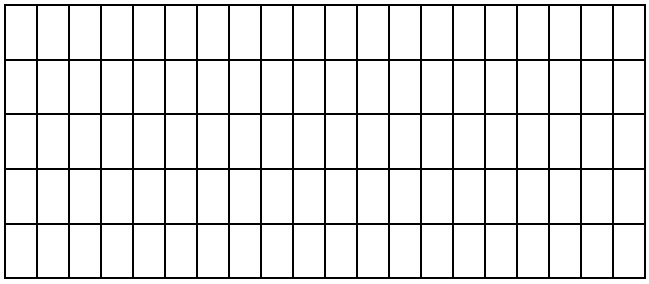
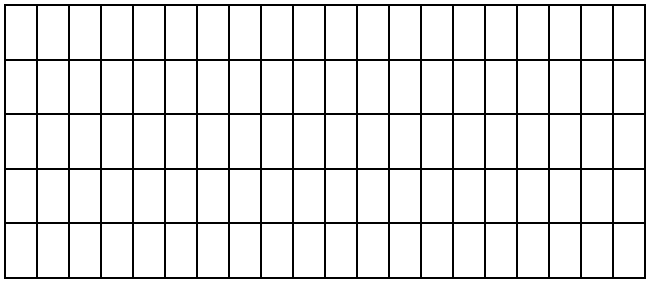
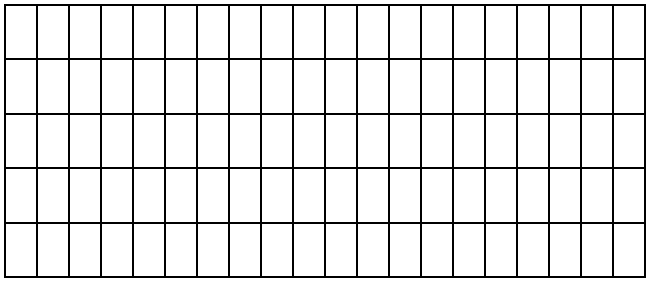
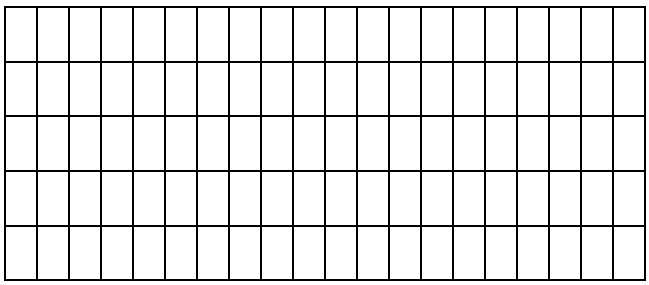
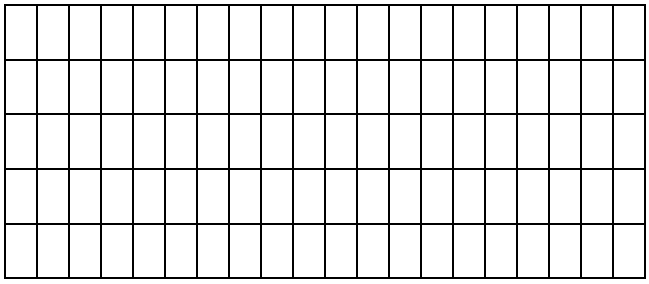
Once again, we have a grid with 8 on one side, and can keep filling rows until we use up all 42 squares.

41	42						

Practice 5: Division with Remainders

Problem	Grid	Answer, with R, and fraction
17 ÷ 3		
12 ÷ 5		
11 ÷ 3		
7 ÷ 2		
14 ÷ 4		

Answers page 55

Problem	Grid	Answer, with R, and fraction
15 ÷ 7		
9 ÷ 5		
13 ÷ 3		
16 ÷ 5		
16 ÷ 7		

Answers page 56

Division as sharing

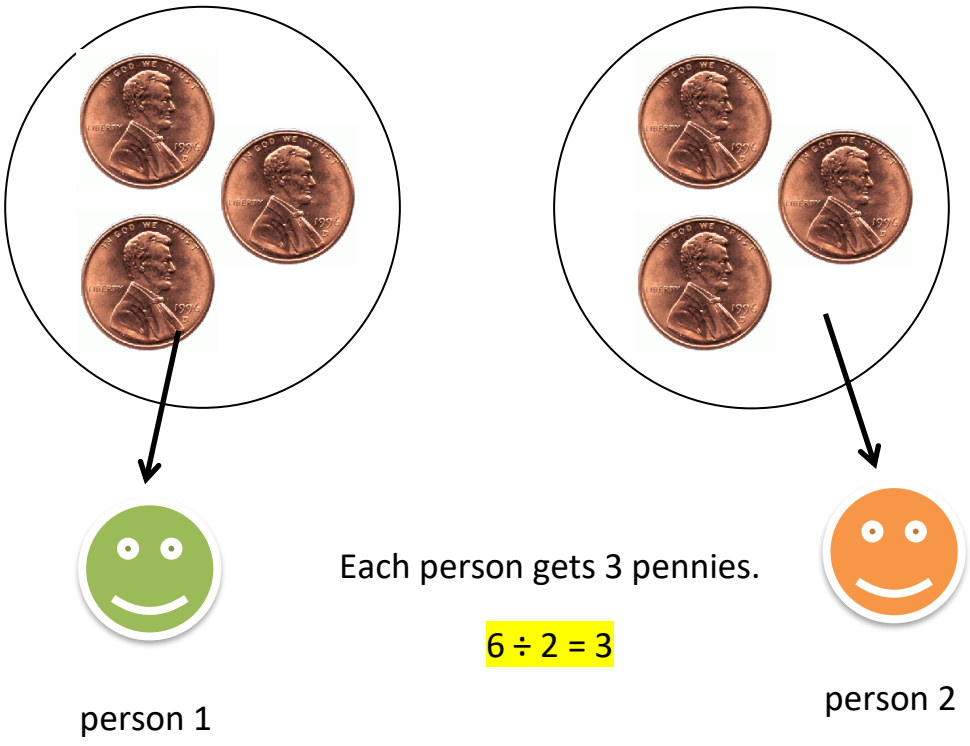
There are several ways to understand the concept of division. Sharing is one way, and we will start with it.

This is one way to look at the expression: 6 ÷ 2 = 3:

If I share 6 objects equally between 2 people, how many objects does each person get?

Each person gets 3 objects.

6 ÷ 2 can mean 6 pennies shared by two people

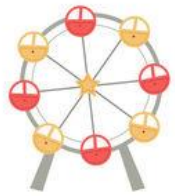


Practice 6: Word problems using division as sharing

Let's Eat!

Drue and his 4 friends had enough money to buy 11 cheeseburgers. How many cheeseburgers will each of the 5 friends have?
Brittney has 22 brownies that she plans to share with her 3 friends when they come over after school. How many brownies will each of the 4 girls have?
Bryan's mom left 7 sandwiches in the refrigerator for Bryan and his friends to eat after school. If 2 friends came home with Bryan, how many sandwiches will each of the 3 boys have?
Jacob and 4 of his friends ordered a pizza. If the pizza has 8 slices, how many slices will each of the 5 boys have?
Bria found 20 bags of chips in her pantry. If she shares will 7 friends, how many bags will each of the 8 girls have?

Answers page 57



The Carnival

Some friends are going to a carnival that has several different rides. Each of the rides requires a different amount of tickets.

The Ferris Wheel requires 3 tickets. If Taylor has 20 tickets, how many times can she ride the Ferris Wheel?
The roller coaster takes 5 tickets. If Angel has 17 tickets, how many times can he ride the roller coaster?
The Spider takes 4 tickets. If Michael has 15 tickets, how many times can he ride the Spider?
The Twister takes 3 tickets. If Donna has 19 tickets, how many times can she ride the Twister?
The Fun House takes 2 tickets. If Kaila has 22 tickets, how many times can she go in the Fun House?

Answers page 58

Happy Halloween!

Corey is having some friends over for a Halloween party and he is putting together some "treat bags" filled with various kinds of candy. He needs to fill 9 bags.

He has 35 pieces of bubble gum. How many pieces should he put in each bag?
He has 20 Hershey Kisses. How many should he put in each bag?
He has 42 Jolly Ranchers. How many should he put in each bag?
He has 37 Star Bursts. How many should he put in each bag?
He has 50 miniature Reese's Peanut Butter Cups. How many should he put in each bag?

This is a lot of candy. I hope they don't eat it all at once. And for sure, Corey should not each all the left overs himself!

Answers page 59

Cookies!!

Lindsey is trying to decide what kind of cookies to make when 7 friends come over to spend the night. Since she will eat cookies, too, there will be 8 girls total.

If she makes the chocolate chip recipe, which makes 50 cookies, how many will each girl have?
If she makes the oatmeal raisin recipe, which makes 42 cookies, how many will each girl have?
If she makes the snickerdoodle recipe, which makes 58 cookies, how many will each girl have?
If she makes the peanut butter recipe, which makes 35 cookies, how many will each girl have?
If she makes the sugar cookie recipe, which makes 50 cookies, how many will each girl have?

Answers page 60

Clean Up

Mrs. Washington is cleaning out her garage.

She found 45 books that she plans to donate to the 7 elementary school in the Pattonville School District. How many books will each school receive?
She found 15 picture frames that she will give to her 2 daughters, Savanna and Lashay. How many picture frames will each daughter receive?
She found 12 blankets, which she plans to donate to 4 different charities. How many blankets will each charity receive?
She has 13 boxes of junk that needs to go to the dump. If she can put 5 boxes in her car at a time, how many trips will she need to make?
Mrs. Washington paid Chad, Justin, and Leo to help her clean the garage. She gave Chad $28 in <u>one dollar bills</u> to split among the 3 boys. How much does each boy receive?

Answers page 61

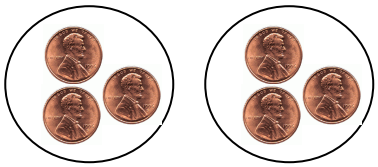
Summary of the three ways to look at Division

There are (at least) three ways to look at any division problem. For an example, we will look at 6 ÷ 3.

1. **Sharing:** Six divided by three means six objects divided evenly into three groups, as shown.

It answers the question: If I share 6 objects equally between 3 people, how many objects does each person get? Note, this is read left to right.

2. Grouping: Six divided by three means how many groups of three are in six:



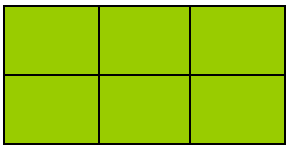
There are two groups of three in six. It is this way of looking at division that works well for division of fractions, coming soon.

It answers the question: How many groups of three are in six? Note, this is reading right to left.

3. Inverse of multiplication: 3 * some number = 6. It answers the question: What number times 3 equals 6?

We looked at multiplication as a rectangular array, and will look at division the same way.

It answers the question**:** 3 * some number = 6.



What number times 3 equals 6?

In all cases the answer is the same - two, of course.
All three of these views are important. Given any situation involving division, one of these must make sense.

The symbols representing Division

There are three main ways to show division with symbols.

Notation	Symbols read…
$8 \div 2$	left to right →
$2\overline{)\,8}$	right to left ←
$\frac{8}{2}$	up to down ↓

If you have been confused about division in the past, this could be one of the reasons. Things will become clear to you now!

Note: $a \div b$ is <u>not</u> the same as $b \div a$.

As a specific example, $8 \div 2$ is <u>not</u> the same as $2 \div 8$.

Practice 7: Three ways of Division

Write each of the following division problems three different ways.

5 divided by 2	$5 \div 2$	$2\overline{)5}$	$\frac{5}{2}$
3 divided by 2			
$2 \div 5$			
$\frac{8}{3}$			
$5\overline{)6}$			
$6\overline{)5}$			
$\frac{7}{3}$			
15 divided by 3			

Answers page 62

Is 6 ÷ 2 is the same as 2 ÷ 6? _____

Reading from left to right, write in words what 6 ÷ 2 says.

Reading from left to right, write in words what 2 ÷ 6 says.

Draw a picture to show 6 ÷ 2
Draw a picture to show 2 ÷ 6

The meaning of Division

What does it mean to say 8 divided by 4?
What does 7 divided by 3 mean?
Explain how division and multiplication are connected. Give an example.

DIVISION ALGORITHMS

We present the Ladder Method, largely a place value method, that we recommend. It reinforces place value and makes a lot of sense. It relies on continued subtraction. (The traditional long division algorithm is a completely specialized technique that does not generalize, and is contrary to the philosophy of the program. We recognize that some students know and prefer a more traditional algorithm such as long and short division, and then those students should use that algorithm.)

Ladder Division

This method relies on continued subtraction.

We provide several examples of ladder division. Hopefully you can see that there are many ways to solve any division problem using this method. Basically, it works by subtracting away groups until there is nothing left!

One of the best parts of ladder division is that you can use the multiplication facts you are comfortable with. There is more than one way to solve a division problem using the ladder method. The following five examples demonstrate this. Examples 1 and 4 shows two different solutions; of course, both ways of solving give the same answer.

Example 1. **12 ÷ 3**

Start with 12 m&m's.

This is how Student A solves the problem:

This student knows multiplication with 4's

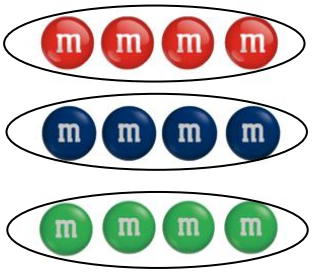
4 groups of **3**

$3\overline{)12}$

- 12

0 remaining

Take away



This is how Student B solves the problem:

This student keeps taking away 3's

$3\overline{)12}$

1 group of 3 - 3

9

1 group of 3 - 3

6

2 groups of 3 - 6

0 remaining

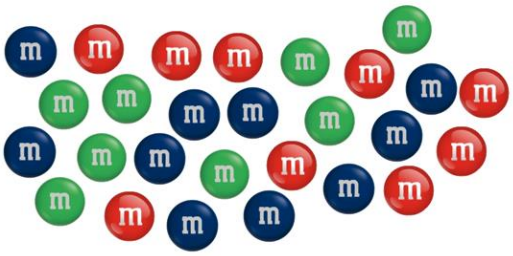
4 groups of **3**

Take away

So 12 ÷ **3** = **4** and **4** ∗ **3** = 12

Example 2. **27 ÷ 3**

Start with 27 m&m's.



This student knows multiplication with 5's

$$3\overline{)\,27}$$

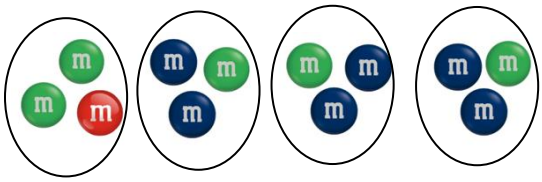
5 groups of 3 $\quad - 15$

12 Remaining

4 groups of 3 $\quad - 12$

$\quad\quad 0$

Take away

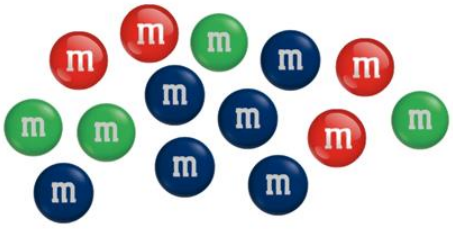


9

So, 9 * 3 = 27, and 27 ÷ 3 = 9

Example 3. 14 ÷ 5

Start with 14 m&m's.



This student knows multiplication with 5's

$5\overline{)14}$

2 groups of 5

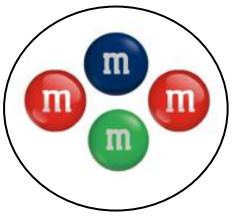
- 10

4

There are no groups of 5 in 4

Take away

Remaining



So, 14 ÷ 5 = 2 with 4 remaining.

There are 2 groups of 5 and 4 ones in 14.

Example 4. **735 ÷ 7**

This is how Student A solves the problem:

	7)**735**
100 groups of **7**	- 700
	35
5 groups of **7**	- 35
	0
_____	
105	

This is how Student B solves the problem:

	7)**735**
50 groups of **7**	- 350
	385
50 groups of **7**	- 350
	35
5 groups of **7**	- 35
	0
_____	
105	

So, 105 * 7 = 735, and 735 ÷ 7 = 105

Example 5. **632 ÷ 8**

	8) **632**
70 groups of 8	- 560
	72
9 groups of 8	- 72
	0
79	

So, 79 * 8 = 632, and 632 ÷ 8 = 79

Practice 8: Ladder Division method

Example:

$7\overline{)735}$

10 groups of 700 -700

35

5 groups of 7 -35

0

105

So, 105 * 7 = 735, and 735 ÷ 7 = 105

Note: There are different ways to answer these.

a. $8\overline{)632}$	b. $7\overline{)245}$
c. $9\overline{)423}$	d. $3\overline{)213}$

Answers page 64

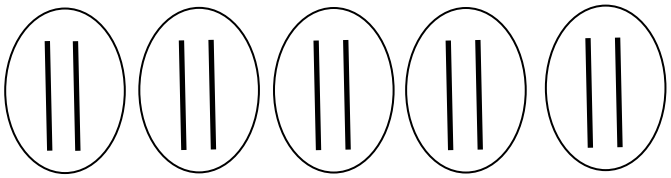
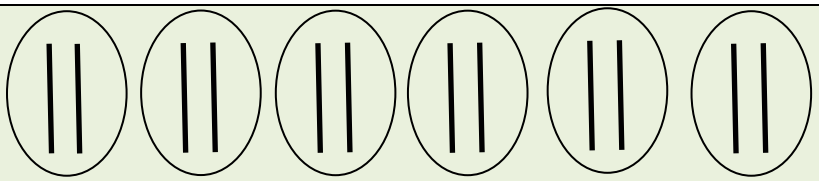
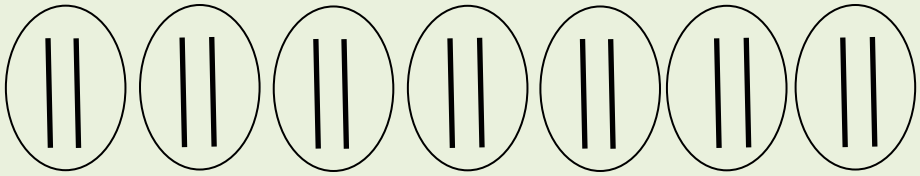
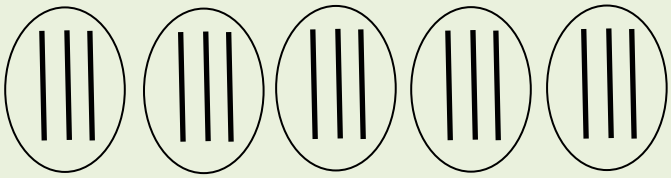
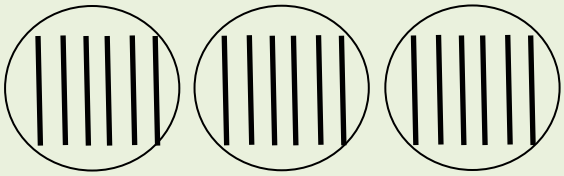
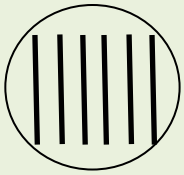
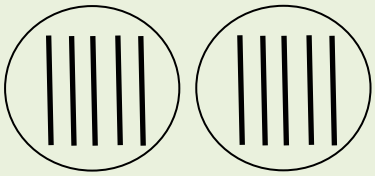
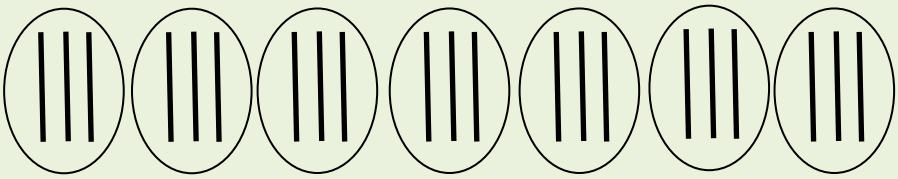
e. $6\overline{)432}$	f. $6\overline{)282}$
g. $8\overline{)756}$	h. $4\overline{)248}$
i. $5\overline{)320}$	j. $2\overline{)124}$

k. $2\overline{)212}$	l. $3\overline{)624}$
m. $4\overline{)832}$	n. $4\overline{)428}$
o. $6\overline{)6648}$	p. $7\overline{)735}$

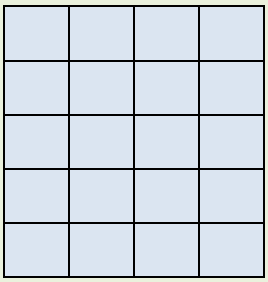
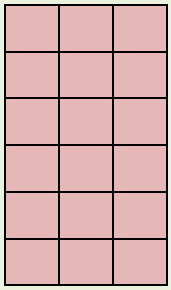
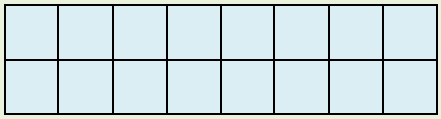
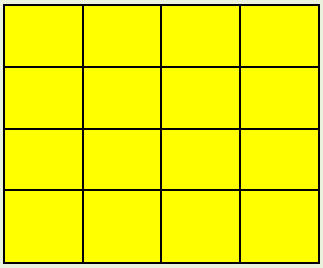
MATH WHISPERER

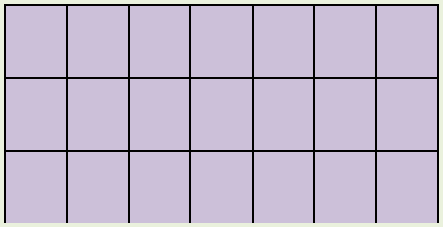
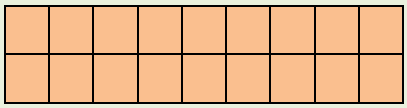
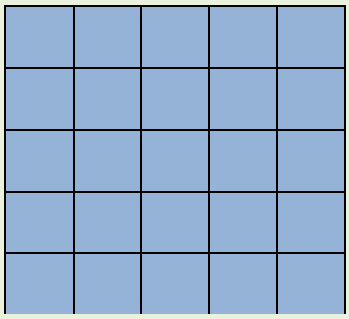
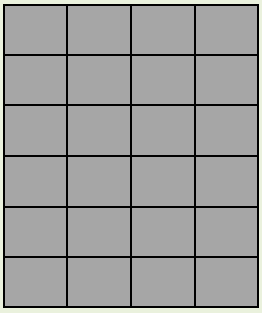
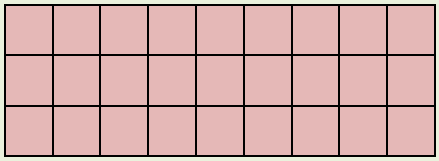
Where math makes sense

Practice 1: Division with popsicle sticks

10 ÷ 5 = 2 10 popsicle sticks divided into 5 piles equals 2.	
12 ÷ 6 = 2 12 popsicle sticks divided into 6 piles equals 2.	
14 ÷ 7 = 2 14 popsicle sticks divided into 7 piles equals 2.	
15 ÷ 5 = 3 15 popsicle sticks divided into 5 piles equals 3.	
18 ÷ 3 = 6 18 popsicle sticks divided into 3 piles equals 6.	
6 ÷ 1 = 6 popsicle sticks divided into 1 pile equals 1.	
10 ÷ 2 = 5 10 popsicle sticks divided into 2 piles equals 5.	
21 ÷ 7 = 3 21 popsicle sticks divided into 7 piles equals 3.	
15 ÷ 3 = 5 15 popsicle sticks divided into 3 piles equals 5.	

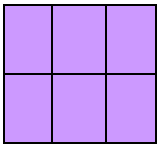
Practice 2: Multiplication and Division number families

Multiplication or division fact	Grid	Other members of the fact family
$4 * 5 = 20$		$5 * 4 = 20$ $20 \div 4 = 5$ $20 \div 5 = 4$
$3 * 6 = 18$		$6 * 3 = 18$ $18 \div 6 = 3$ $18 \div 3 = 6$
$1 * 9 = 9$		$9 * 1 = 9$ $9 \div 1 = 9$ $9 \div 9 = 1$
$16 \div 2 = 8$		$16 \div 8 = 2$ $2 * 8 = 16$ $8 * 2 = 16$
$16 \div 4 = 4$		$4 * 4 = 16$

Multiplication or division fact	Grid	Other members of the fact family
7 * 3 = 21		3 * 7 = 21 21 ÷ 3 = 7 21 ÷ 7 = 3
18 ÷ 2 = 9		18 ÷ 9 = 2 9 * 2 = 18 2 * 9 = 18
25 ÷ 5 = 5		5 * 5 = 25
24 ÷ 6 = 4		24 ÷ 4 = 6 4 * 6 = 24 6 * 4 = 24
27 ÷ 3 = 9		27 ÷ 9 = 3 3 * 9 = 27 9 * 3 = 27

Practice 3: Multiplication and Division are a number family

These are the multiplication and division number family statement for this grid.



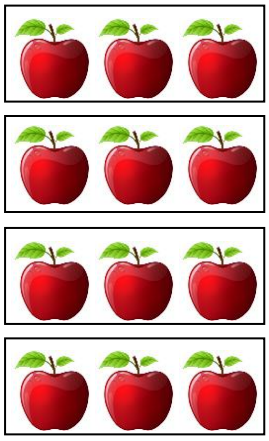
$2 * 3 = 6$ $3 * 2 = 6$ $6 \div 2 = 3$ $6 \div 3 = 2$

Write the other 3 members of each multiplication/division number family.

$4 * 5 = 20$	$5 * 4 = 20$	$20 \div 5 = 4$	$20 \div 4 = 5$
$8 \div 4 = 2$	$8 \div 2 = 4$	$4 * 2 = 8$	$2 * 4 = 8$
$3 * 7 = 21$	$7 * 3 = 21$	$21 \div 3 = 7$	$21 \div 7 = 3$
$2 \overline{)12} = 6$	$12 \div 6 = 2$	$2 * 6 = 12$	$6 * 2 = 12$
$56 \div 8 = 7$	$56 \div 7 = 8$	$7 * 8 = 56$	$8 * 7 = 56$
$a \div b = c$	$a \div c = b$	$b * c = a$	$c * b = a$

Practice 4: Word Problems using Division

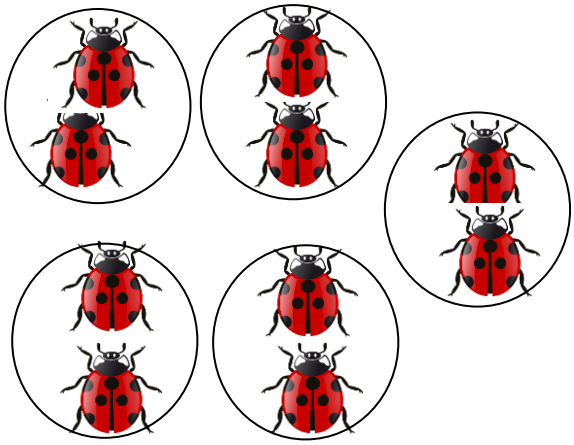
Divide 12 apples into groups of 3 apples each. How many groups are there?
What division fact does this represent?



There are 4 groups

$12 \div 3 = 4$

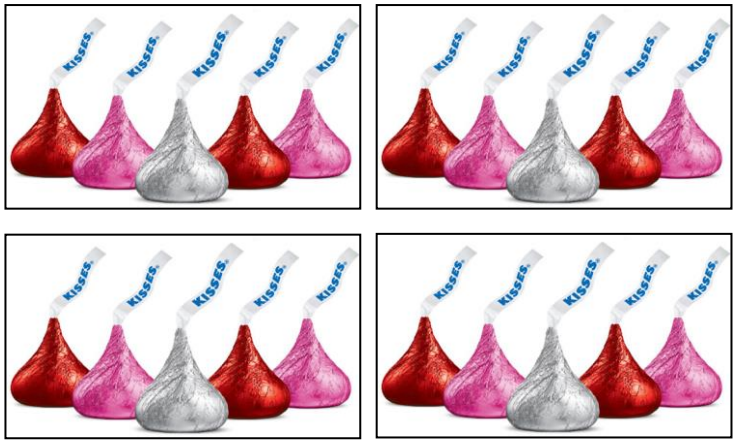
Divide 10 bugs into five groups. How many bugs in each group?
What division fact does this represent?



There are 2 bugs in each group

$10 \div 5 = 2$

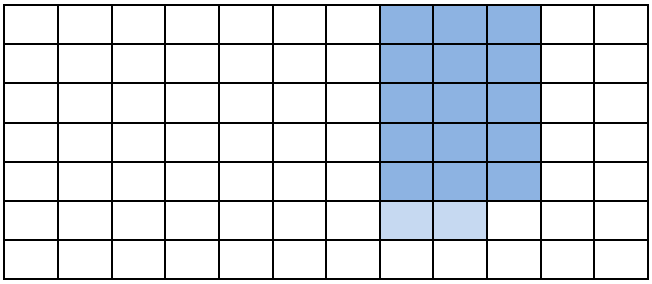
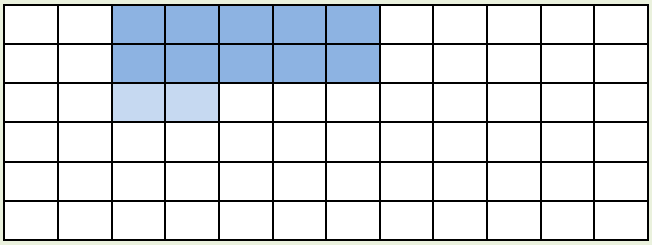
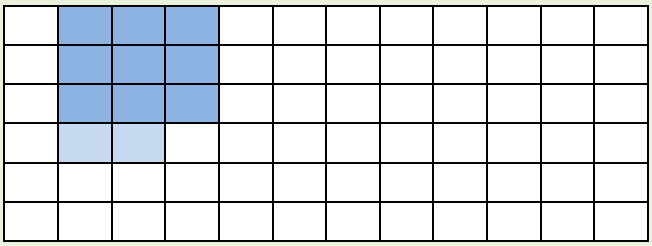
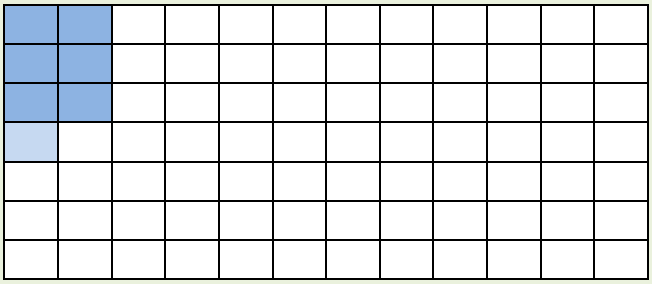
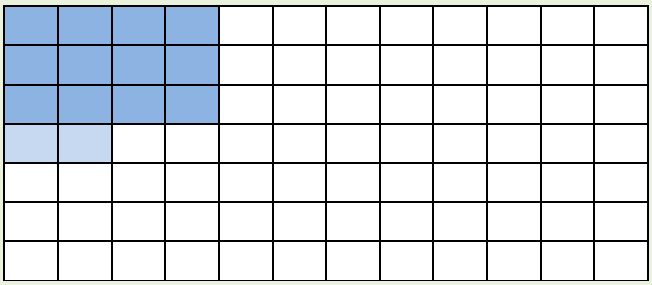
Divide 20 candy kisses into four equal piles. What division fact does this represent?

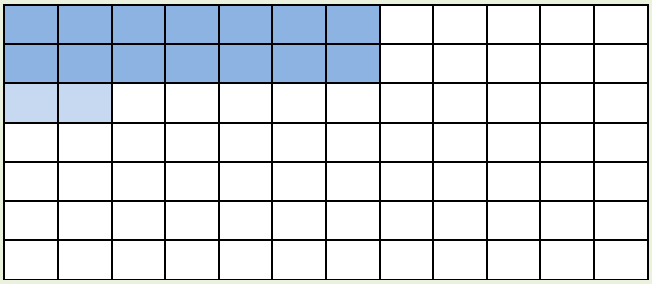
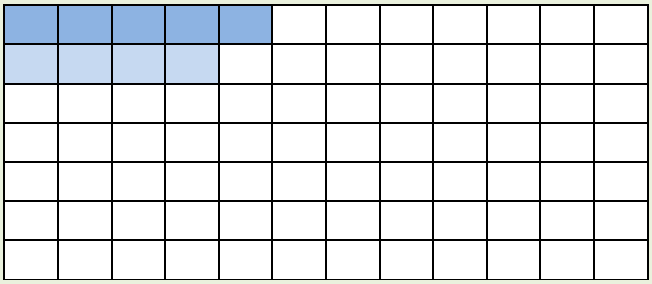
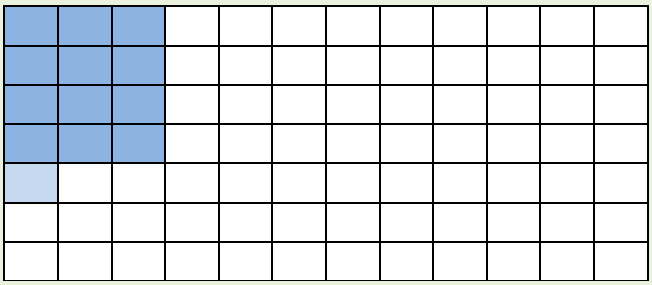
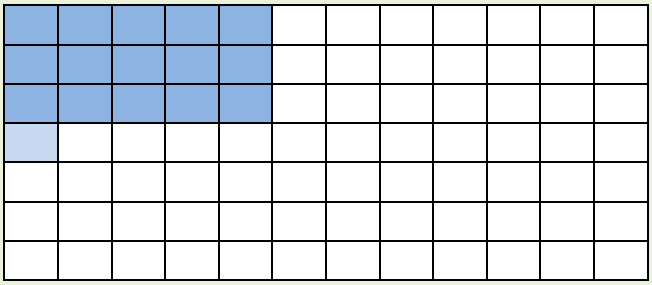
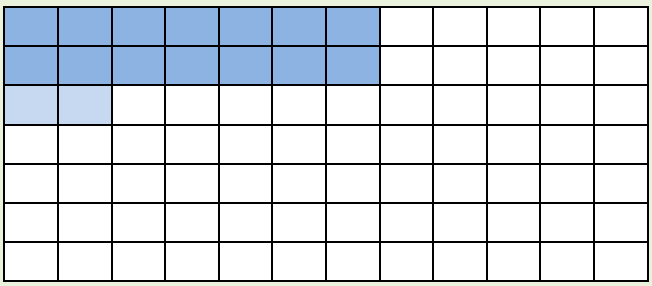


There are 5 candy kisses in each pile.

$20 \div 4 = 5$

Practice 5: Division with Remainders

Problem	Grid	Answer, with R, and fraction
17 ÷ 3		5 R2 $5\ \frac{2}{3}$
12 ÷ 5		2 R2 $2\ \frac{2}{5}$
11 ÷ 3		3 R2 $3\ \frac{2}{3}$
7 ÷ 2		3 R1 $3\ \frac{1}{2}$
14 ÷ 4		3 R2 $3\ \frac{2}{4} = 3\ \frac{1}{2}$

Problem	Grid	Answer, with R, and fraction
15 ÷ 7		2 R1 $2\ \frac{1}{7}$
9 ÷ 5		1 R4 $1\ \frac{4}{5}$
13 ÷ 3		4 R1 $4\ \frac{1}{3}$
16 ÷ 5		3 R1 $3\ \frac{1}{5}$
16 ÷ 7		2 R2 $2\ \frac{2}{7}$

Practice 6: Word problems using Division as sharing

Let's Eat!

Drue and his 4 friends had enough money to buy 11 cheeseburgers.
How many cheeseburgers will each of the 5 friends have?

$11 \div 5 = 2\frac{1}{5}$ Each person gets $2\frac{1}{5}$ cheeseburger. Then they can share the one cheeseburger and have $\frac{1}{5}$ of it.

Brittney has 22 brownies that she plans to share with her 3 friends when they come over after school. How many brownies will each of the 4 girls have?

$22 \div 4 = 5\frac{2}{4} = 5\frac{1}{2}$

Each girl gets $5\frac{1}{2}$ brownies.

Bryan's mom left 7 sandwiches in the refrigerator for Bryan and his friends to eat after school. If 2 friends came home with Bryan, how many sandwiches will each of the 3 boys have?

Each boy gets $2\frac{1}{3}$ sandwiches.

Jacob and 4 of his friends ordered a pizza.
If the pizza has 8 slices, how many slices will each of the 5 boys have?

Each boy will get $1\frac{3}{5}$ slices. (They will still be hungry!)

Bria found 20 bags of chips in her pantry.
If she shares will 7 friends, how many bags will each of the 8 girls have?

Each girl will get $2\frac{1}{2}$ bags of chips.

The Carnival

Some friends are going to a carnival that has several different rides. Each of the rides requires a different amount of tickets.

The Ferris Wheel requires 3 tickets.
If Taylor has 20 tickets, how many times can she ride the Ferris Wheel?

Taylor can ride 6 times, which will cost her $3 * 6 = 18$ tickets. It's a shame Taylor doesn't have 21 tickets – then she could ride 7 times, as 7 rides $*$ 3 tickets/ride = 21 tickets.

The roller coaster takes 5 tickets.
If Angel has 17 tickets, how many times can he ride the roller coaster?

Angel can ride 3 times. 3 rides $*$ 5 tickets per ride = 15 tickets. He will have 2 tickets left over.

The Spider takes 4 tickets.
If Michael has 15 tickets, how many times can he ride the Spider?

Michael can ride 3 times. 4 tickets/ride $*$ 3 rides equal 12 tickets.

The Twister takes 3 tickets.
If Donna has 19 tickets, how many times can she ride the Twister?

Donna can ride 6 times. 3 tickets per ride $*$ 6 rides = 18 tickets. She will have one ticket left over and probably feel kind of dizzy from 6 rides.

The Fun House takes 2 tickets.
If Kaila has 22 tickets, how many times can she go in the Fun House?

Kaila can go in 11 times. She will have no tickets left over, because $11 * 2 = 22$.

Happy Halloween!

Corey is having some friends over for a Halloween party and he is putting together some "treat bags" filled with various kinds of candy. He needs to fill 9 bags.

He has 35 pieces of bubble gum. How many pieces should he put in each bag? 35 ÷ 9 = 3, with a remainder of 8. So, each bag gets 3 pieces of bubble gum.
He has 20 Hershey Kisses. How many should he put in each bag? 20 ÷ 9 = 2, with a remainder of 2. So, each bag gets 2 Hershey kisses.
He has 42 Jolly Ranchers. How many should he put in each bag? Each bag will have 4 Jolly Ranchers. 4 ∗ 9 = 36. 42 – 36 = 6, so there will be 6 Jolly Ranchers left over.
He has 37 Star Bursts. How many should he put in each bag? He should put 4 Star Bursts in each bag. That will take care of 9 bags ∗ 4 per bag = 36 Star Bursts.
He has 50 miniature Reese's Peanut Butter Cups. How many should he put in each bag? He should put 5 in each bag. 5 in each bag ∗ 9 bags = 45 Peanut Butter Cups in all.

Cookies!!

Lindsey is trying to decide what kind of cookies to make when 7 friends come over to spend the night. Since she will eat cookies, too, there will be 8 girls total.

If she makes the chocolate chip recipe, which makes 50 cookies, how many will each girl have?

50 ÷ 8 = 6, with a remainder of 2. So, each girl will get $6\ \frac{2}{8}$ or $6\ \frac{1}{4}$ cookies.

If she makes the oatmeal raisin recipe, which makes 42 cookies, how many will each girl have?

42 ÷ 8 = 5, with a remainder of 2. So, each girl will get $5\ \frac{2}{8}$ or $5\ \frac{1}{4}$ cookies.

If she makes the snickerdoodle recipe, which makes 58 cookies, how many will each girl have?

Each girl will have 7 ¼ cookies, because 7 cookies per girl ∗ 8 girls = 56 cookies. 58 – 56 = 2 cookies remaining. Those two cookies shared by 8 girls means 2 ÷ 8 or = $\frac{2}{8}$ = $\frac{1}{4}$ cookie also.

If she makes the peanut butter recipe, which makes 35 cookies, how many will each girl have?

Each girl will get $4\ \frac{3}{8}$ cookies. 4 cookies per girl ∗ 8 girls = 32 cookies. The remaining 35 – 32 = 3 cookies can be shared evenly if each girl gets $\frac{3}{8}$ cookie.

If she makes the sugar cookie recipe, which makes 50 cookies, how many will each girl have?

Each girl will get $6\ \frac{1}{4}$ cookies. 6 cookies per girl ∗ 8 girls = 48 cookies. The two remaining cookies can be evenly divided into fourths, as $\frac{1}{4}$ ∗ 8 = 2.

Clean Up

Mrs. Washington is cleaning out her garage.

She found 45 books that she plans to donate to the 7 elementary school in the Pattonville School District. How many books will each school receive?

Each school will receive 6 books. 6 books ∗ 7 books per school = 42 books. She will have 3 books remaining, as 45 – 42 = 3.

She found 15 picture frames that she will give to her 2 daughters, Savanna and Lashay. How many picture frames will each daughter receive?

Each daughter will receive 7 frames. 15 ÷ 2 = 7, with a remainder of 1.

She found 12 blankets, which she plans to donate to 4 different charities. How many blankets will each charity receive?

3 blankets will go to each charity. 3 blankets for each charity ∗ 4 charities = 12 blankets.

She has 13 boxes of junk that needs to go to the dump. If she can put 5 boxes in her car at a time, how many trips will she need to make?

She will need to make 3 trips. Trip one: take 5 boxes, 8 boxes remain

Trip two: take 5 boxes, 3 boxes remain

Trip three: take the last 3 boxes

Practice 7: Three ways of Division

Write each of the following division problems three different ways.

5 divided by 2	$5 \div 2$	$2\overline{)5}$	$\frac{5}{2}$
3 divided by 2	$3\overline{)2}$	$3 \div 2$	$\frac{3}{2}$
$2 \div 5$	$5\overline{)2}$	$\frac{5}{2}$	2 divided by 5
$\frac{8}{3}$	8 divided by 3	$3\overline{)8}$	$\frac{8}{3}$
$5\overline{)6}$	6 divided by 5	$\frac{6}{5}$	$6 \div 5$
$6\overline{)5}$	5 divided by 6	$\frac{5}{6}$	$5 \div 6$
$\frac{7}{3}$	7 divided by 3	$3\overline{)7}$	$7 \div 3$
15 divided by 3	$3\overline{)15}$	$15 \div 3$	$\frac{15}{3}$

Is 6 ÷ 2 is the same as 2 ÷ 6? No

Reading from left to right, write in words what 6 ÷ 2 says.

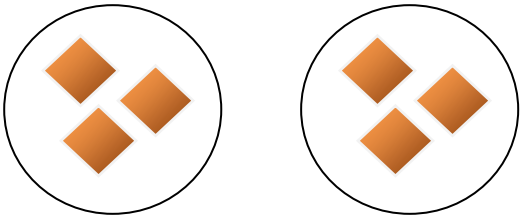
6 divided by 2

Reading from left to right, write in words what 2 ÷ 6 says.

2 divided by 6

Draw a picture to show 6 ÷ 2

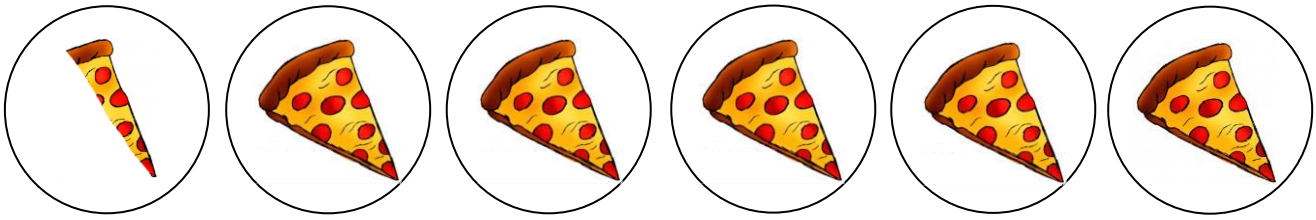
Here is 1 object: 



6 objects divided into 2 piles

Draw a picture to show 2 ÷ 6

Here is one object like a pizza: 



1/3 of the pizza

Practice 8: Ladder Division method

Example:

	7)**735**
100 groups of 7	- 700
	35
5 groups of 7	- 35
	0
	105

So, 105 * 7 = 735, and 735 ÷ 7 = 105

NOTE: There are lots of ways to solve this.

a. 79)632 56 72 72 0	8)632 70 groups of 8 560 72 9 groups of 8 72 0 79	8)632 60 groups of 8 480 152 10 groups of 8 80 72 6 groups of 8 48 24 3 groups of 8 24 0 79
b. 35)245 21 35 35 0	7)245 30 groups of 7 210 35 5 groups of 7 35 0 35	7)245 20 groups of 7 140 105 10 groups of 7 70 35 5 groups of 7 35 0 35

NOTE: There are many ways to solve these.

c.

$$\begin{array}{r} 47 \\ 9\overline{)423} \\ 36 \\ \hline 63 \\ 63 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 9\overline{)423} \\ 180 \\ \hline 243 \\ 180 \\ \hline 63 \\ 45 \\ \hline 18 \\ 18 \\ \hline 0 \end{array}$$

20 groups of 9
20 groups of 9
5 groups of 9
2 groups of 9

47

d.

$$\begin{array}{r} 71 \\ 3\overline{)213} \\ 21 \\ \hline 3 \\ 3 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 71 \\ 3\overline{)213} \\ 210 \\ \hline 3 \\ 3 \\ \hline 0 \end{array}$$

70 groups of 3
1 group of 3

71

$$\begin{array}{r} 3\overline{)213} \\ 60 \\ \hline 153 \\ 60 \\ \hline 93 \\ 30 \\ \hline 3 \\ 3 \\ \hline 0 \end{array}$$

20 groups of 3
20 groups of 3
30 groups of 3
1 group of 3

71

e.

$$\begin{array}{r} 72 \\ 6\overline{)432} \\ 42 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6\overline{)432} \\ 420 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$$

70 groups of 6
2 groups of 6

72

$$\begin{array}{r} 6\overline{)432} \\ 300 \\ \hline 132 \\ 60 \\ \hline 72 \\ 60 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$$

50 groups of 6
10 groups of 6
10 groups of 6
2 groups of 6

72

NOTE: There are many ways to solve these.

f.

```
      47
   6)282
     24
     42
     42
      0
```

```
          6)282
40 groups of 6   240
                  42
7 groups of 6     42
                   0
47
```

```
          6)282
20 groups of 6   120
                 162
20 groups of 6   120
                  42
2 groups of 6     12
                  30
5 groups of 6     30
                   0
47
```

g.

```
      94
   8)752
     72
     32
     32
      0
```

```
          8)752
90 groups of 8   720
                  32
4 groups of 8     32
                   0
94
```

```
          8)752
40 groups of 8   320
                 432
40 groups of 8   320
                 112
10 groups of 8    80
                  32
4 groups of 8     32
                   0
94
```

h.

```
      62
   4)248
     24
      8
      8
      0
```

```
          4)248
60 groups of 4   240
                   8
2 groups of 4      8
                   0
62
```

```
          4)248
50 groups of 4   200
                  48
10 groups of 4    40
                   8
2 groups of 4      8
                   0
62
```

NOTE: There are many ways to solve these.

i.

```
      64
5 ) 320
    30
     20
     20
      0
```

```
                5 ) 320
60 groups of 5    300
                   20
4 groups of 5      20
                    0
____
64
```

```
                5 ) 320
20 groups of 5    100
                  320
20 groups of 5    100
                  220
20 groups of 5    100
                   20
4 groups of 5      20
                    0
____
64
```

j.

```
      62
2 ) 124
    12
     4
     4
     0
```

```
                2 ) 124
60 groups of 2    120
                    4
2 groups of 2       4
                    0
____
62
```

```
                2 ) 124
50 groups of 2    100
                   24
10 groups of 2     20
                    4
2 groups of 2       4
                    0
____
62
```

k.

```
      106
2 ) 212
    2
    01
      12
      12
       0
```

```
                2 ) 212
100 groups of 2   200
                   12
6 groups of 2      12
                    0
____
106
```

```
                2 ) 212
50 groups of 2    100
                  112
50 groups of 2    100
                   12
5 groups of 2      10
                    2
1 group of 2        2
                    0
____
106
```

NOTE: There are many ways to solve these.

i.

$$\begin{array}{r} 64 \\ 5\overline{)320} \\ 30 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$$\begin{array}{r} \\ 5\overline{)320} \\ 300 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

60 groups of 5

4 groups of 5

64

$$\begin{array}{r} \\ 5\overline{)320} \\ 100 \\ \hline 320 \\ 100 \\ \hline 220 \\ 100 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

20 groups of 5

20 groups of 5

20 groups of 5

4 groups of 5

64

j.

$$\begin{array}{r} 62 \\ 2\overline{)124} \\ 12 \\ \hline 4 \\ 4 \\ \hline 0 \end{array}$$

$$\begin{array}{r} \\ 2\overline{)124} \\ 120 \\ \hline 4 \\ 4 \\ \hline 0 \end{array}$$

60 groups of 2

2 groups of 2

62

$$\begin{array}{r} \\ 2\overline{)124} \\ 100 \\ \hline 24 \\ 20 \\ \hline 4 \\ 4 \\ \hline 0 \end{array}$$

50 groups of 2

10 groups of 2

2 groups of 2

62

k.

$$\begin{array}{r} 106 \\ 2\overline{)212} \\ 2 \\ \hline 01 \\ 12 \\ \hline 12 \\ \hline 0 \end{array}$$

$$\begin{array}{r} \\ 2\overline{)212} \\ 200 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$$

100 groups of 2

6 groups of 2

106

$$\begin{array}{r} \\ 2\overline{)212} \\ 100 \\ \hline 112 \\ 100 \\ \hline 12 \\ 10 \\ \hline 2 \\ 2 \\ \hline 0 \end{array}$$

50 groups of 2

50 groups of 2

5 groups of 2

1 group of 2

106

NOTE: There are many ways to solve these.

l.
```
      208
   3 )624
      6
      02
       0
       24
       24
        0
```

```
          208
   3 )624
200 groups of 3   600
                   24
8 groups of 3      24
                    0
208
```

```
          208
   3 )624
100 groups of 3   300
                  324
100 groups of 3   300
                   24
5 groups of 3      15
                    9
3 groups of 3       9
                    0
208
```

m.
```
      208
   4 )832
      8
      03
       0
       32
       32
        0
```

```
          208
   4 )832
200 groups of 4   800
                   32
8 groups of 4      32
                    0
208
```

```
          208
   4 )832
150 groups of 4   600
                  232
50 groups of 4    200
                   32
5 groups of 4      20
                   12
3 groups of 4      12
                    0
208
```

n.
```
      107
   4 )428
      4
      02
       0
       28
       28
        0
```

```
          107
   4 )428
100 groups of 4   400
                   28
7 groups of 4      28
                    0
107
```

```
          107
   4 )428
20 groups of 4     80
                  348
50 groups of 4    200
                  148
30 groups of 4    120
                   28
7 group of 4       28
                    0
107
```

NOTE: There are many ways to solve these.

o.

```
      1 1 0 8
   ┌─────────
6 ) 6 6 4 8
    6
    ─
    0 6
      6
      ─
      0 4
        0
        ─
        4 8
        4 8
        ───
          0
```

```
                  6 ) 6 6 4 8
1000 groups of 6    6 0 0 0
                      6 4 8
100 groups of 6       6 0 0
                        4 8
8 groups of 6           4 8
                        ───
                          0
____
1108
```

```
                  6 ) 6 6 4 8
1000 groups of 6    6 0 0 0
                      6 4 8
100 groups of 6       6 0 0
                        4 8
5 groups of 6           3 0
                        ───
                        1 8
3 groups of 6           1 8
____
1108
```

p.

```
      1 0 5
   ┌─────────
7 ) 7 3 5
    7
    ─
    0 3
      0
      ─
      3 5
      3 5
      ───
        0
```

```
                  7 ) 7 3 5
100 groups of 7     7 0 0
                      3 5
5 groups of 7         3 5
____                  ───
105                     0
```

```
                  7 ) 7 3 5
50 groups of 7      3 5 0
                    3 8 5
50 groups of 7      3 5 0
                      3 5
5 groups of 7         3 5
____                  ───
105                     0
```

Help for Helpers

I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as "I know you can do this. I believe in you." And then leave the student alone to do the work.

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn't going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don't generally want to ask our boss or friend: "Am I right? As an adult, we have to know the answer is right ourselves. The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student's life. You will gain confidence in your students as you watch them be successful on their own.