MATH WHISPERER

Where math makes sense

Research Edition

The Operations Series

Multiplication

Math Whisperer is a program created and designed for math to make sense, so all students can learn math. For more information, please go to www.mathwhisperer.com

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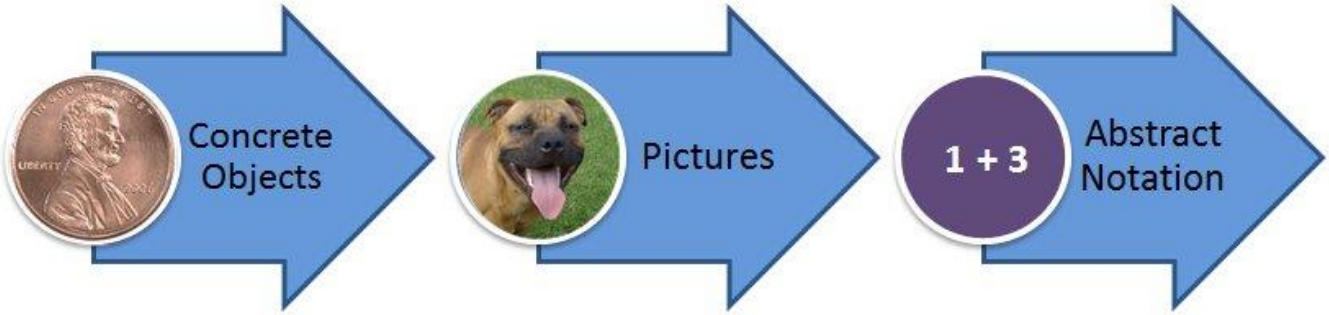
DEDICATION

Math Whisperer materials are dedicated to each person who wants to be successful in math, including those who have struggled in the past. Our goal for our students is that they know the math they need to lead the lives they want.

Contents

1. Introduction

Math Whisperer lessons are based on scientific research about how people learn math. Math is actually supposed to make sense. When you start with hands-on objects, math can make sense.



You are probably used to starting with the third step of abstract notation, which means using symbols and maybe a formula. Some people are able to start at this third step, using a formula. Maybe they even understand why the formula works. Maybe they don't, but they get the right answers. These people will benefit from the hands-on objects, also, as they will understand the math at a deeper level. This three step progression works for everybody.

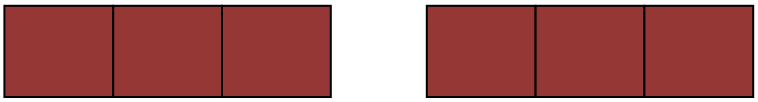
It may feel silly to you to use hands-on objects. My advice to you is: Try it, please. You will see for yourself how well the three step progression works. You are much more likely to remember the formulas this way. And if you forget them, you can reinvent them for yourself. Won't it feel great to never have to learn this again? The math will stick with you with the three step progression.

Hello. I'm Bernice, founder of Math Whisperer. I've worked with lots of students just like you, and they were all able to learn the math they wanted and needed to learn. So can you.

What is Multiplication

Multiplication means “groups of.” For example, $2 * 3$ means “two groups of 3.”.

This is “two groups of 3” squares:



And this is “2 groups of 3 pennies”:

Reading left to right , $2 * 3$ means two groups of three.

$2 * 3$ means 2 groups of 3	$3 * 2$ means 3 groups of 2
	

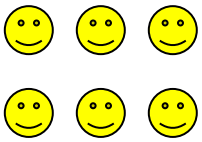
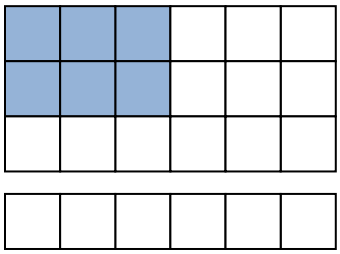
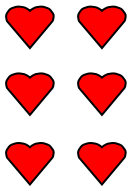
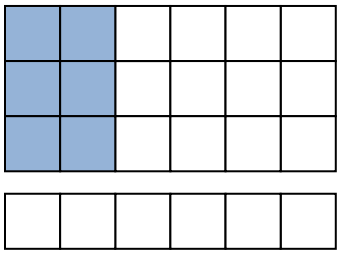
Both $2 * 3 = 6$ and $3 * 2 = 6$ so they are now different

$3 *$ 4 means _____

$4 * 3$ means _____

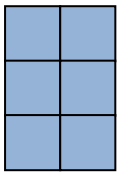
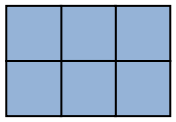
Multiplication means “groups of”

Here are some ways to look at multiplication, with symbols, words, pictures, and a grid.

Symbols	With words	Picture	Grid	Product
$2 * 3$	2 groups of 3			6
$3 * 2$	3 groups of 2			6

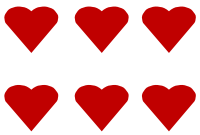
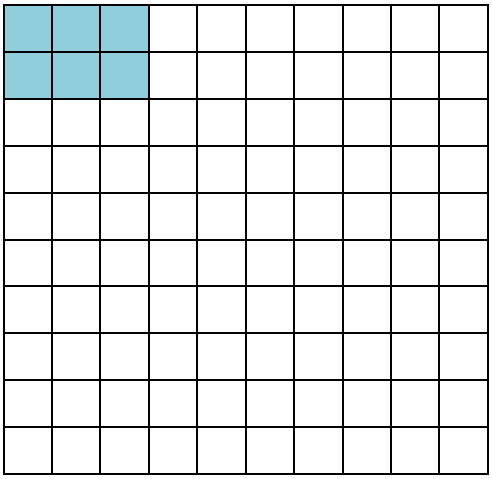
NOTE: There isn’t a right way or a wrong way to do the grids for $2 * 3$ and $3 * 2$. The important point is that they are different.

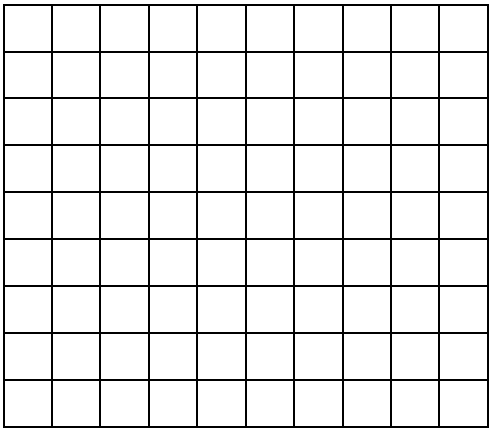
I could have done it this way.

$2 * 3$  and $3 * 2$ 

Practice 1: Multiplication means "groups of"

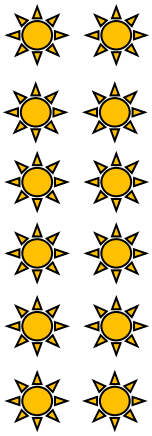
Now it's your turn. Use any symbols you like.

Expression	Symbols	Product	Grid
$2 * 3$		6	

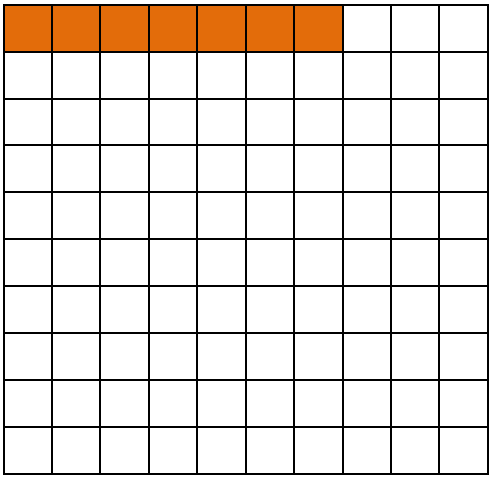
Expression	Symbols	Product	Grid
$3 * 4$			

Expression	Symbols	Product	Grid
$6 * 7$			

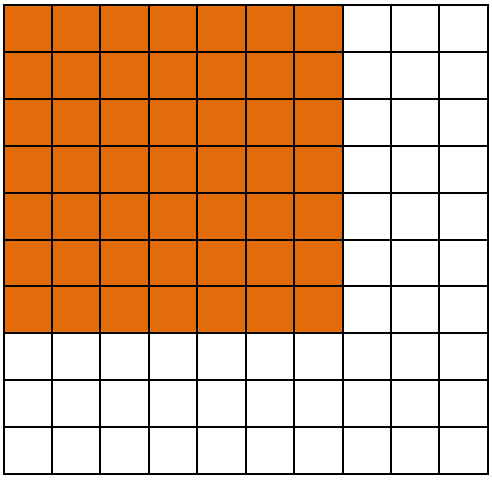
Expression	Symbols	Product	Grid
			

Expression	Symbols	Product	Grid
			

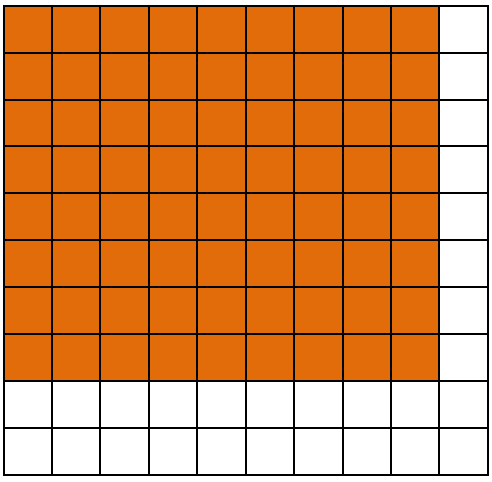
Expression	Symbols	Product	Grid



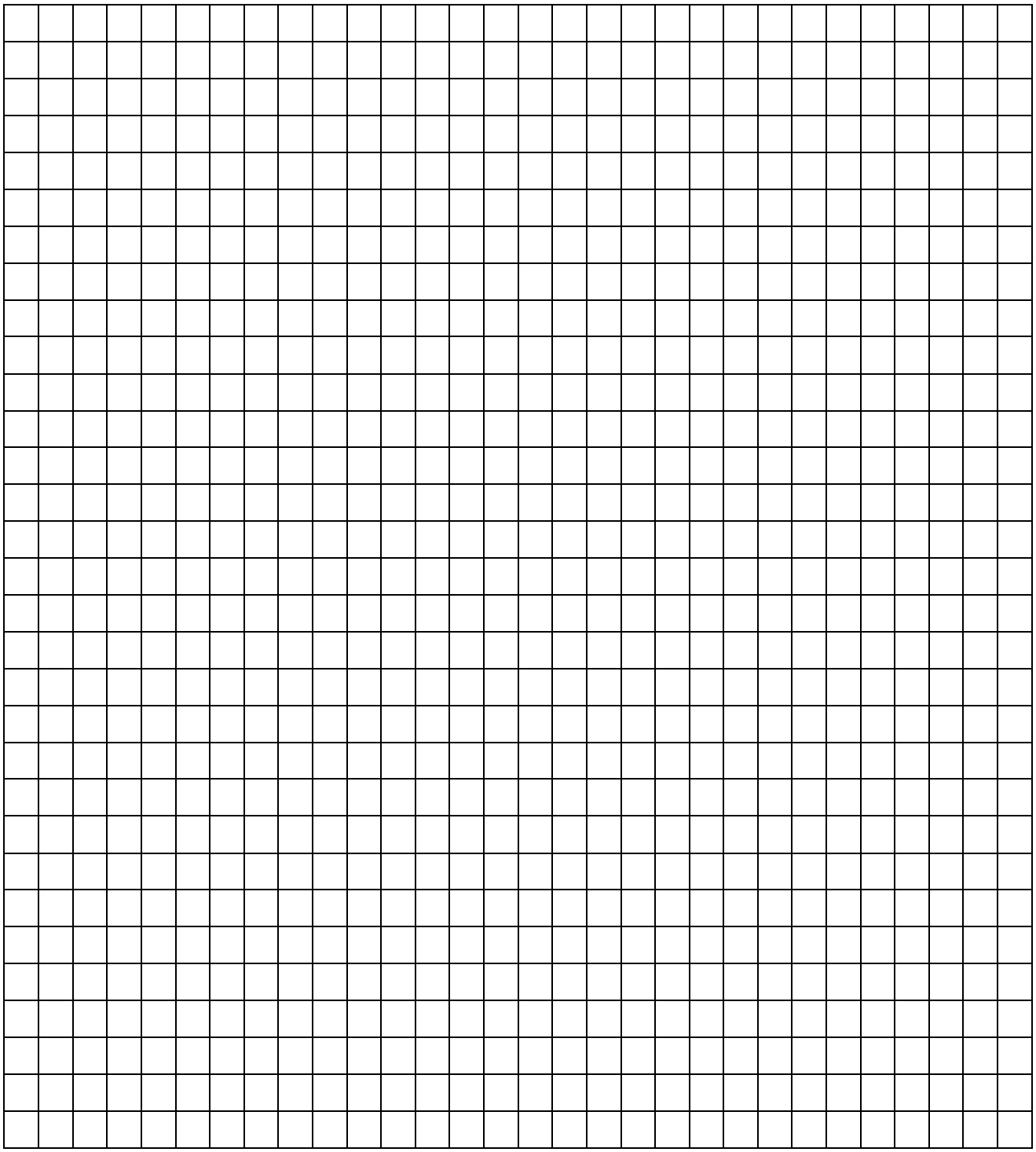
Expression	Symbols	Product	Grid



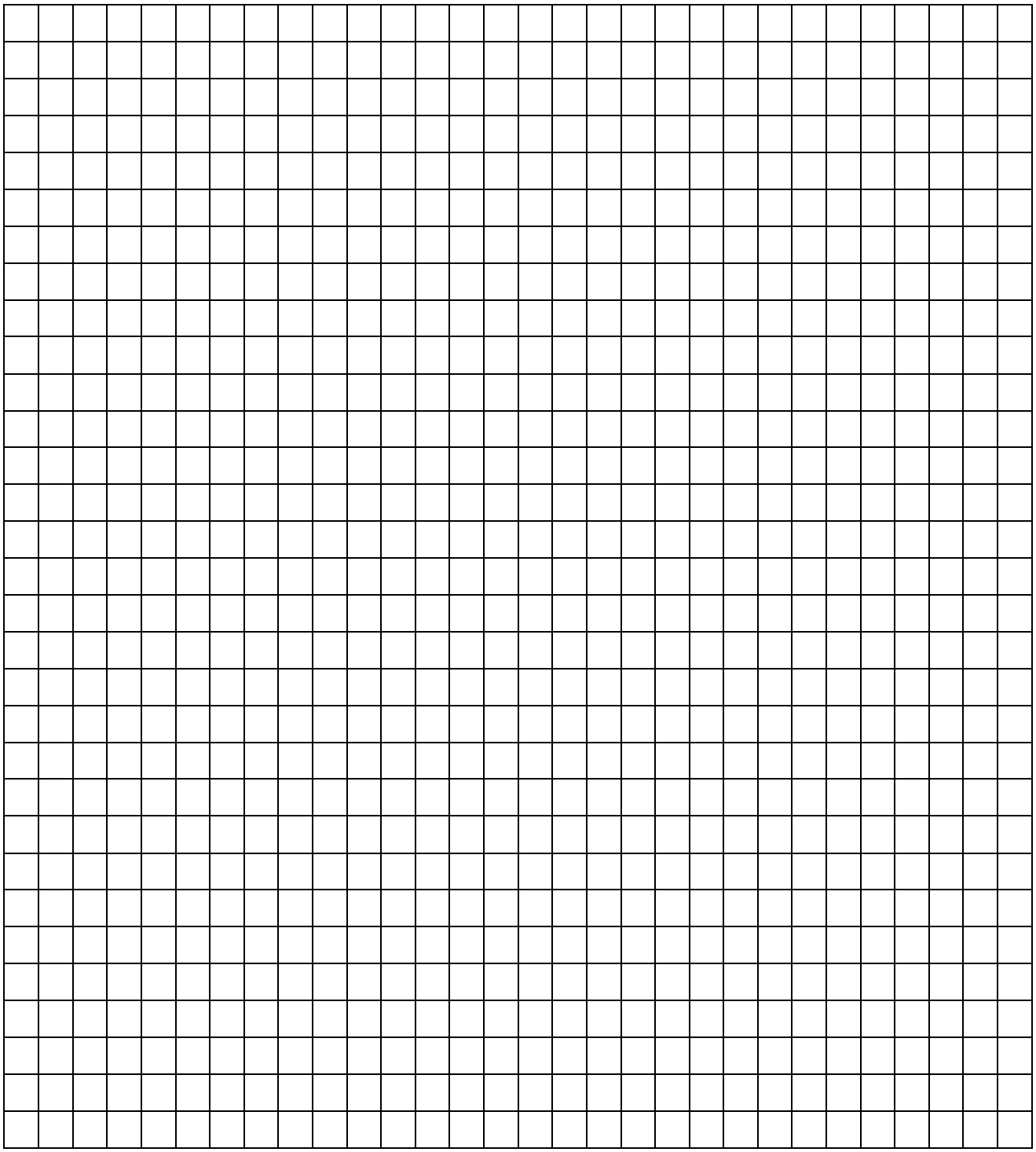
Expression	Symbols	Product	Grid



Show all the ways to have a product of 16. Another way to say this is to show all the different rectangles with 16 squares inside.



Show all the ways to have a product of 24. Another way to say this is to show all the different rectangles with 24 squares inside.



Multiplication facts

I have good news and not-quite so good news.
The not-quite so good news is that you really do have to memorize your multiplication facts. You will need them all the time in math, especially in fractions. The good news is that fractions will be SO much easier for you when you know them.

Getting started:

Let's find out what multiplication facts you do remember. The Multiplication facts check on page 15 can be used for this. Once you know which facts you do recall correctly, you can focus on the ones that are more difficult for you to remember. The following pages will provide strategies for those pesky, but important, facts.

There are different symbols for multiplication.

2 times 3 can be written as 2 x 3 or 2 ∗ 3.

All multiplication facts can be figured out using either pictures or a grid. It just takes longer. Memorizing up to 12's really speeds things up.

Practice 2: The facts I need practice with

0 x 0 =	1 x 0 =	2 x 0 =	3 x 0 =	4 x 0 =	5 x 0 =
0 x 1 =	1 x 1 =	2 x 1 =	3 x 1 =	4 x 1 =	5 x 1 =
0 x 2 =	1 x 2 =	2 x 2 =	3 x 2 =	4 x 2 =	5 x 2 =
0 x 3 =	1 x 3 =	2 x 3 =	3 x 3 =	4 x 3 =	5 x 3 =
0 x 4 =	1 x 4 =	2 x 4 =	3 x 4 =	4 x 4 =	5 x 4 =
0 x 5 =	1 x 5 =	2 x 5 =	3 x 5 =	4 x 5 =	5 x 5 =
0 x 6 =	1 x 6 =	2 x 6 =	3 x 6 =	4 x 6 =	5 x 6 =
0 x 7 =	1 x 7 =	2 x 7 =	3 x 7 =	4 x 7 =	5 x 7 =
0 x 8 =	1 x 8 =	2 x 8 =	3 x 8 =	4 x 8 =	5 x 8 =
0 x 9 =	1 x 9 =	2 x 9 =	3 x 9 =	4 x 9 =	5 x 9 =
0 x 10 =	1 x 10 =	2 x 10 =	3 x 10 =	4 x 10 =	5 x 10 =

Multiplication Facts Check – I need to concentrate on these

6 x 0 =	7 x 0 =	8 x 0 =	9 x 0 =	10 x 0 =
6 x 1 =	7 x 1 =	8 x 1 =	9 x 1 =	10 x 1 =
6 x 2 =	7 x 2 =	8 x 2 =	9 x 2 =	10 x 2 =
6 x 3 =	7 x 3 =	8 x 3 =	9 x 3 =	10 x 3 =
6 x 4 =	7 x 4 =	8 x 4 =	9 x 4 =	10 x 4 =
6 x 5 =	7 x 5 =	8 x 5 =	9 x 5 =	10 x 5 =
6 x 6 =	7 x 6 =	8 x 6 =	9 x 6 =	10 x 6 =
6 x 7 =	7 x 7 =	8 x 7 =	9 x 7 =	10 x 7 =
6 x 8 =	7 x 8 =	8 x 8 =	9 x 8 =	10 x 8 =
6 x 9 =	7 x 9 =	8 x 9 =	9 x 9 =	10 x 9 =
6 x 10 =	7 x 10 =	8 x 10 =	9 x 10 =	10 x 10 =

Practice 3: The Multiplication chart

The Multiplication Chart can help you understand and remember multiplication facts.

1. Color in the multiplication facts you know.

Multiplication Chart

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Practice 4: Multiplication by 0 and 1

Multiplication by 0

3 ∗ 0 means 3 groups of 0. This means 3 groups of nothing. 3 ∗ 0 must = 0.

0 ∗ 3 means 0 groups of 3. This means no groups of 3. 0 ∗ 3 must = 0.

For the general case, where x is any number:

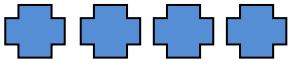
x ∗ 0 means x groups of 0. x ∗ 0 = 0

0 ∗ x means 0 groups of x. 0 ∗ x = 0

Hint: 0 ∗ anything = 0

Multiplication by 1

4 ∗ 1 means 4 groups of 1. 

1 ∗ 4 means 1 group of 4. 

For the general case, where x is any number, x ∗ 1 = x

And 1 ∗ x = x.

a. $1 * 4 =$

b. $4 * 0 =$

c. $8 * 1 =$

d. $7 * 1 =$

e. $0 * 6 =$

f. $5 * 0 =$

g. $4 * 1 =$

h. $3 * 1 =$

i. $1 * 2 =$

j. $1 * 0 =$

k. $0 * 1 =$

l. $1 * 1 =$

n. $845 * 0 =$

p. $0 * 43 =$

q. $1 * 87 =$

r. $987 * 0 =$

s. $67 * 1 =$

t. $9 * 0 =$

u. $0 * 54 =$

v. $0 * z =$

w. $y * 1 =$

x. any number $* 0 =$

z. $1 *$* any number =

aa. $1 * 23 =$

Practice 5: Multiplication by 2

Let's look at some multiplication facts with the hundreds number chart.

There is a pattern.

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Notice that all products with 2 as a factor end in : 0, 2, 4. 6, 8

Vocabulary note: In the equation $2 * 3 = 6$, 2 and 3 are factors, and 6 is the product.

If we look at all the cases where 2 is a factor, this is what the multiplication chart looks like:

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Since 2 is a factor in every case, the same pattern of all products ending in 0, 2, 4, 6 or 8 is still true.

Practice 6: Multiplication by 3

There is a weird fact about the product when 3 is a factor. All the digits in the product add to a number that is evenly divisible by 3.

I may be the Math Whisperer, but I don't know why this is true. I just know that it is.

Note: If this seems too complicated for you for now, just skip it

Here is an example: $3 * 5 = 15$

Adding the digits in the 15, 1 + 5 = 6, which is evenly divisible by 3.

Here is a counter-example: $7 * 5 = 35$. 3 + 5 = 8, which is not evenly divisible by 3.

Here is a more complicated example: $12 * 5 = 60$. 3 is a factor in 12, so we could re-write this:

$12 * 5 =$ $3 * 4$ $* 5$ $= 60$

and the digits in the product of 60 add to 6, (6 to = 6) which is evenly divisible by 3.

Reminder: In $3 * 4 = 12$ 3 and 4 are the factors and 12 is the product.

factors and product	sum of digits in product	evenly divisible by 3
$3 * 4 = 12$	$1 + 2 = 3$	yes
$3 * 8 =$		
$3 * 7 =$		
$9 * 3 =$		
$11 * 3 =$		
$11 * 6 =$		
$4 * 6 =$		
$1 * 3 =$		
$7 * 5 =$		
$7 * 6 =$		
$6 * 5 =$		
$2 * 12 =$		
$2 * 18 =$		
$2 * 15 =$		
$9 * 5 =$		
$9 * 10 =$		
$4 * 7 =$		
$6 * 6 =$		
$10 * 4 =$		

Practice 7: Multiplication by 5

The multiplication chart shows where 5 is a factor in blue and where $10 = 2 * 5$ is a factor in purple.

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

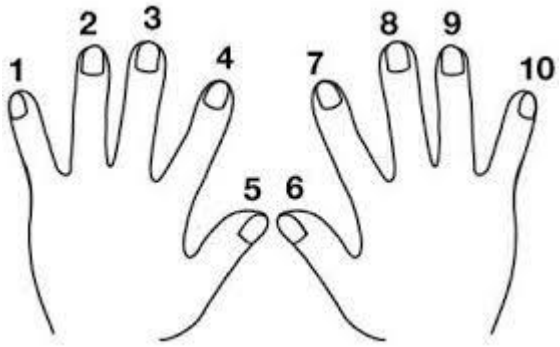
The pattern to notice is that whenever 5 is a factor, the product ends in either 0 or 5.

Practice 8: Multiplication by 9

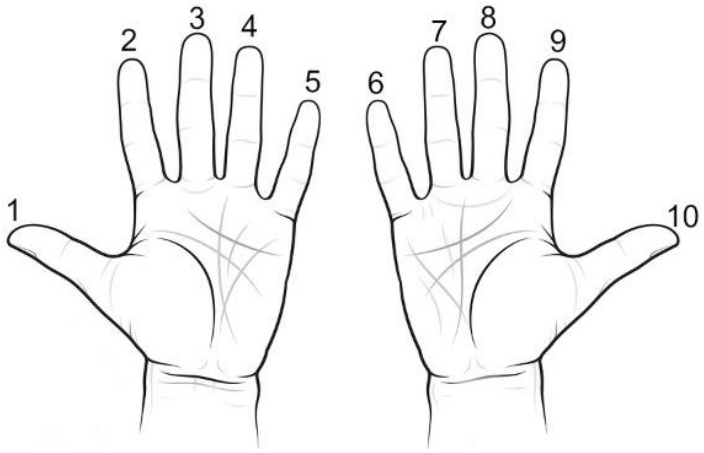
There is a trick using your fingers to quickly find any multiplication by 9, up to $9 * 9$. It's better to memorize, but to be honest, 9's can be tough. It's good to have a back-up plan!

To see how this works, hold your hands the way that is most comfortable for you.

This is my favorite way:



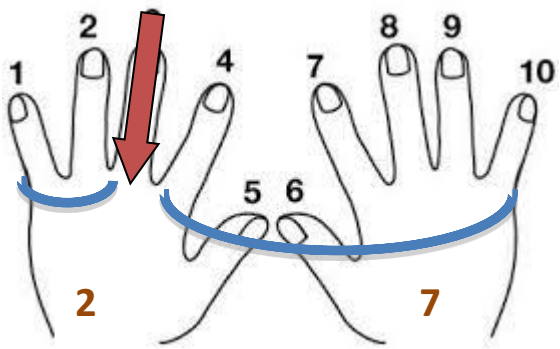
This way works too:



Here is how fingers show the multiplication by 9 facts:

This is 3 * 9 = 27

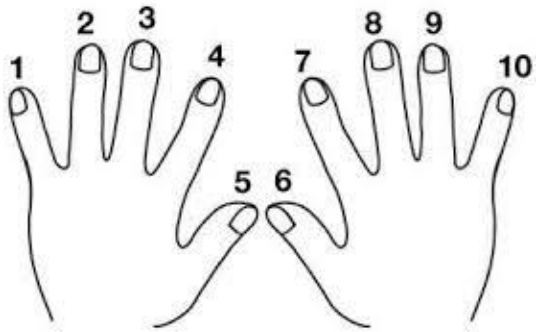
(graphic shows third finger bent down, with arrow between the 3 in 3 * 9 and the third finger. The answer is read left to right, as **27.**



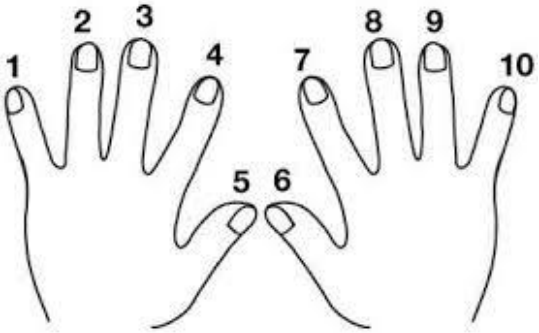
The left fingers are the tens place, and all fingers to the right are ones place.

Show the following multiplication facts with your fingers and draw what you see.

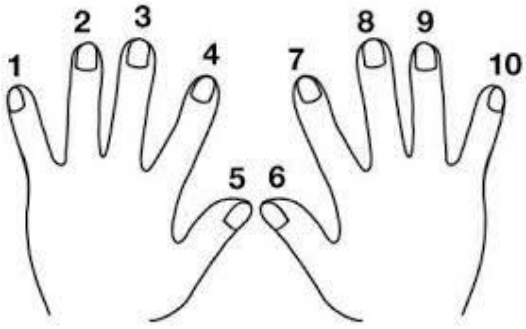
2 * 9



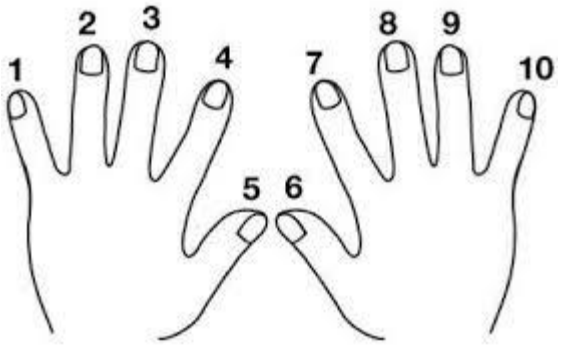
4 * 9



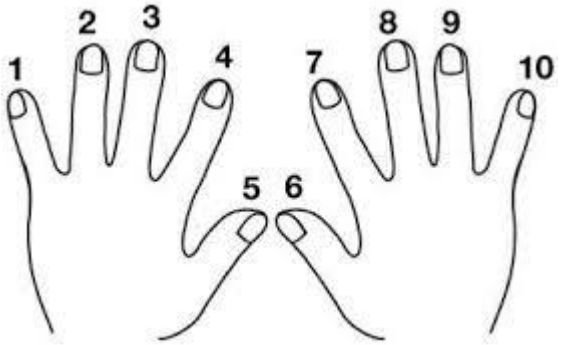
9 * 5



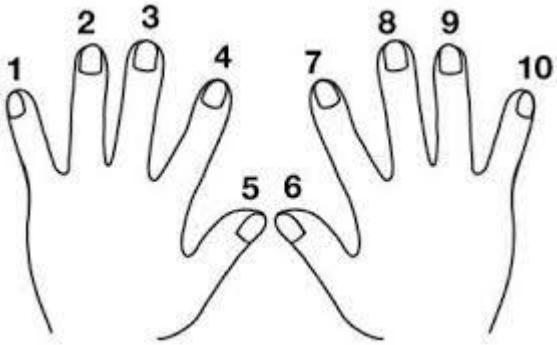
9 * 6



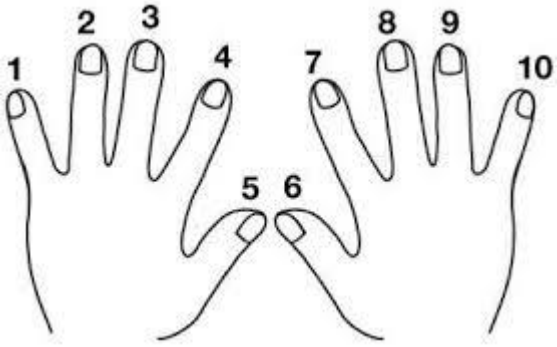
7 * 9



9 * 8



9 * 9



Practice 9: Multiplication by 10

Here are some multiplication facts with 10:

$1 * 10 = 10$

$2 * 10 = 20$

$3 * 10 = 30$

Do you see the pattern? All you have to do is to put a "0" on the one's place.

a. $4 * 10 =$
b. $5 * 10 =$
c. $6 * 10 =$
d. $7 * 10 =$
e. $8 * 10 =$
f. $9 * 10 =$
g. $10 * 10 =$
h. $72 * 10 =$
i. $70 * 10 =$
j. $114 * 10 =$
k. $86 * 10 =$
l. $90 * 10 =$
m. $999 * 10 =$
n. $1000 * 10 =$
o. $40 * 10 =$
p. $41 * 10 =$

Practice 10: Multiplication using a ∗ b = b ∗ a

Can you use a ∗ b = b ∗ a to remember multiplication facts you have trouble with?

For example, if you have trouble with 8 ∗ 2, you may be fine with 2 * 8. So you can switch 8 ∗ 2 to 2 ∗ 8.

A. List multiplication facts that you can use a ∗ b = b ∗ a to help you remember:

B. Find the products. You may want to use a ∗ b = b ∗ a to find them. It's up to you!

a. 8 ∗ 2 =

b. 9 ∗ 3 =

c. 7 ∗ 4 =

d. 6 ∗ 3 =

e. 8 ∗ 4 =

f. 7 ∗ 5 =

g. 9 ∗ 2 =

h. 3 ∗ 7 =

i. 9 ∗ 4 =

j. 9 ∗ 6 =

k. 6 ∗ 2 =

l. 5 ∗ 8 =

m. 5 ∗ 3 =

n. 9 ∗ 7 =

o. 8 ∗ 5 =

p. 7 ∗ 3 =

q. 5 ∗ 4 =

r. 8 ∗7 =

s. 8 ∗ 6 =

t. 6 ∗ 7 =

u. 7 ∗ 8 =

v. 9 ∗ 5 =

w. 6 ∗ 5 =

x. 4 ∗ 3 =

y. 8 ∗ 3 =

z. 7 ∗ 6 =

Practice 11: Mixed Multiplication facts practice

Fact	How do you know you are right?

a. $2 * 7 =$

b. $3 * 7 =$

c. $5 * 4 =$

d. $2 * 10 =$

e. $9 * 5 =$

f. $7 * 10 =$

The connection between Multiplication and Addition

Multiplication and addition are related. Both are ways to put things together and count them.

Here is an example: $3 * 5$ means 3 groups of five

This is the same as repeated addition :

Multiplication is more efficient. That means it takes less time to multiply than to do repeated addition.

With the example of $3 * 5$, if you have memorized that $3 * 15 = 15$, that is one step.

If you add repeatedly, you do $5 + 5 = 10$, $10 + 5 = 15$, and that is two steps.

So it is one step quicker to multiply here.

Practice 12: Multiplication is more efficient than Addition

Practice: Compare multiplication with repeated addition : $8 * 7 = ?$

Either count the steps for each part, or time yourself.

A. Do you remember the multiplication fact $8 * 7$?

Let's hope so: $8 * 7 =$ _____

Number of steps _____steps

Or time to solve _____seconds

B. Repeated addition of 8 "7"s

$7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 =$

Number of steps _____steps

Or time to solve _____seconds

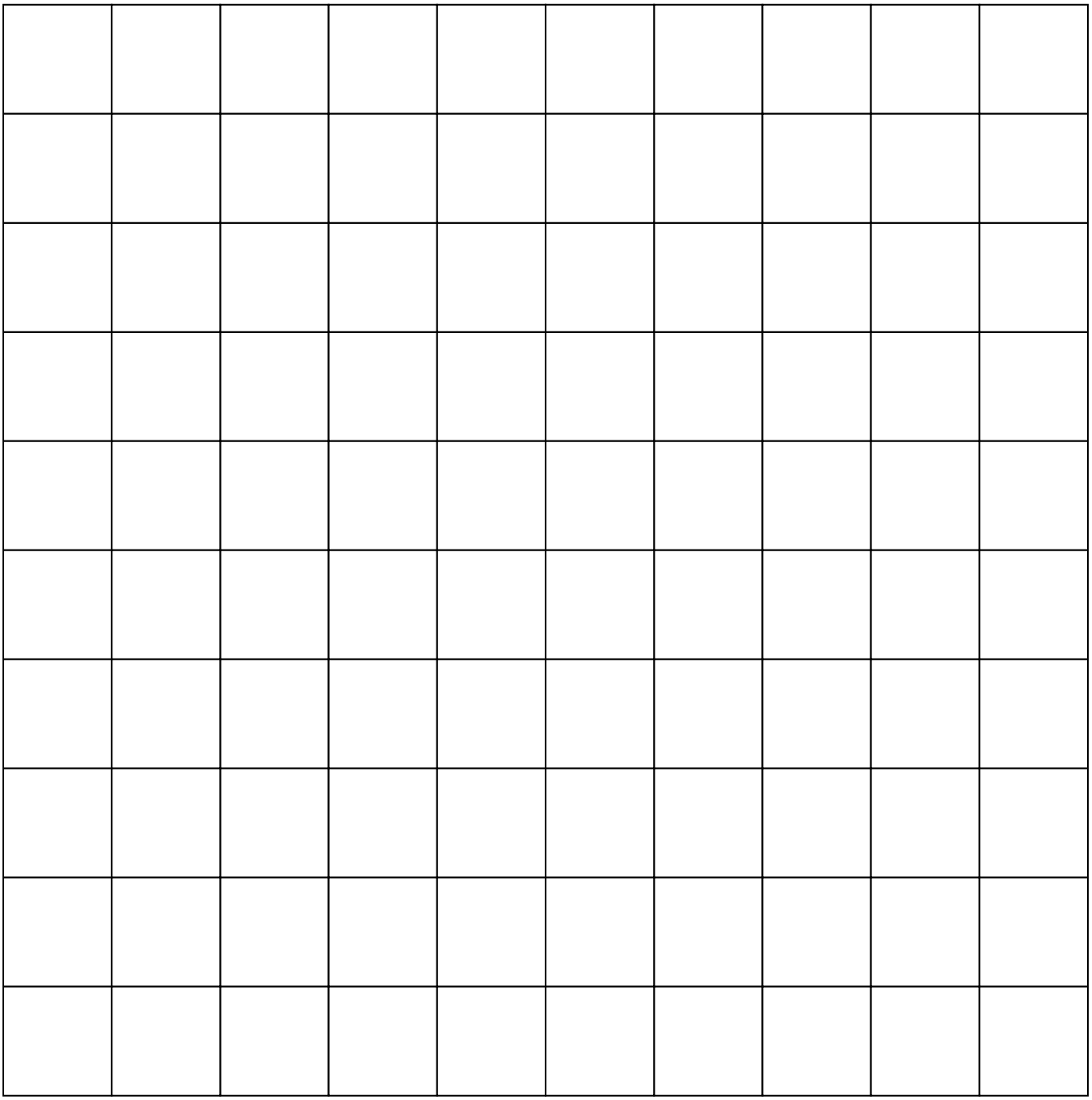
C. Repeated addition of 7 "8"s (using $a * b = b * a$)

$8 + 8 + 8 + 8 + 8 + 8 + 8 =$

Number of steps _____steps

Or time to solve _____seconds

Check your answer: by creating a rectangle to show $7 * 8$



Symbols for Multiplication

It is very important for you to move away from the symbol “x” for “times”, because x is also the standard variable for algebra. Other symbols will be • and parentheses and having variables next to each other.

Example: These all mean 2 times 3:	Symbols
2 • 3	•
2 * 3	*
(2) (3)	( ) ( )
2 (3)	x ( )
(2) 3	( ) *

And these all mean a times b:

a • b	•
a * b	*
ab	
(a) (b)	( ) ( )
a (b)	x ( )
(a) b	( ) *

Practice 13: Multiplication Symbols

Write down all the symbols you can think of to show the following multiplication statements.

Remember that the symbols are read from left to right. So “a groups of b” could be written a • b.

1. 3 groups of 2

2. 4 groups of 5

3. 3 groups of 7

4. 3 groups of x

5. 5 groups of 2

6. w groups of 2

7. x groups of y

Practice 14: More symbols for Multiplication

Find the products.

1. $1 \bullet 4 =$
2. $(5)\ (8) =$
3. $4 * 5 =$
4. $7 * 6 =$
5. $3\ (7) =$
6. $(2)\ (8) =$
7. $7 * 8 =$
8. $9 * 3 =$
9. $5 \bullet 3 =$
10. $7 \bullet 2 =$
11. $4 * 9 =$
12. $7\ (4) =$
13. $(6)\ (6) =$
14. $7 \bullet 8 =$
15. $0 * 7 =$
16. $3 \bullet 4 =$
17. $(4)\ (9) =$
18. $2 * 9 =$
19. $9 \bullet 0 =$
20. $8 * 6 =$
21. $6\ (2) =$
22. $(7)\ (5) =$
23. $3 \bullet 9 =$
24. $2 * 3 =$
25. $4 \bullet 8 =$
26. $3 * 7 =$
27. $4\ (4) =$
28. $(5)\ (6) =$
29. $3 \bullet 8 =$
30. $7 * 2 =$

Two Digit by Two Digit Multiplication

Using the concept of multiplication, that "a • b" means "a groups of b," the problem 24 x 13 can be seen as 24 groups of 13.

If you were taught how to solve multiplication problems in an American school, the chances are 99.9% that you learned the following method:

```
  13
x 24
----
  52
+260
----
 312
```

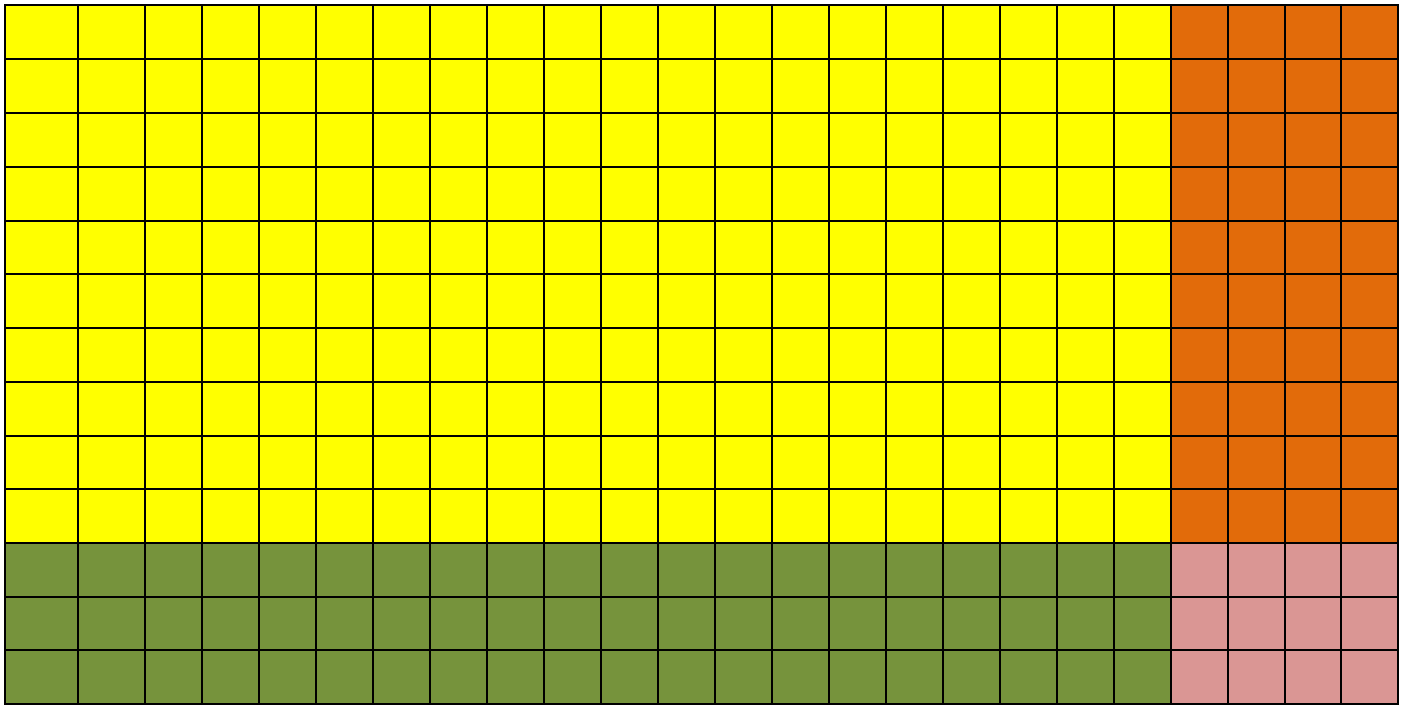
Multiply the 4 times the 13 = 52. Then put a "0" for a place holder, then multiply the 2 times the 13 (get 26) and add the two products

The above method is efficient, and many students are able to remember it. However, a reasonable percentage of students forget the "0" place holder and try to solve it this way:

```
  13
x 24
----
  52
+ 26
----
  78
```

This is an example of not understanding to put in a "0" place holder

The French algorithm for multiplication is very helpful for students who have trouble with the place holder concept. Another name for the French method is the method of partial products. Here is how the French multiply, and we will refer to the picture of “24 groups of 13.”



x	24	
	13	
	200	1 ten times 2 tens, these are the yellow squares
	40	3 ones times 2 tens, these are the green squares
	60	1 ten times 4 ones, these are the orange squares
+	12	3 ones times 4 ones, these are the pink squares
	312	

Notice our efficient American algorithm combines the first two steps, 200 + 60 = 260. Second step 40 + 12 = 52.

Because the French algorithm is based on place value, it offers you an understanding of what the numbers mean.

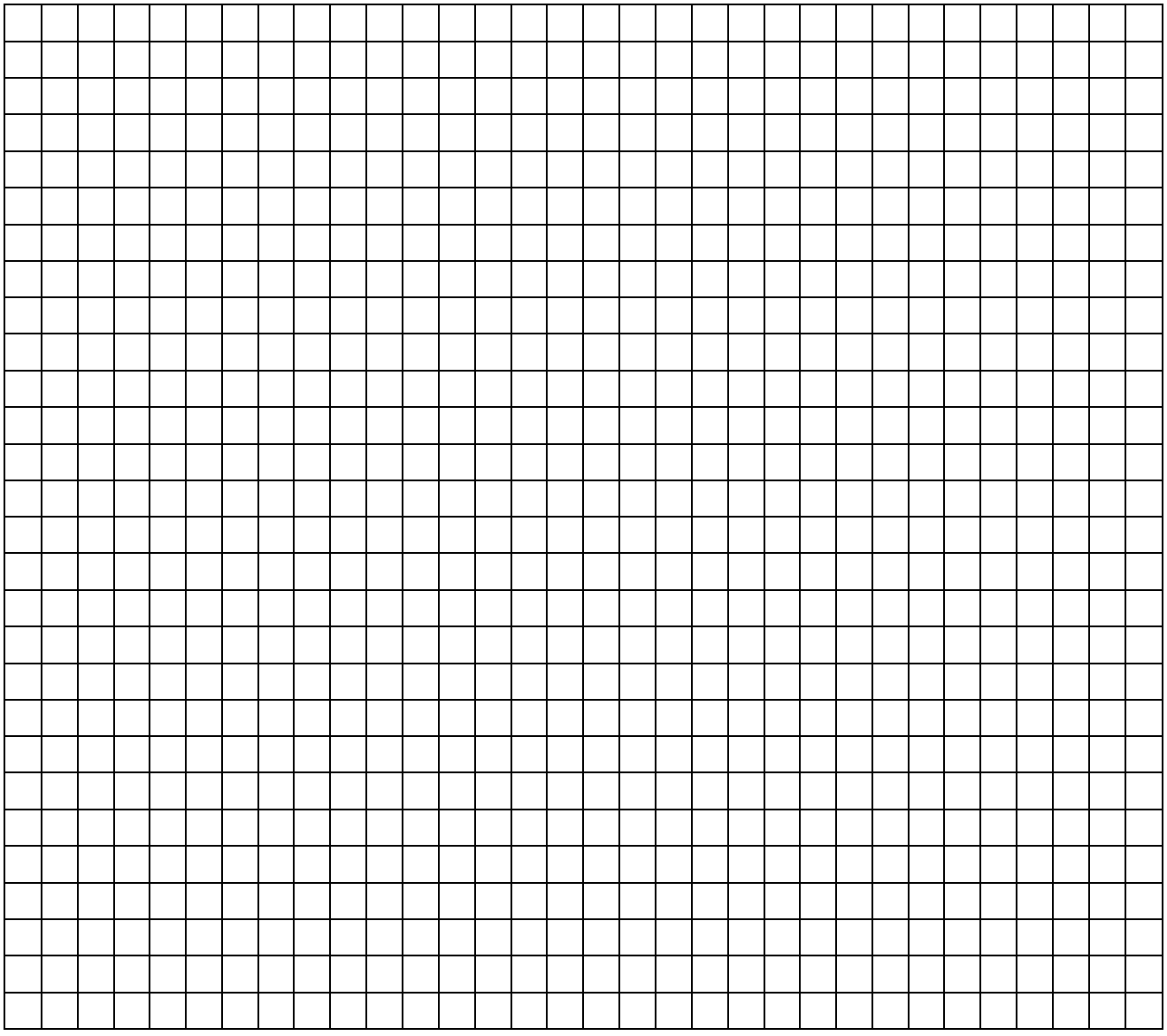
Practice 15: Two Digit by Two Digit

Find the product by using the French method and also with the graph paper.

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline \end{array}$$

_____	_____	tens	times	_____	tens
_____	_____	tens	times	_____	ones
_____	_____	ones	times	_____	tens
_____	_____	ones	times	_____	ones,

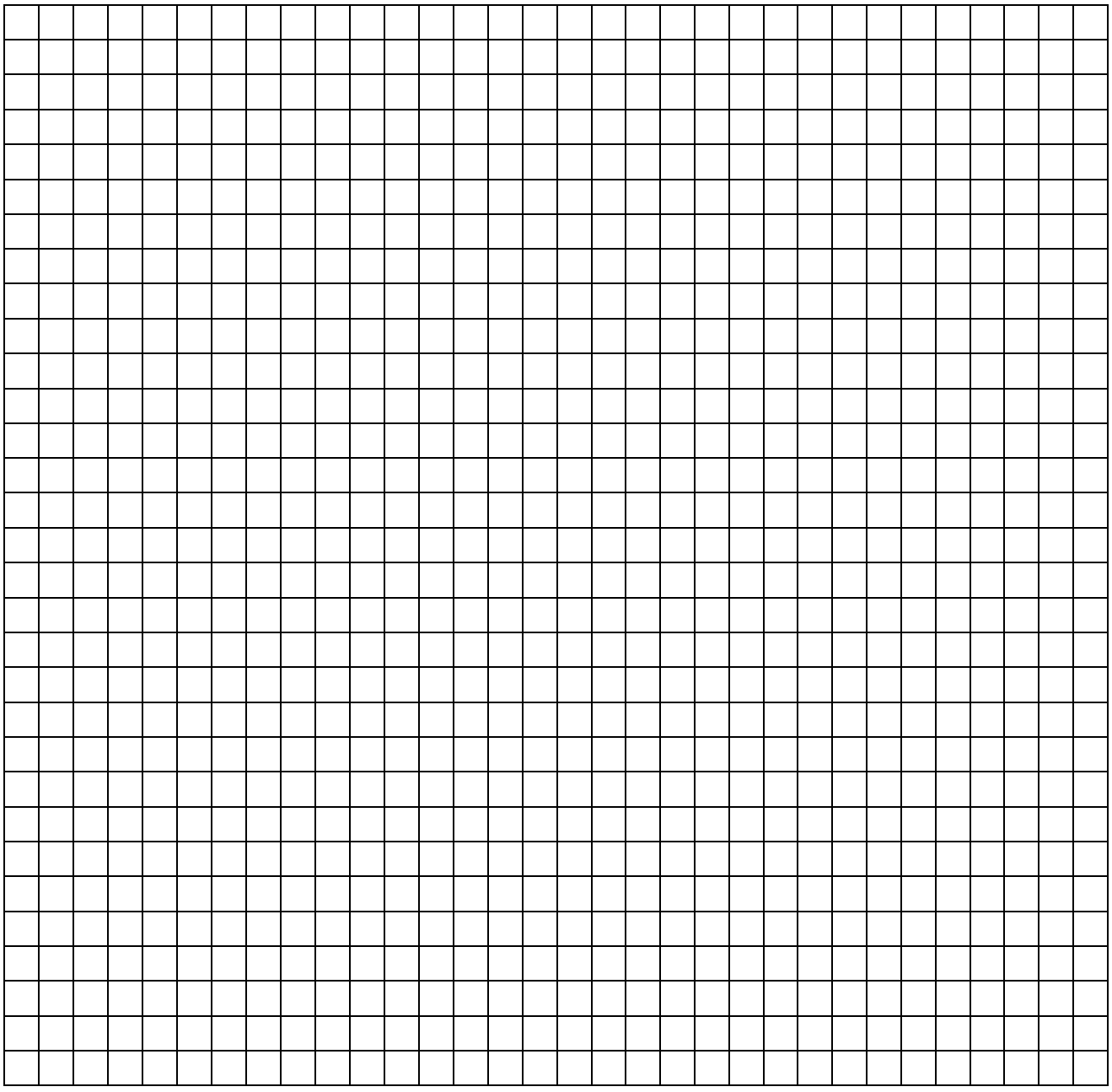
The product



$$\begin{array}{r} 25 \\ \times\ 13 \\ \hline \end{array}$$

_____	_____	tens	times	_____	tens
_____	_____	tens	times	_____	ones
_____	_____	ones	times	_____	tens
_____	_____	ones	times	_____	ones,

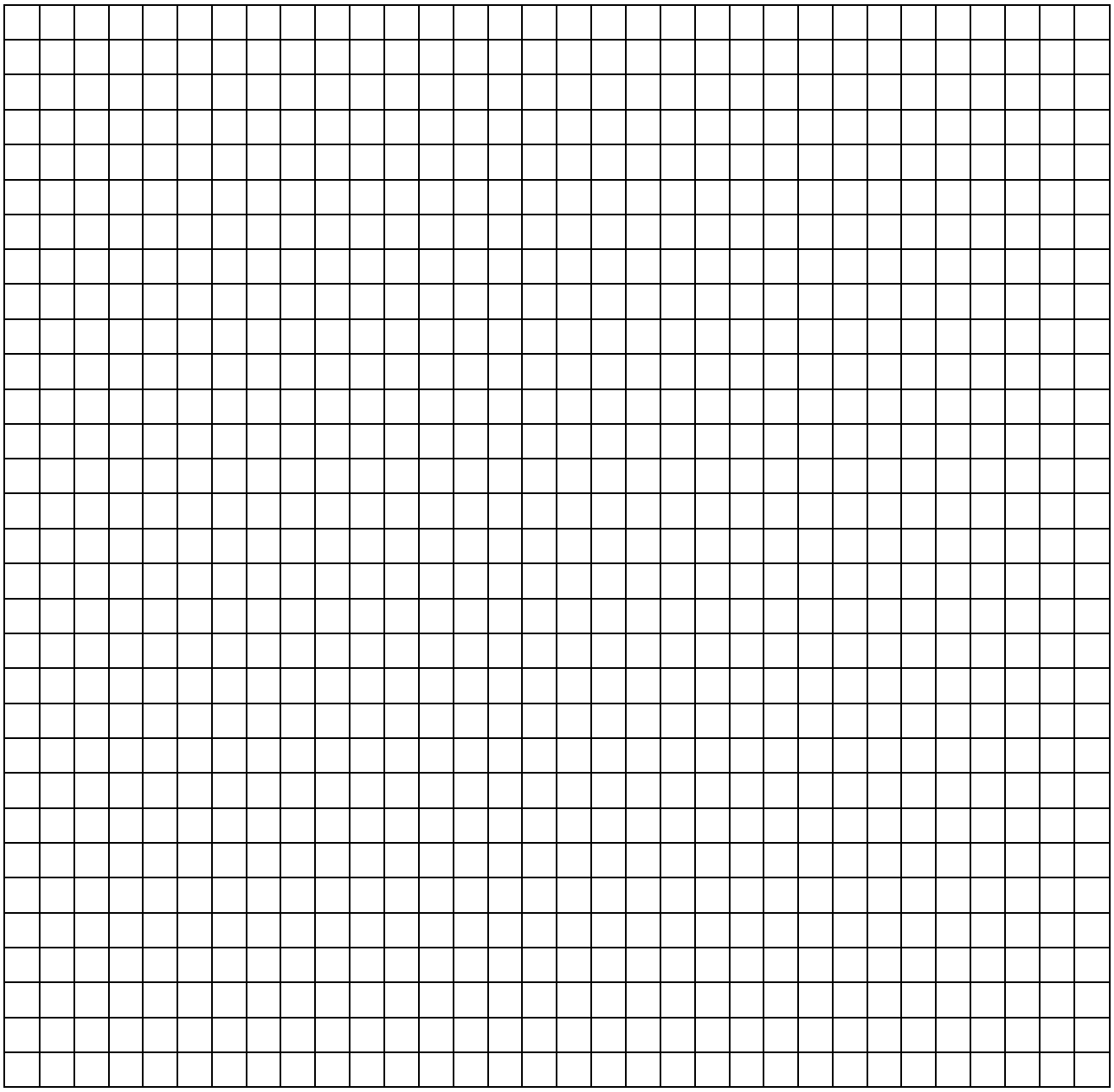
The product



$$\begin{array}{r} 31 \\ \times\ 15 \\ \hline \end{array}$$

_____	_____	tens	times	_____	tens
_____	_____	tens	times	_____	ones
_____	_____	ones	times	_____	tens
_____	_____	ones	times	_____	ones,

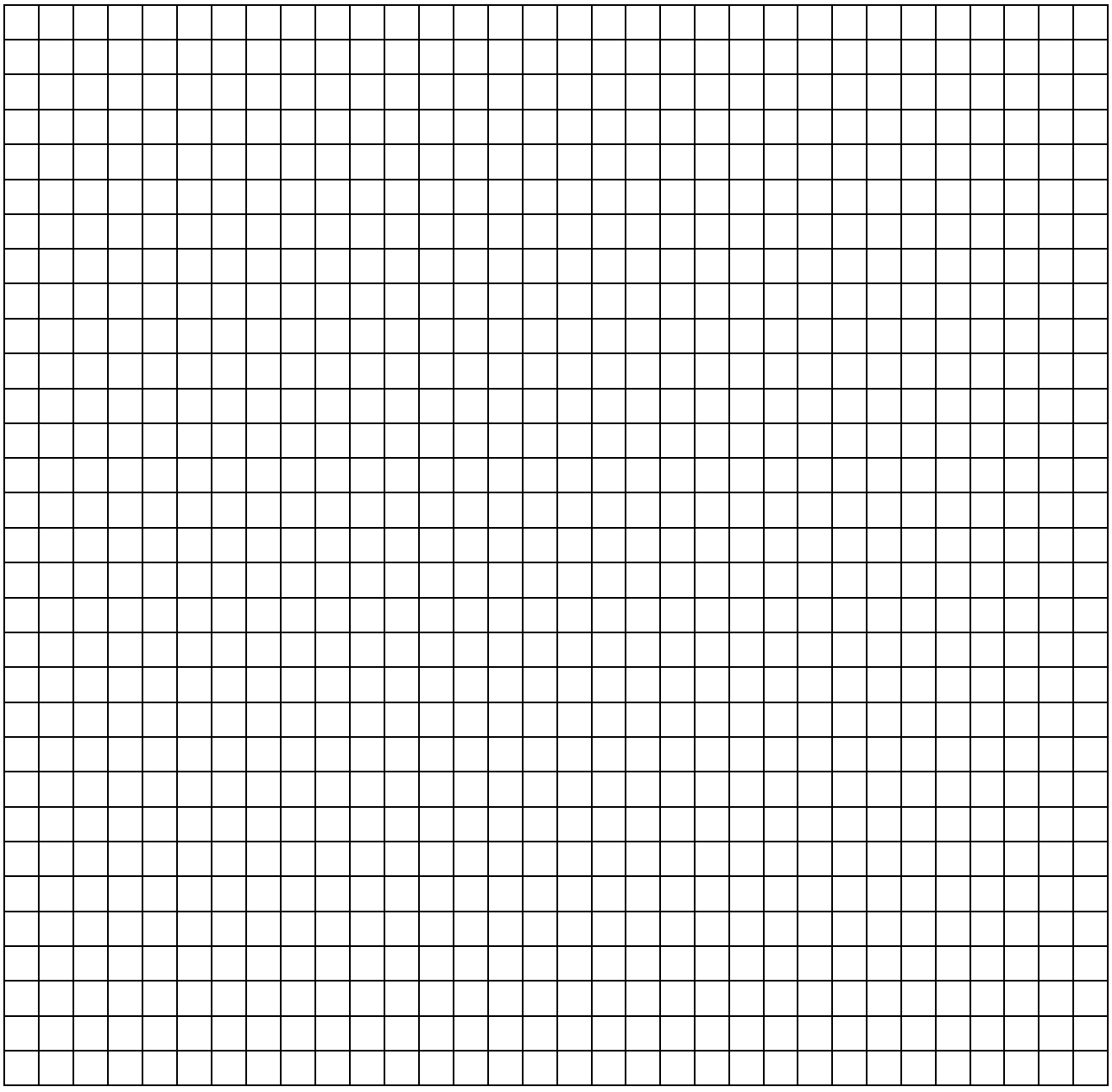
The product



18
x 17

_____ _____ tens times _____ tens
_____ _____ tens times _____ ones
_____ _____ ones times _____ tens
_____ _____ ones times _____ ones,

The product



Practice 16: Multiplication with units/labels

For the multiplications shown below, supply what there are 12 of.
Multiplication A is completed as an example.

Some of these might be too challenging, but do your best. You will be able to them all at some point.

A. 3 * 4 blocks totals 12 blocks

B. 3 * 4 apples totals _____

C. 3 apples * 4 totals _____

D. 3 * 4 feet totals _____

E. 3 * 4 fifths totals _____

F. 3 fourths * 4 totals _____

G. 3 feet * 4 feet totals _____

H. 3 fifths * 4 fifths totals _____

I. 3 apples * 4 apples totals _____

J. 3 apples * 4 oranges totals _____

K. 3 fifths * 4 feet totals _____

L. 3 feet per second * 4 seconds totals _____

M. 3 feet per second * 4 feet totals_____

Now look back at your answers. Which of these makes sense?

Here is what is amazing: The numbers multiply and so do the units. It's like you have two multiplication problems. For problem A, you multiply the numbers. For problem G, you multiply the units.

This turns out to be very very powerful in many places. It can help you:

With fractions
With measurements
All over the place in science, especially chemistry
And more!!!

Practice 17: Multiplication by tens and scientific notation

You will need a calculator to check your answers

Use a calculator to do these multiplications:

$7 * 100 =$

$7 * 1{,}000 =$

$7 * 10{,}000 =$

$7 * 100{,}000 =$

$5 * 100 =$

$5 * 1{,}000 =$

$5 * 10{,}000 =$

$5 * 100{,}000 =$

$70 * 200 =$

$700 * 200 =$

$7{,}000 * 200 =$

$7{,}000 * 2{,}000 =$

Example:

There is a way to keep track of all the zeros using “powers of ten.” This is a symbolic way to keep easy track of all the zeros.

Here are some examples:

$10 = 10^1$
$100 = 10^2$
$1{,}000 = 10^3$
$10{,}000 = 10^4$

$1{,}000{,}000 = 10^6$ one million
$1{,}000{,}000{,}000 = 10^9$ one billion

Using exponents is such an easier way to keep track of the zeros. It is like an accordion.

one million = 1,000,000

10^6

We need a bit of vocabulary to go farther. Here is an example:

$100 = 10^2$

The "2" is an "exponent". It is smaller than the 1 and the 0, and goes above.

Here is the important point from all of this:

The number of zeros on the right hand side that show place value is equal to the exponent of the ten.

Looking at some examples, this is the case.

$10 = 10^1$ 1 zero

$100 = 10^2$ 2 zeros

$1{,}000 = 10^3$ 3 zeros

$10{,}000 = 10^4$ 4 zeros

$100{,}000 = 10^5$ 5 zeros

$1{,}000{,}000 = 10^6$ 6 zeros

…

$1{,}000{,}000{,}000 = 10^9$ 9 zeros

Based on these examples, I hope you can use the fact that the number of zeros on the right hand side, that show place value, is equal to the exponent of the ten.

This is not a formal proof like a mathematician would do. Our goal is to be able to use this to make multiplication easier for you.

What this says: $1{,}000 = 10^3$ 3 zeros This is one thousand

What this does not say: 10,005 **does not =** 10^3 This is ten thousand and five.

The zeros have to all be on the right hand side and show place value.

This is read as "10 to the second power" in **formal** language.

Informally we say:

10^2	10 squared	2 is the exponent
10^3	10 cubed	3 is the exponent
10^4	10 to the fourth	4 is the exponent
10^5	10 to the fifth	5 is the exponent
one million = 10^6	10 to the sixth	6 is the exponent
one billion = 10^9	10 to the ninth	9 is the exponent

factors	product	number of zeros in factor	number of zeros in product
$7 * 100$			
$7 * 1{,}000$			
$7 * 10{,}000$			
$7 * 100{,}000$			
$5 * 100$			
$5 * 1{,}000$			
$5 * 10{,}000$			
$5 * 100{,}000$			
$70 * 200$			
$70 * 2{,}000$			
$700 * 200$			
$7{,}000 * 2{,}000$			
$50 * 700$			
$500 * 700$			
$5{,}000 * 7{,}000$			
$50 * 2$			
$50 * 20$			

Multiplying with exponents:
If you are dealing with large numbers, multiplying with exponents can make your life a lot easier.

Examples:

$1{,}000 * 100 = 10^3 * 10^2 = 10^5$

$1{,}000{,}000 * 1{,}000 = 10^9 * 10^3 = 10^{12}$

Notice that you just add the exponents!!

In general $10^a * 10^b = 10^{(a+b)}$

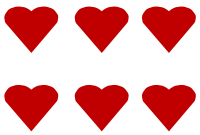
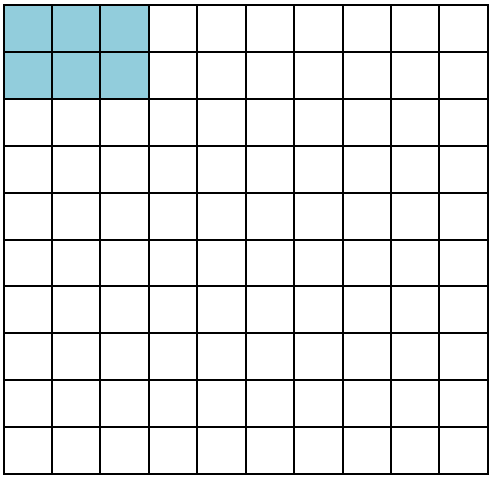
Solutions / Answers

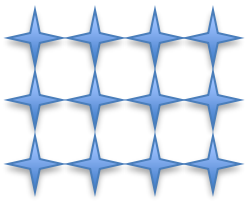
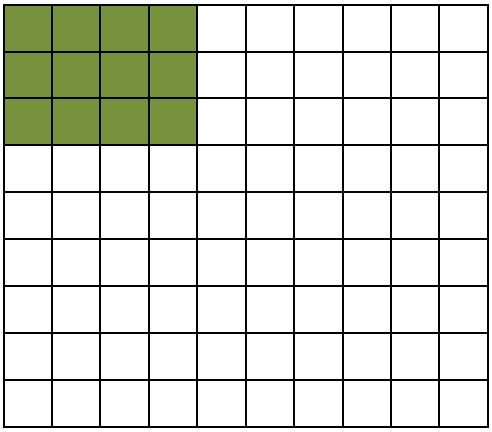
MATH WHISPERER

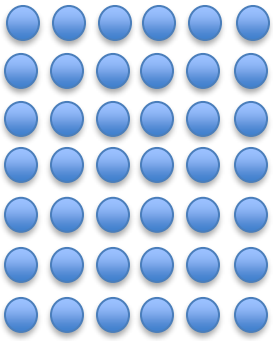
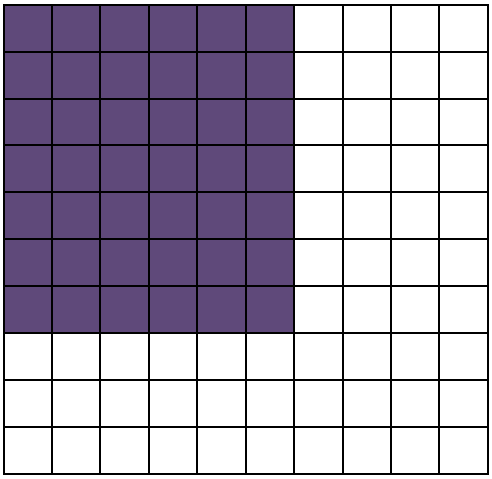
Where math makes sense

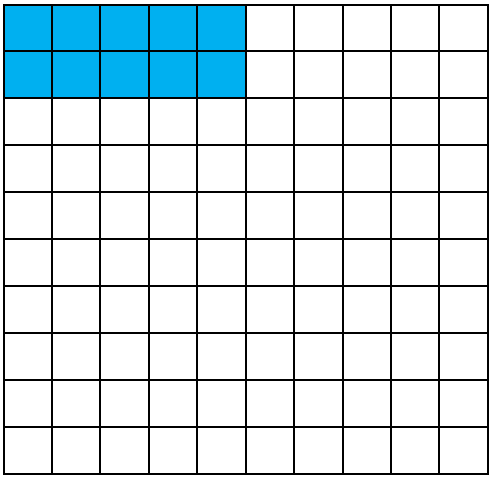
Practice 1: Multiplication means “groups of”

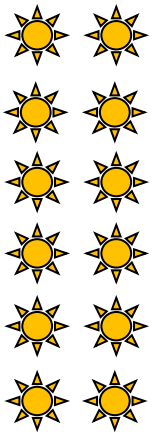
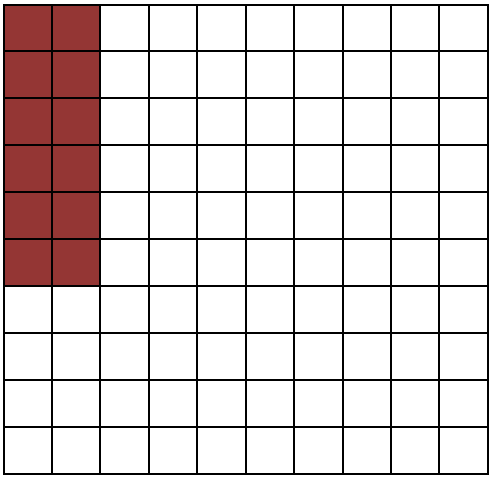
Now it’s your turn:

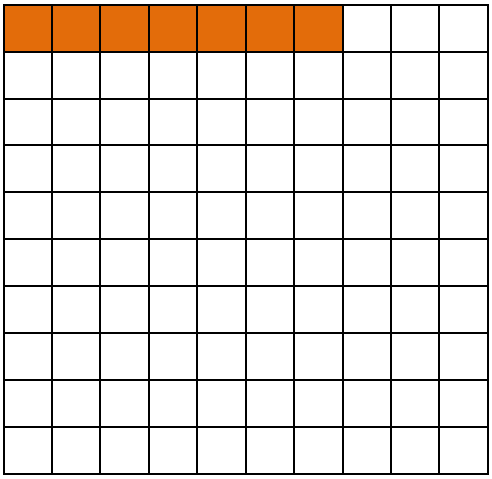
Expression	Symbols	Product	Grid
2 * 3		6	

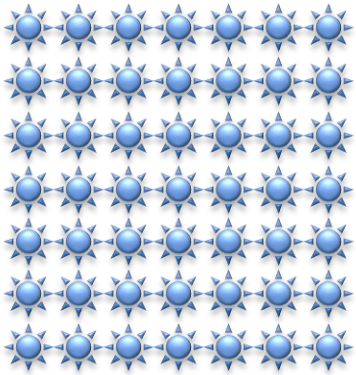
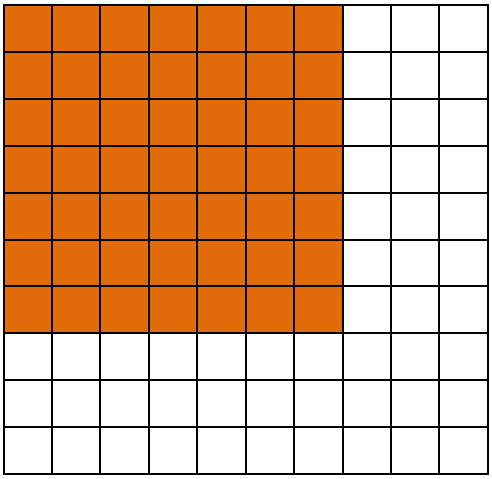
Expression	Symbols	Product	Grid
3 * 4		12	

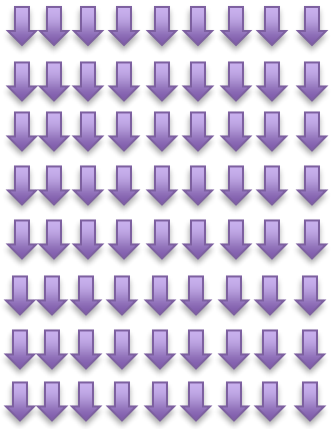
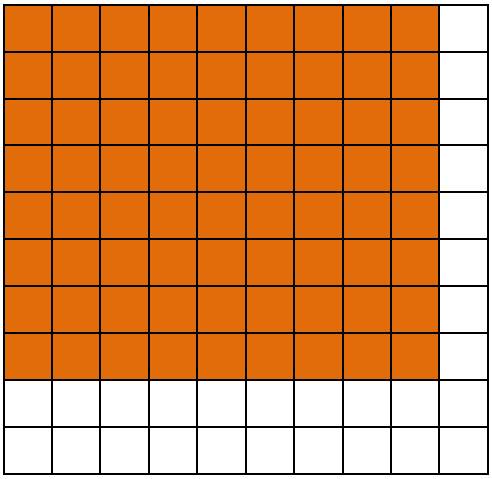
Expression	Symbols	Product	Grid
6 * 7		42	

Expression	Symbols	Product	Grid
5 * 2		10	

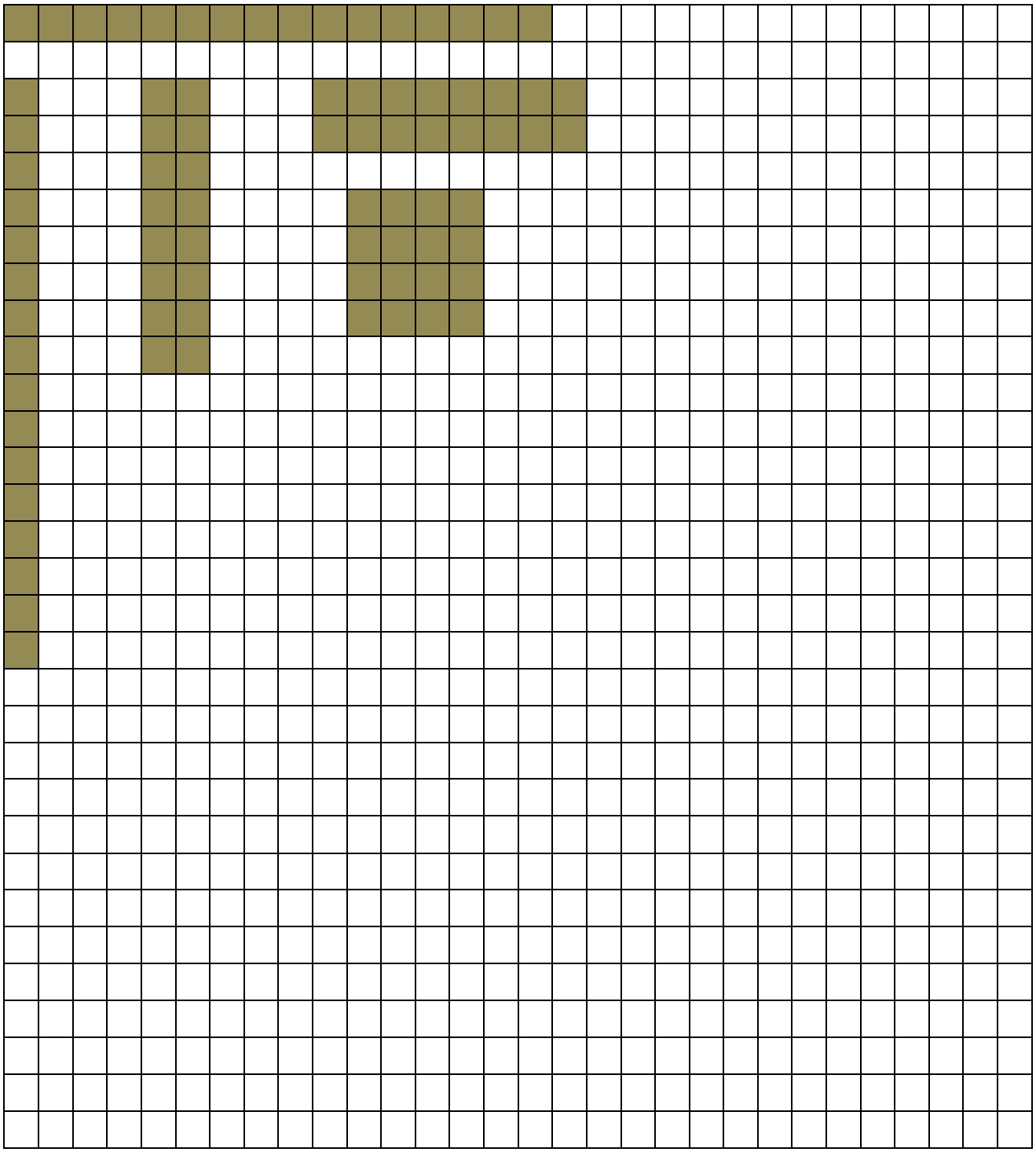
Expression	Symbols	Product	Grid
2 * 6			

Expression	Symbols	Product	Grid
7 * 1		7	

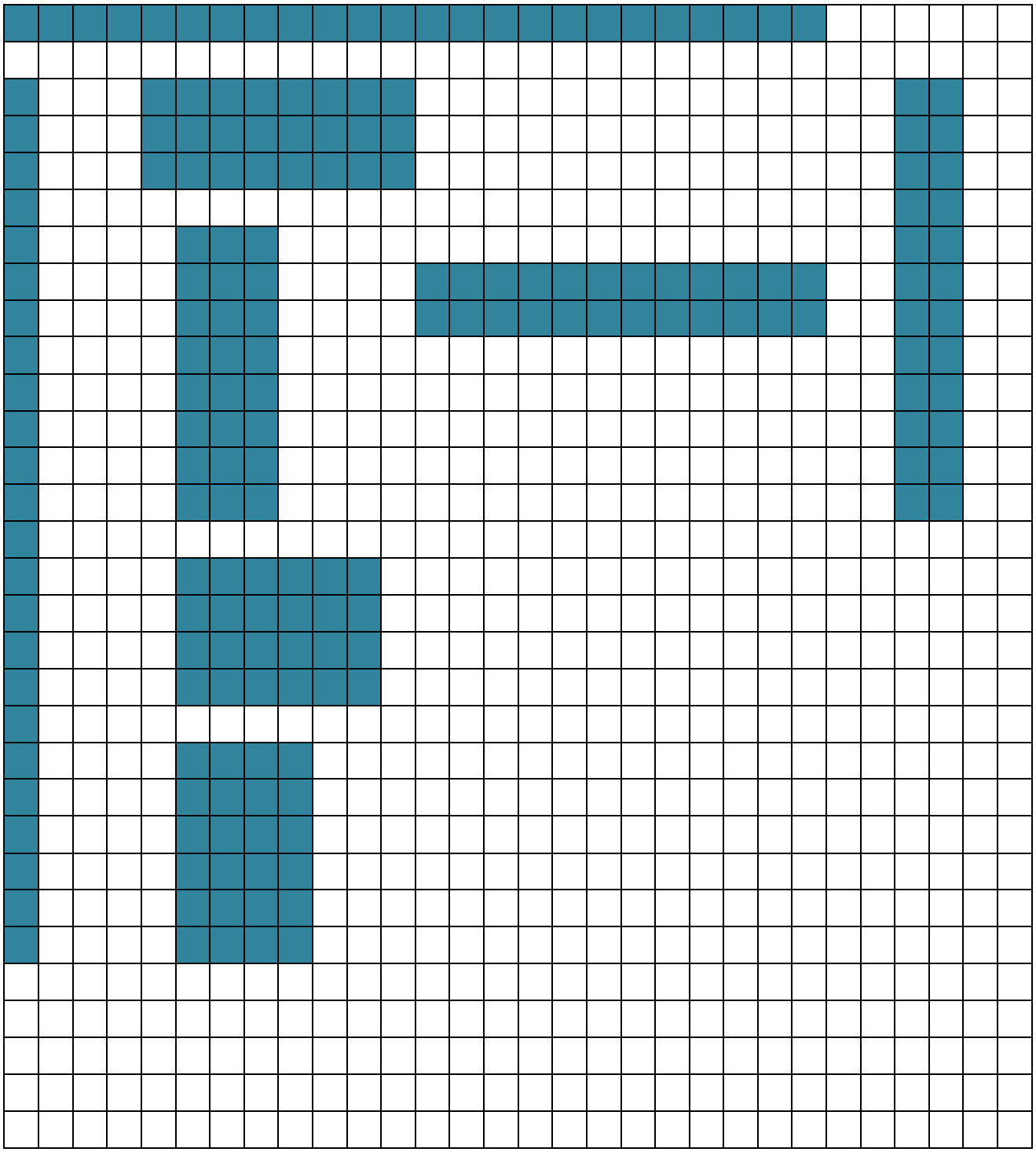
Expression	Symbols	Product	Grid
7 * 7		49	

Expression	Symbols	Product	Grid
9 * 8		72	

Show all the ways to have a product of 16. Another way to say this is to show all the different rectangles with 16 squares inside.



Show all the ways to have a product of 24. Another way to say this is to show all the different rectangles with 24 squares inside.



Practice 2: The facts I need practice with

0 x 0 = 0	1 x 0 = 0	2 x 0 = 0	3 x 0 = 0	4 x 0 = 0	5 x 0 = 0
0 x 1 = 0	1 x 1 = 1	2 x 1 = 2	3 x 1 = 3	4 x 1 = 4	5 x 1 = 5
0 x 2 = 0	1 x 2 = 2	2 x 2 = 4	3 x 2 = 6	4 x 2 = 8	5 x 2 = 10
0 x 3 = 0	1 x 3 = 3	2 x 3 = 6	3 x 3 = 9	4 x 3 = 12	5 x 3 = 15
0 x 4 = 0	1 x 4 = 4	2 x 4 = 8	3 x 4 = 12	4 x 4 = 16	5 x 4 = 20
0 x 5 = 0	1 x 5 = 5	2 x 5 = 10	3 x 5 = 15	4 x 5 = 20	5 x 5 = 25
0 x 6 = 0	1 x 6 = 6	2 x 6 = 12	3 x 6 = 18	4 x 6 = 24	5 x 6 = 30
0 x 7 = 0	1 x 7 = 7	2 x 7 = 14	3 x 7 = 21	4 x 7 = 28	5 x 7 = 35
0 x 8 = 0	1 x 8 = 8	2 x 8 = 16	3 x 8 = 24	4 x 8 = 32	5 x 8 = 40
0 x 9 = 0	1 x 9 = 9	2 x 9 = 18	3 x 9 = 27	4 x 9 = 36	5 x 9 = 45
0 x 10 = 0	1 x 10 = 10	2 x 10 =	3 x 10 = 30	4 x 10 = 40	5 x 10 = 50

Multiplication Facts Check – I need to concentrate on these

6 x 0 = 0	7 x 0 = 0	8 x 0 = 0	9 x 0 = 0	10 x 0 = 0
6 x 1 = 6	7 x 1 = 7	8 x 1 = 8	9 x 1 = 9	10 x 1 = 10
6 x 2 = 12	7 x 2 = 14	8 x 2 = 16	9 x 2 = 18	10 x 2 = 20
6 x 3 = 18	7 x 3 = 21	8 x 3 = 24	9 x 3 = 27	10 x 3 = 30
6 x 4 = 24	7 x 4 = 28	8 x 4 = 32	9 x 4 = 36	10 x 4 = 40
6 x 5 = 30	7 x 5 = 35	8 x 5 = 40	9 x 5 = 45	10 x 5 = 50
6 x 6 = 36	7 x 6 = 42	8 x 6 = 48	9 x 6 = 54	10 x 6 = 60
6 x 7 = 42	7 x 7 = 49	8 x 7 = 56	9 x 7 = 63	10 x 7 = 70
6 x 8 = 48	7 x 8 = 56	8 x 8 = 64	9 x 8 = 72	10 x 8 = 80
6 x 9 = 54	7 x 9 = 63	8 x 9 = 72	9 x 9 = 81	10 x 9 = 90
6 x 10 = 60	7 x 10 = 70	8 x 10 = 80	9 x 10 = 90	10 x 10 = 100

Practice 4: Multiplication by 0 and 1

a. $1 * 4 = 4$

b. $4 * 0 = 0$

c. $8 * 1 = 8$

d. $7 * 1 = 7$

e. $0 * 6 = 0$

f. $5 * 0 = 0$

g. $4 * 1 = 4$

h. $3 * 1 = 3$

i. $1 * 2 = 2$

j. $1 * 0 = 0$

k. $0 * 1 = 0$

l. $1 * 1 = 1$

m. $732 * 1 = 732$

n. $845 * 0 = 0$

p. $0 * 43 = 0$

q. $1 * 87 = 87$

r. $987 * 0 = 0$

s. $67 * 1 = 67$

t. $9 * 0 = 0$

u. $0 * 54 = 0$

v. $0 * z = z$

w. $y * 1 =$

x. any number $*$ 0 = 0

z. 1 $*$ any number = any number

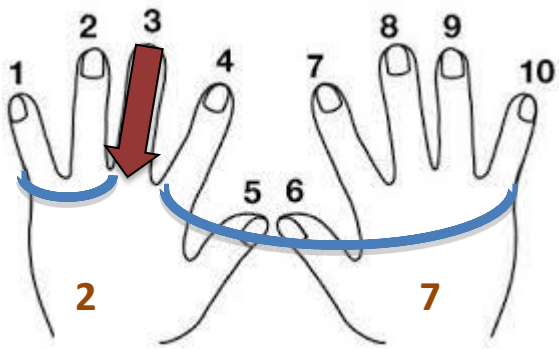
aa. $1 * 23 = 23$

bb. $0 * 7 = 0$

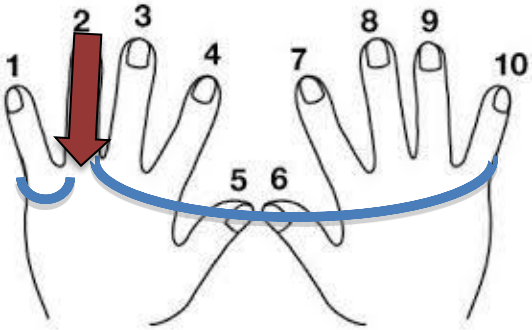
Practice 6: Multiplication by 3

factors and product	sum of digits	evenly divisible by 3
3 * 4 = 12	1 + 2 = 3	yes
3 * 8 = 24	2 + 4 = 6	yes
3 * 7 = 21	2 + 1 = 3	yes
9 * 3 = 27	2 + 7 = 9	yes
11 * 3 = 33	3 + 3 = 6	yes
11 * 6 = 66	6 + 6 = 12 (can do 1 + 2 = 3)	yes
4 * 6 = 24	2 + 4 = 6	yes
1 * 3 = 3	3	yes
7 * 5 = 35	3 + 5 = 8	no
7 * 6 = 42	4 + 2 = 6	yes
6 * 5 = 30	3 + 0 = 3	yes
2 * 12 = 24	2 + 4 = 6	yes
2 * 18 = 36	3 + 6 = 9	yes
2 * 15 = 30	3 + 0 = 3	yes
9 * 5 = 45	4 + 5 = 9	yes
9 * 10 = 90	9 + 0 = 9	yes
4 * 7 = 28	2 + 8 = 10	no
6 * 6 = 36	3 + 6 = 9	yes
10 * 4 = 40	4 + 0 = 4	no

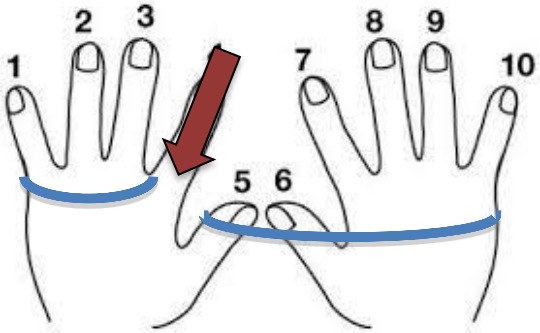
Practice 8: Multiplication by 9



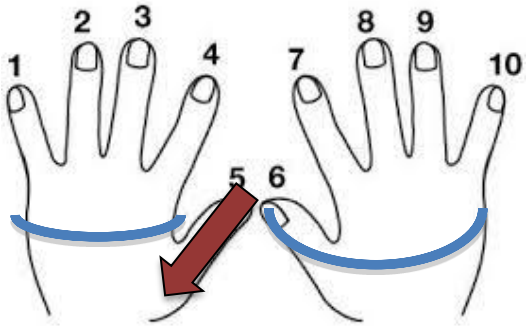
Show the following multiplication facts with your fingers and draw what you see.



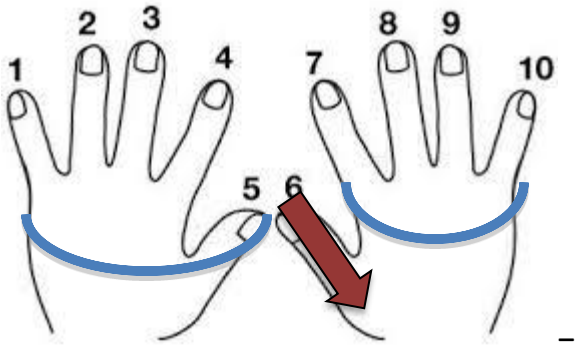
2 * 9 = 18



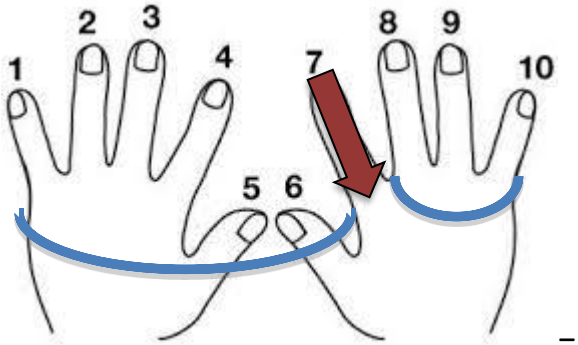
4 * 9 = 36



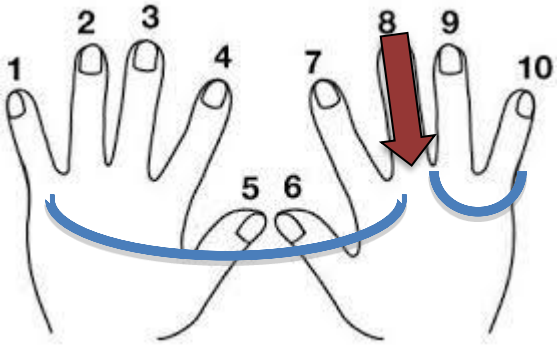
9 * 5 = 5 * 9 = 45



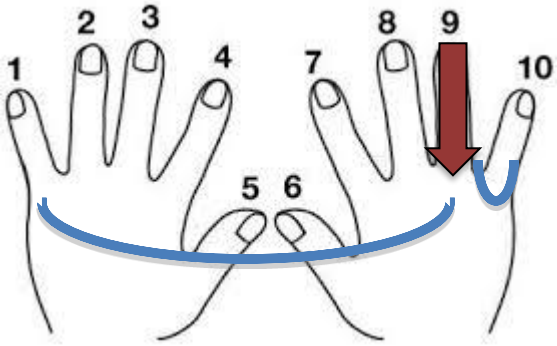
9 * 6 = 6 * 9 = 54



7 * 9 = 63



9 * 8 = 72



9 * 9 = 81

Practice 9: Multiplication by 10

Here are some multiplication facts with 10:

$1 * 10 = 10$

$2 * 10 = 20$

$3 * 10 = 30$

Do you see the pattern? All you have to do is to put a "0" on the one's place.

q. $4 * 10 = 40$

r. $5 * 10 = 50$

s. $6 * 10 = 60$

t. $7 * 10 = 70$

u. $8 * 10 = 80$

v. $9 * 10 = 90$

w. $10 * 10 = 100$

x. $72 * 10 = 720$

y. $70 * 10 = 700$

z. $114 * 10 = 1140$

aa. $86 * 10 = 860$

bb. $90 * 10 = 900$

cc. $999 * 10 = 9990$

dd. $1000 * 10 = 10{,}000$

ee. $40 * 10 = 400$

ff. $41 * 10 = 410$

Practice 10: Multiplication using $a * b = b * a$

Can you use $a * b = b * a$ to remember multiplication facts you have trouble with?

For example, if you have trouble with $8 * 2$, you may be fine with $2 * 8$. So you can switch $8 * 2$ to $2 * 8$.

C. List multiplication facts that you can use $a * b = b * a$ to help you remember:

D. Find the products. You may want to use $a * b = b * a$ to find them. It's up to you!

a. $8 * 2 = 16$

b. $9 * 3 = 27$

c. $7 * 4 = 28$

d. $6 * 3 = 18$

e. $8 * 4 = 32$

f. $7 * 5 = 35$

g. $9 * 2 = 18$

h. $3 * 7 = 21$

i. $9 * 4 = 36$

j. $9 * 6 = 54$

k. $6 * 2 = 12$

l. $5 * 8 = 40$

m. $5 * 3 = 15$

n. $9 * 7 = 63$

o. $8 * 5 = 40$

p. $7 * 3 = 21$

q. $5 * 4 = 20$

r. $8 * 7 = 56$

s. $8 * 6 = 48$

t. $6 * 7 = 42$

u. $7 * 8 = 56$

v. $9 * 5 = 45$

w. $6 * 5 = 30$

x. $4 * 3 = 12$

y. $8 * 3 = 24$

z. $7 * 6 = 42$

Practice 11: Mixed multiplication facts practice

Fact	How do you know you are right?
a. $2 * 7 = 14$	I use 7 + 7 = 14 to check. I have memorized the facts.
b. $3 * 7 = 21$	I memorized it, and I check 2 + 1 = 3, evenly divisible by 3.
c. $5 * 4 = 20$	I know $2 * 5 = 10$, so $4 * 5 = 2 * (2 * 5) = 2 * 10 = 20$.
d. $2 * 10 = 20$	I put a 0 on the end = 20.
e. $9 * 5 = 45$	I used my fingers = 45. I check that the sum of digits is evenly divisible by 3. 4+ 5 = 9.
f. $7 * 10 = 70$	I put a 0 on the end of 7 = 70. So $7 * 10 = 70$

Practice 12: Multiplication is more efficient than addition

Practice: Compare multiplication with repeated addition : $8 * 7 = ?$

Either count the steps for each part, or time yourself.

A. Do you remember the multiplication fact $8 * 7$?

Let's hope so: $8 * 7 =$ 56

Number of steps 1 steps I memorized

Or time to solve _____seconds

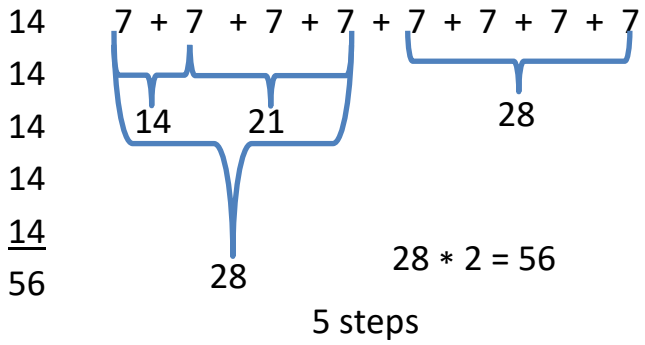
B. Repeated addition of 8 "7"s

7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 =

14 14 14 14

Number of steps 5 steps

Or time to solve _____seconds

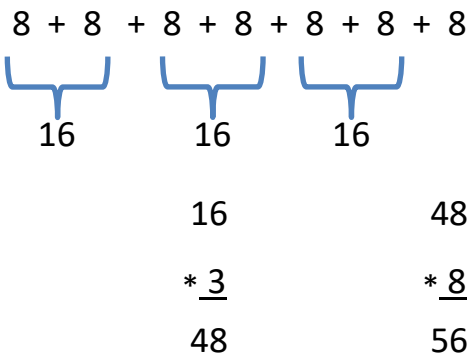


C. Repeated addition of 7 "8"s (using $a * b = b * a$)

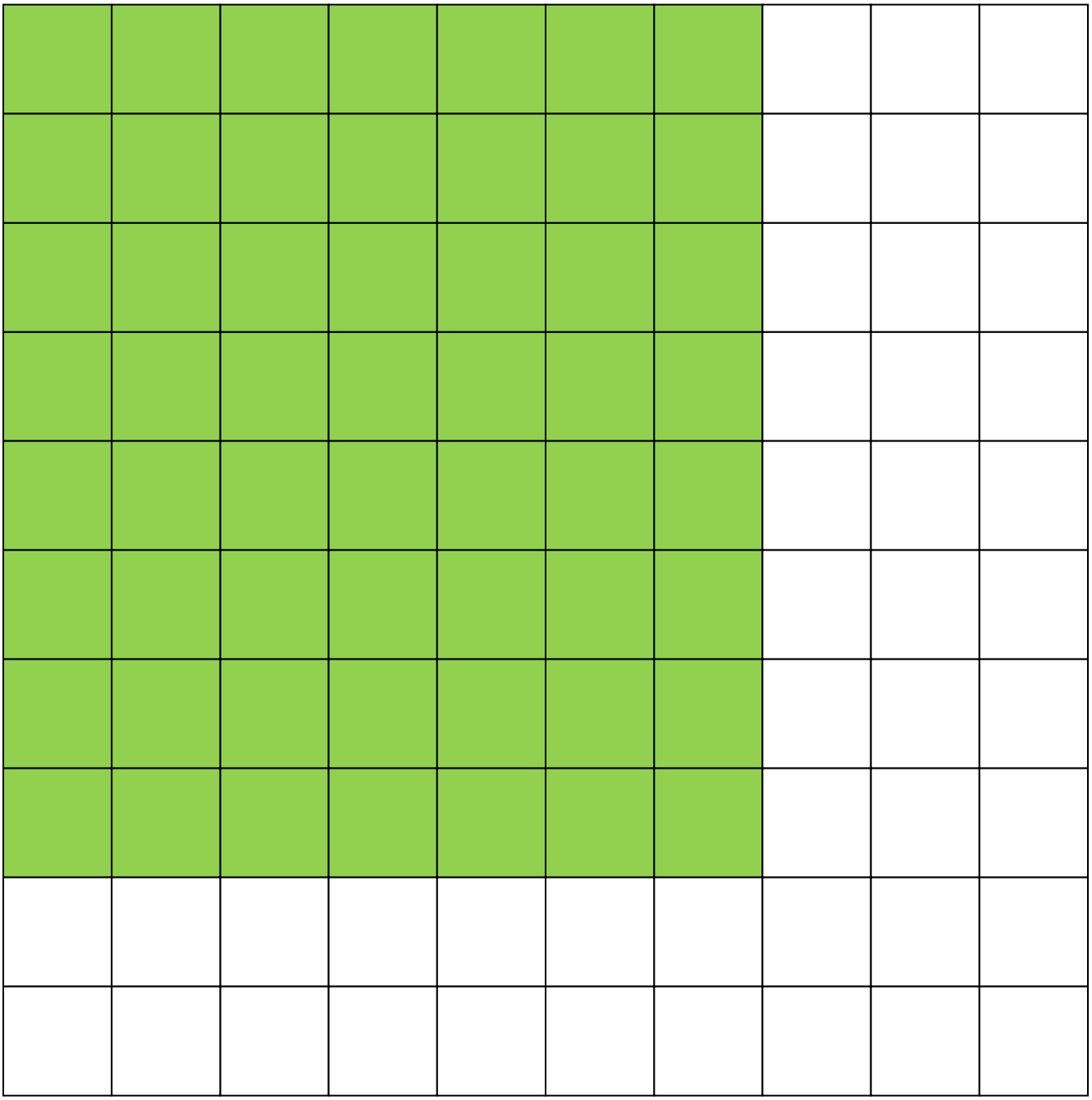
8 + 8 + 8 + 8 + 8 + 8 + 8 =

Number of steps 5 steps

Or time to solve _____seconds



Check your answer:



Practice 13: Multiplication Symbols

Write down all the symbols you can think of to show the following multiplication statements.

Remember that the symbols are read from left to right. So a groups of b could be written a • b.

1. 3 groups of 2 3 •2, 3∗ 2, (3) (2), (3) 2, 3 (2)

2. 4 groups of 5 4 • 5, 4 ∗ 5, (4) (5), 4 (5), (4) 5

3. 3 groups of 7 3 • 7, 3 ∗ 7, (3) (7), 3 (7), (3) 7

4. 3 groups of x 3 • x, 3 ∗ x, 3x, 3 (x), (3) (x), (3) x

5. 5 groups of 2 5 • 2, 5 ∗ 2, (5) (2), 5 (2), (5) 2

6. w groups of 2 w • 2, w ∗ 2, (w) (2), w (2), (w) 2

   More traditional, use a • b = b • a, so w • 2 = 2 • w = 2w

7. x groups of y x • y, x ∗ y, (x) (y), x (y), (x) y

Practice 14: More symbols for Multiplication

Find the products.

1. $1 \bullet 4 = 4$
2. $(5)\,(8) = 40$
3. $4 * 5 = 20$
4. $7 * 6 = 42$
5. $3\,(7) = 21$
6. $(2)\,(8) = 16$
7. $7 * 8 = 56$
8. $9 * 3 = 27$
9. $5 \bullet 3 = 15$
10. $7 \bullet 2 = 14$
11. $4 * 9 = 36$
12. $7\,(4) = 28$
13. $(6)\,(6) = 36$
14. $7 \bullet 8 = 56$
15. $0 * 7 = 0$
16. $3 \bullet 4 = 12$
17. $(4)\,(9) = 36$
18. $2 * 9 = 18$
19. $9 \bullet 0 = 0$
20. $8 * 6 = 48$
21. $6\,(2) = 12$
22. $(7)\,(5) = 35$
23. $3 \bullet 9 = 27$
24. $2 * 3 = 6$
25. $4 \bullet 8 = 32$
26. $3 * 7 = 21$
27. $4\,(4) = 16$
28. $(5)\,(6) = 30$
29. $3 \bullet 8 = 24$
30. $7 * 2 = 14$

Practice 15: Two Digit by Two Digit

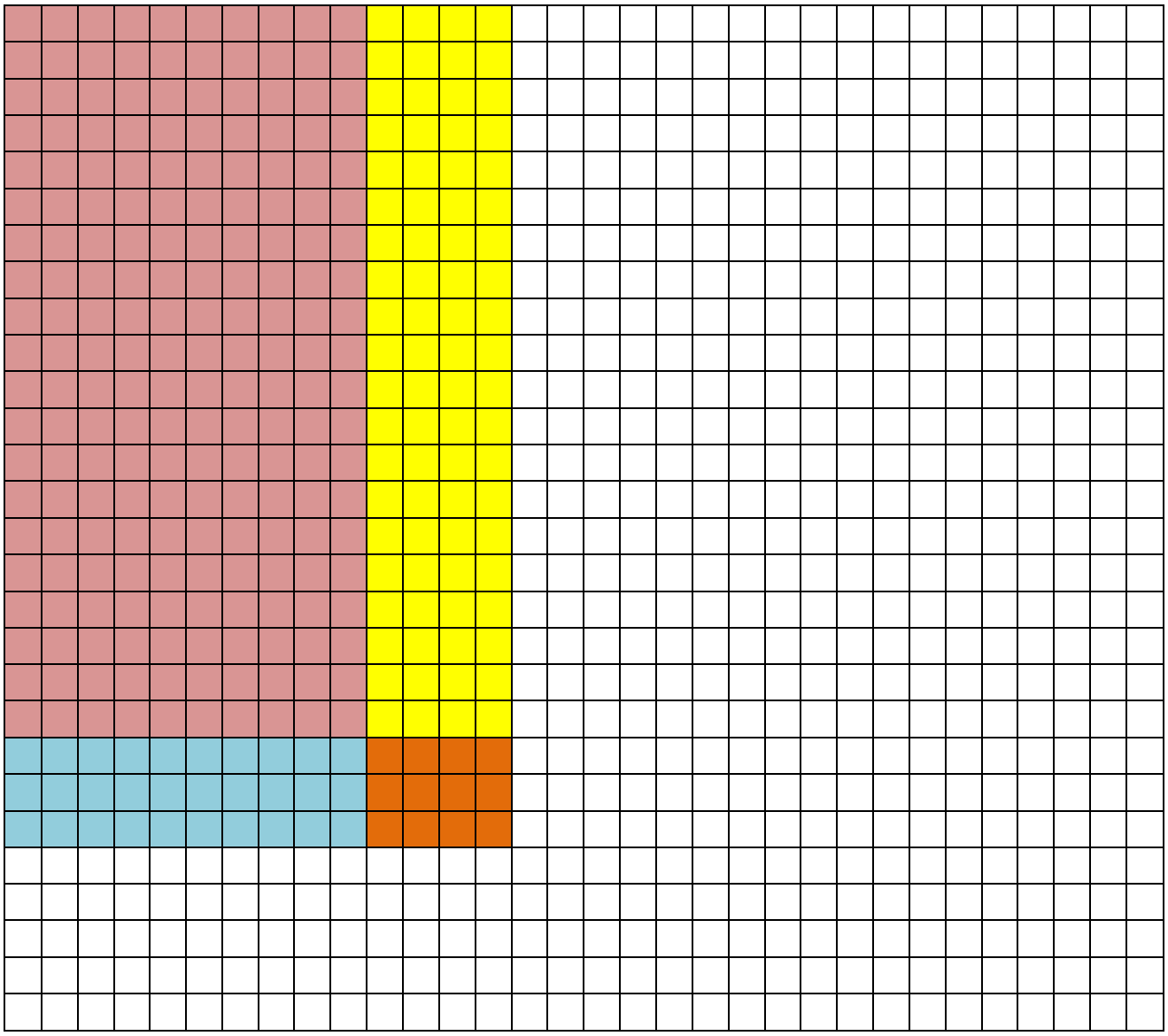
Find the product by using the French method and also with the graph paper.

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline \end{array}$$

200	1	tens	times	2	tens
80	2	tens	times	4	ones
30	3	ones	times	1	tens
12	4	ones	times	3	ones,

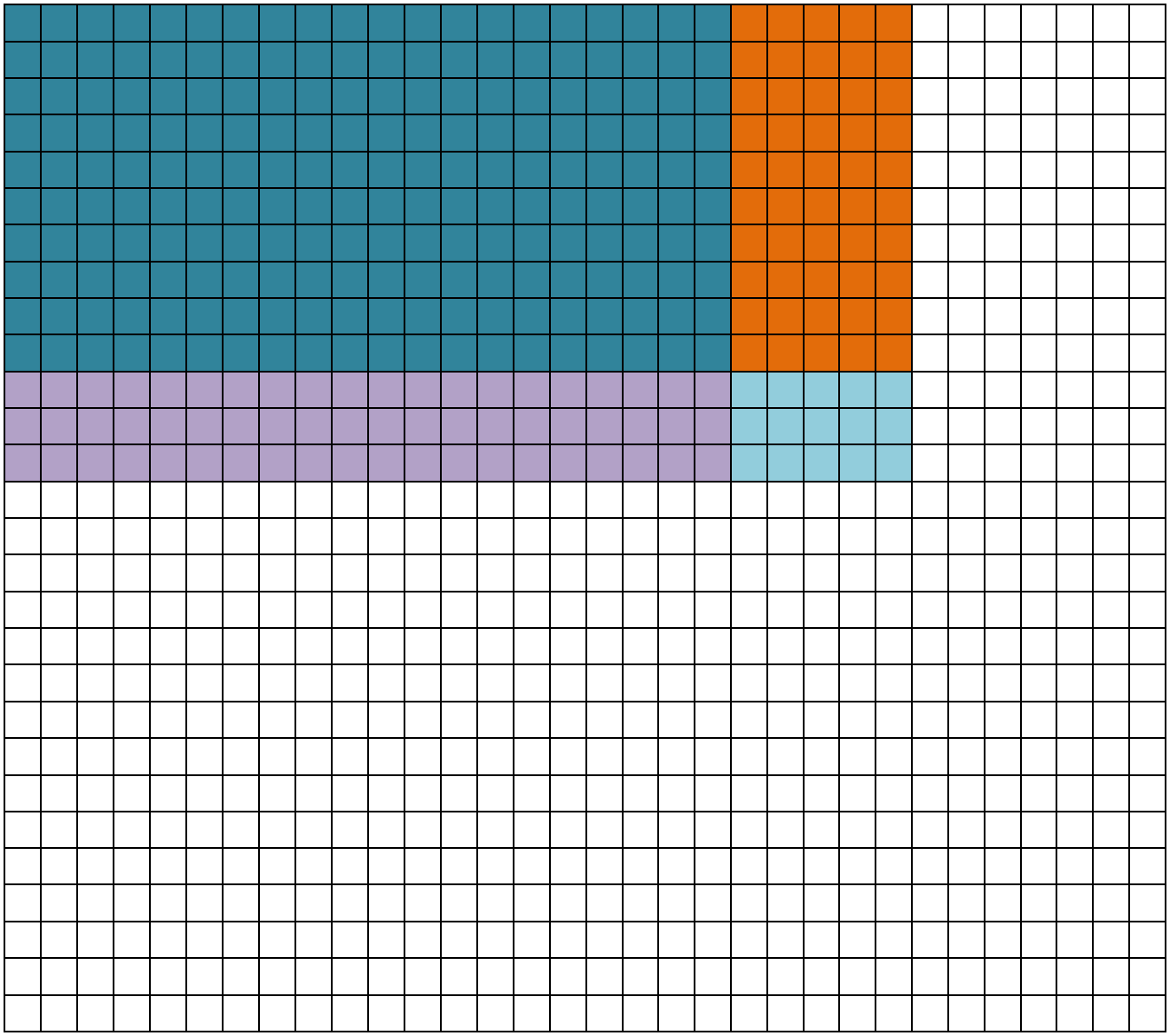
322 The product

$$\begin{array}{r} 23 \\ 14 \\ \hline 92 \\ 230 \\ \hline 322 \end{array}$$



25					
x 13					
200	1	tens	times	2	tens
50	1	tens	times	5	ones
60	3	ones	times	2	tens
15	3	ones	times	5	ones,
325	The product				

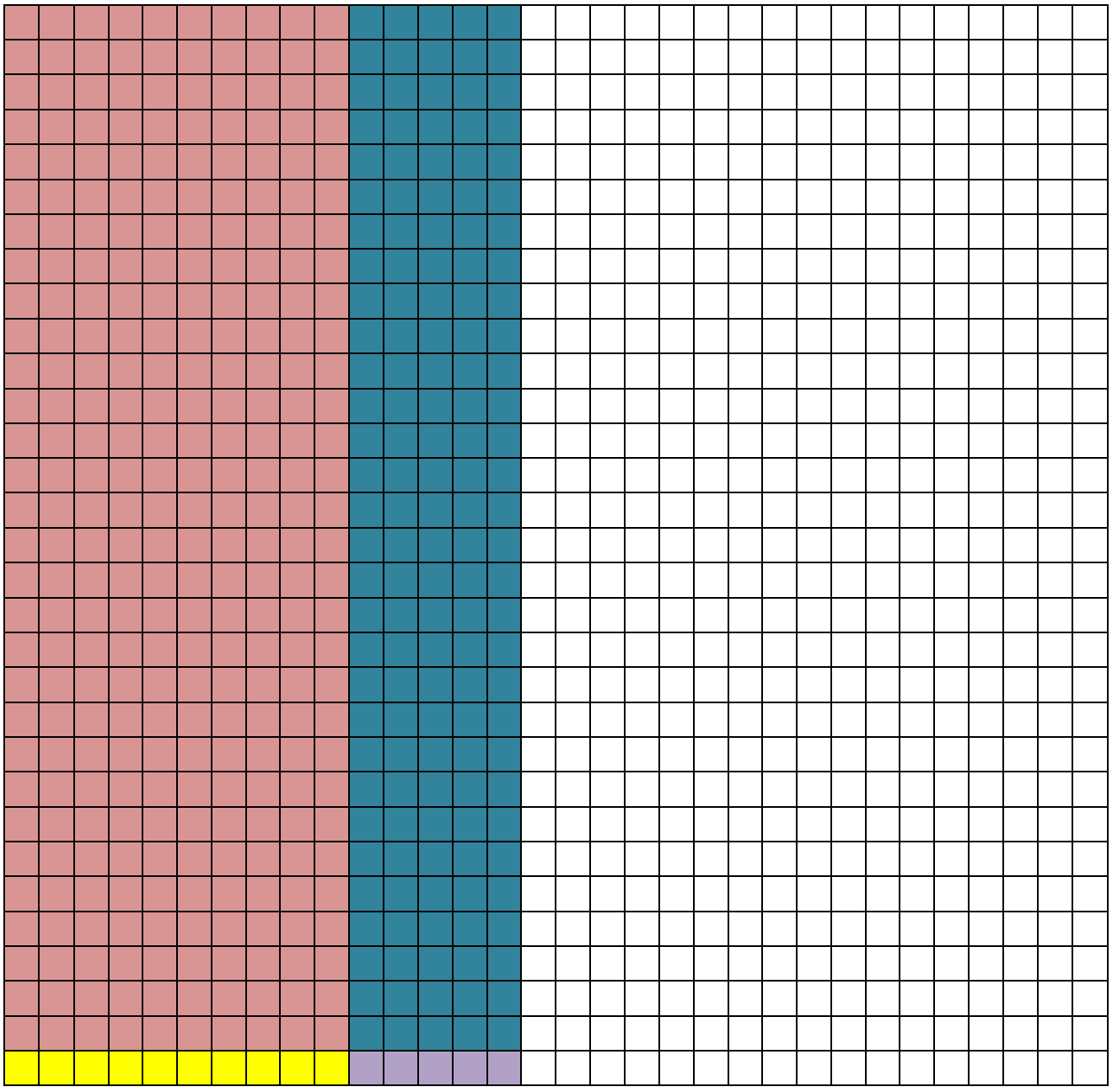
25
13
75
250
325



	31				
x	15				
300	3	tens	times	1	tens
150	3	tens	times	5	ones
10	1	ones	times	1	tens
5	5	ones	times	1	ones,

465 The product

15
31
15
450
465

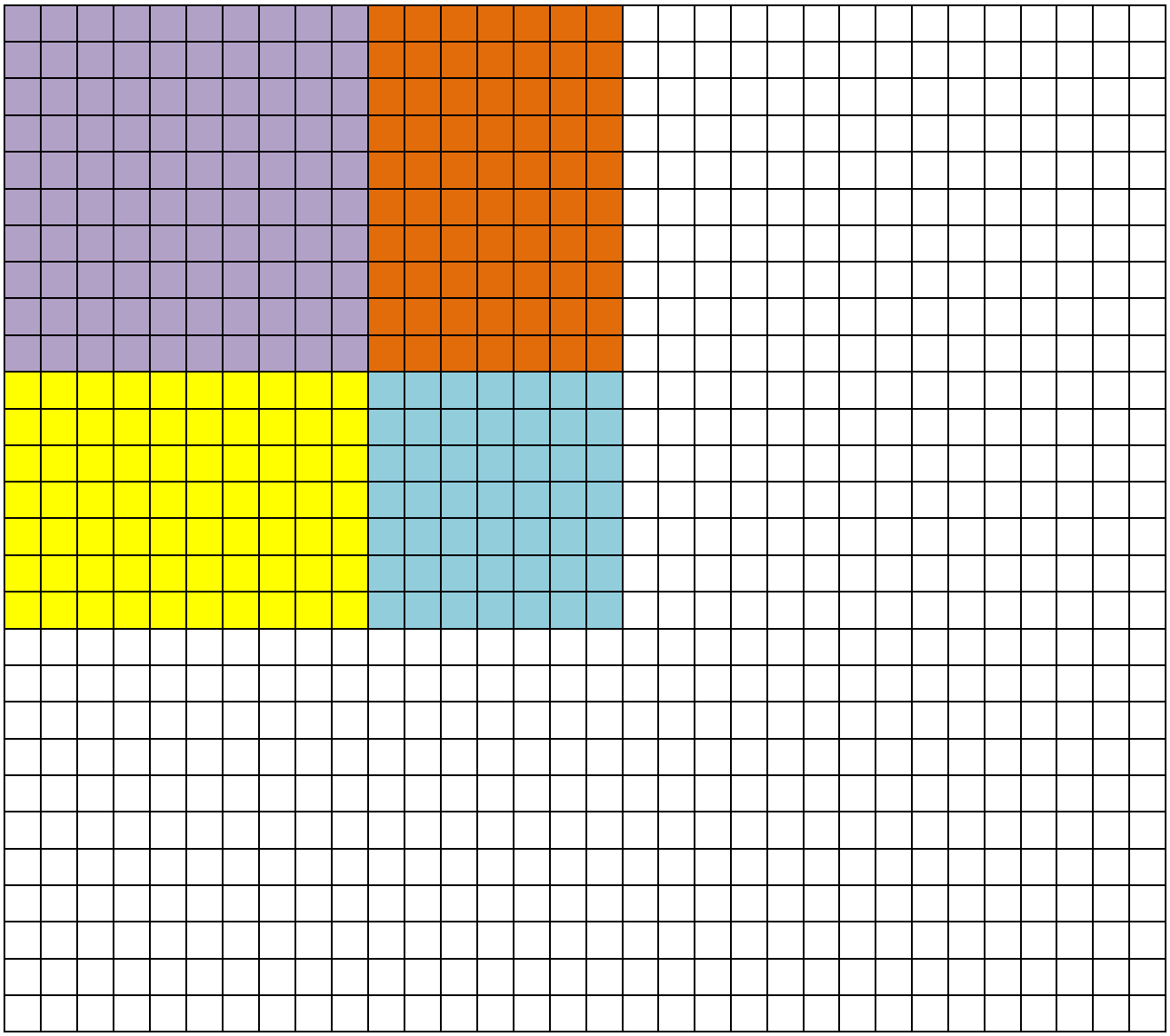


18
x 17

100	1	tens	times	1	tens
80	8	tens	times	8	ones
70	1	ones	times	1	tens
56	7	ones	times	8	ones,

306 The product

18
17
126
180
306



Practice 16: Multiplication with units/labels

For the multiplications shown below, supply what there are 12 of.
Multiplication A is completed as an example.

A. 3 ∗ 4 blocks totals 12 blocks

B. 3 ∗ 4 apples totals 12 apples

C. 3 apples ∗ 4 totals 12 apples

D. 3 ∗ 4 feet totals 12 feet

E. 3 ∗ 4 fifths totals 12 fifths

F. 3 fourths ∗ 4 totals 12 fourths

G. 3 feet ∗ 4 feet totals 12 feet * feet = 12 square feet

H. 3 fifths ∗ 4 fifths totals 12 fifths * fifths = 12 twenty fifths

I. 3 apples ∗ 4 apples totals doesn't make sense

J. 3 apples ∗ 4 oranges totals doesn't make sense

K. 3 fifths ∗ 4 feet totals 12 fifths feet = 144 square inches

L. 3 feet per second ∗ 4 seconds totals 12 feet/sec * sec = 12 feet

M. 3 feet per second ∗ 4 feet totals 12 square feet per second

Practice 17: Multiplication by tens and scientific notation

You will need a calculator to check your answers

Use a calculator to do these multiplications:

$7 * 100 =$	700
$7 * 1{,}000 =$	7,000
$7 * 10{,}000 =$	70,000
$7 * 100{,}000 =$	700,000
$5 * 100 =$	500
$5 * 1{,}000 =$	5,000
$5 * 10{,}000 =$	50,000
$5 * 100{,}000 =$	500,000
$70 * 200 =$	14,000
$700 * 200 =$	140,000
$7{,}000 * 200 =$	1,400,000
$7{,}000 * 2{,}000 =$	14,000,000

Example:

There is a way to keep track of all the zeros using “powers of ten.” This is a symbolic way to keep easy track of all the zeros.

Help for Helpers

I know how much teachers and parents want to help their students be successful at math. It can be upsetting to us as adults to see a student for whom we care being upset. However, the very very best way to help your student is to offer encouragement, such as "I know you can do this. I believe in you." And then leave the student alone to do the work.

As a metaphor, if you yourself want to become physically fit and choose to run a mile, having someone drive you in a car isn't going to really help you long term. Yes, you will cover the distance. But there is no substitute for the physical exertion, the sweating and huffing and puffing. Learning to be successful in math requires mental exertion, self-soothing during the frustrating times, and mental stamina.

The time of being a student is largely to prepare for adulthood. As an adult needing math in real life or on the job, there is no great answer book that falls from the sky. We don't generally want to ask our boss or friend: "Am I right? Am I right?" As an adult, we have to know the answer is right ourselves. The time of being a student is the appropriate time to learn these skills. So, difficult as it may be for you, and it can be very difficult, I respectfully urge you to do nothing except offer encouraging words. These materials are carefully scaffolded and I guarantee you that your student is capable of doing the work himself or herself. The right answer is only half the goal—your student needs to know the answer is right independently.

My heartfelt wishes to you, the parent, teacher, or important grownup in your student's life. You will gain confidence in your students as you watch them be successful on their own.